Changing the Effective Mass to Control Resonance Problems

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This tutorial article tells how to make a calculated change in a resonance frequency of a structure by changing its effective mass. The article includes two documented case histories that illustrate the technique.

Resonant frequency problems are often encountered in mechanical systems. When this occurs, the level of vibration is generally quite high, and this in turn often causes premature failure of machine components. Often, attempts are made to address the problem by reducing the forcing function. Such courses of action include dynamic balancing of rotating elements and aligning coupled components. It is very important to maintain good balance and alignment. However, in the case of a resonant condition, the primary problem is generally not the magnitude of the forcing function. The problem is that a forcing function, which may be of modest magnitude, matches a system resonance or natural frequency. In such a case, the amplification factor of the vibration can be many fold. The amplification factor is the ratio of the peak dynamic displacement imparted on a system by an oscillating force with a given peak magnitude compared with the displacement imparted by a static force of the same magnitude. Figure 1 is a plot of the amplification factor versus resonant frequency ratio for several systems each with a different level of damping. The resonant frequency ratio is the ratio of the forcing frequency to the resonant frequency. A value of one indicates that the system is at resonance.

The simple equation for resonant frequency is:

$$\omega_n = \sqrt{K / M}$$

Where:

ω_n = resonant frequency in radians per second
K = effective spring constant
M = effective mass.

As stated above, this is a simplified equation. It loses accuracy with very large displacements, and it does not account for the effect of damping. However, for most resonant problems involving machinery where the displacement of even severe vibration is still measured in mils, and damping is generally quite low, this equation works fine.

The subject of this article is how to implement a calculated change in the resonant frequency by changing the effective mass. We have the equation for resonance. And, it is assumed that a resonance at the fugitive frequency has been confirmed. However, there are still two unknown parameters and only one equation. In simple systems, the effective mass can be determined directly by weighing the system. In small machines, where an entire pump or motor is the vibrating mass, an often useful approximation is 90% of the total mass. However, in complicated machinery where the system might be a cantilevered bearing pedestal on the side of a large pump or a bearing flange around the input shaft of a gearbox, it is generally not practical to determine the effective mass directly.

This shortcoming can, however, generally be overcome. The reason for this is that it is usually possible to determine the system spring constant. The simplest way to do this is to push on the system in the orientation of interest until a measurable deflection is observed. One simple example would be to pull on the system with a 'come-along' and a crane scale, and measure the resulting deflection with a dial indicator.

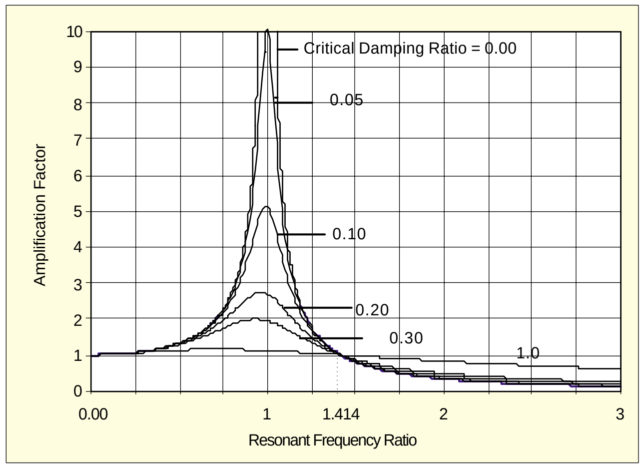

Figure 1. Amplification factor versus resonant frequency ratio for several critical damping factors.

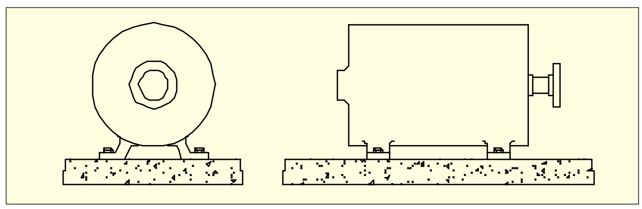

Figure 2. Electric motor that had a resonance at 2×running speed ~7100 CPM. The entire motor vibrates as a single mass.

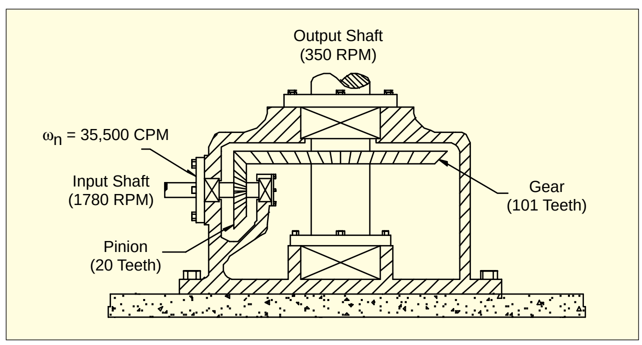

Figure 3. Gearbox that had structural resonances at approximately 36,000 and 44,000 CPM. The resonance was localized to the area of the input shaft bearing housing. The resonance was almost exclusively in the axial orientation relative to the input shaft.

Motor Resonance

It has been determined that a two pole electric motor shown in Figure 2 has a resonance at 7100 CPM, twice running speed, in the horizontal orientation. It was determined that a force of 1100 lbs will cause a displacement of 1 mil. The motor weight is 860 lbs. Determine the effective mass of the system.

Solution:

ω_n = (7100 cycles/min)(1 min/60 sec)(2π radians/cycle) = 744 radians/sec

K = 1100 lbs/[(1 mil)(1 in./1000 mils)] = 1,100,000 lbs/in.

$M = K/\omega_n^2$ = (1,100,000 lbs/in.)/(744/sec)^2 = 1.99 lb sec^2/in.

The weight of this mass can be determined by multiplying the mass by gravitational acceleration, 386 in./sec^2 = (1.99 lb sec^2/in.)(386 in./sec^2) = 767 lbs. If the approximation that the effective mass is 90% of the total mass had been used, the determined weight = (0.9)(860 lbs) = 774 lbs; which for all practical matters is identical to that determined from the spring constant.

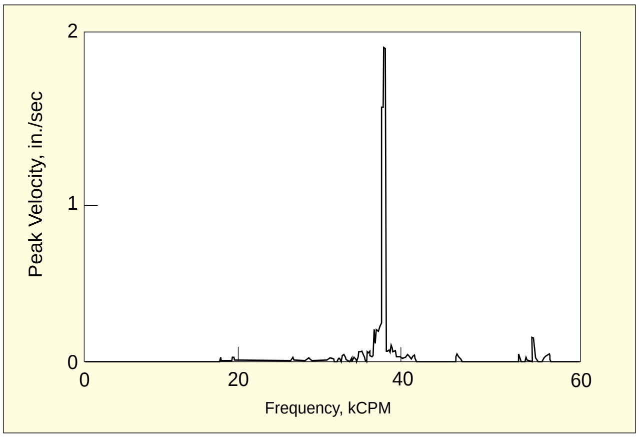

Figure 4. Vibration spectrum of the input bearing in the axial orientation. Note the very high level of vibration at approximately 36,000 CPM. This frequency corresponds to the gear meshing frequency.

Gearbox Resonance

Things do get considerably more complicated when only a part of a machine is involved in a resonant system. For example, a gearbox developed a resonance problem at the gear meshing frequency. The input shaft speed is 1780 RPM and there are 20 teeth on the input pinion as shown by the schematic in Figure 3.

In the area of the input shaft bearing, there is a very high level of vibration at the gear meshing frequencies shown in Figure 4. The existence of a resonance at approximately the gear meshing frequency was confirmed by a bump test as shown in Figure 5. In this case, only a part of the machine is involved in the resonance, and the system is much too complicated to try to derive any sort of an estimate of the effective mass. In such machines, it is possible to derive a value for the spring constant by measuring the deflection imparted by a known force. From the spring constant, the effective mass can be determined.

In this case, however, it was not feasible to impart a static force on the system. In order to overcome this shortcoming, data from the bump test were employed. This was done in two steps. First, system compliance, mils/lb, would have to be determined from existing data. The second step would involve determining the system amplification factor, the ratio of the dynamic to static displacement. From these values, the system spring constant and subsequently the effective mass can be determined.

It is possible to perform a bump test with units of compliance directly. In this case, however, data were recorded in units of mobility, in./sec/lb (IPS/lb). Units of velocity in IPS can be converted to units of displacement by the following equation:

$$D = v / \pi f = 0.3183\ v / f$$

where:

v = velocity, IPS.
= 0.00182 IPS/lb (from the upper trace in Figure 5)
f = frequency, Hz
D = displacement, in. peak to peak
= (0.3183)(0.00182)/(36,011/60)
= 9.65×10^{-7} in./lb
= 9.65×10^{-4} mils/lb

This equation is generally used to convert vibration from units of velocity to displacement. However, there is no reason that it cannot be used to convert a transfer function from units of velocity per unit of applied force to units of displacement per unit of applied force.

Thus, the dynamic compliance will be 9.65×10^{-4} mils/lb or 9.65×10^{-7} in./lb. By inverting this value, the dynamic stiffness K_d can be determined:

$$K_d = \frac{1}{9.65 \times 10^{-7}\ \text{in./lb}} = 1.04 \times 10^6\ \text{lb/in.}$$

The next step is to determine the amplification factor using the following equation:

$$\frac{XK}{F_0} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}}$$

where:

X = dynamic deflection
K = spring constant
F_0 = peak dynamic force
ω = forcing frequency
ω_n = natural or resonant frequency
ξ = critical damping ratio
XK/F_0 = amplification factor.

Everything on the right side of this equation is known except the critical damping ratio. The critical damping ratio is the ratio of actual system damping to that of critical damping. Critical damping is the minimal amount of damping required to prevent vibration when a system is displaced and released.

The critical damping ratio can be determined from the transfer function. Two methods will be employed to determine the critical damping ratio. The first is the half power method. In determining the damping of a system, the parameter Q is often employed. This parameter is a measure of the sharpness of the resonance and is defined as:

$$Q = \frac{1}{2\xi} \text{ or } \xi = \frac{1}{2Q}$$

In using the half power method, the total transfer function is employed. The top trace in Figure 5 is the total transfer function.

The total transfer function will generate a roughly symmetric peak the center of which will be the resonant frequency. The steepness of the peak is a measure of the amount of damping in the system. In the half power method, the frequency of the resonant value is compared with the frequency at which the half power values on either side of the resonance occurs. The half power value is 0.707 multiplied by the peak value. The Q value is:

$$Q = \frac{\omega_n}{\omega_2 - \omega_1}$$

where:

ω_n = resonant frequency, in this case 36,011 CPM
ω_2 = frequency at which the upper half power amplitude occurs, approximately 37,400 CPM
ω_1 = frequency at which the lower half power amplitude occurs, approximately 33,800 CPM

Thus:

$$Q = \frac{36{,}011}{37{,}400 - 33{,}800} = 10.1$$

and

$$\xi = \frac{1}{2Q} = 0.049$$

The second method employs the sloped reactive portion of the transfer function. For transfer functions in which amplitude is measured in units of velocity, the sloped reactive component is the imaginary portion. For transfer functions that measure amplitude in units of either displacement or acceleration, the sloped reactive component would be the real portion. The lower trace in Figure 5 is the imaginary portion of the transfer function. From such a function, the approximate value of the Q parameter is:

$$Q = \frac{\left(\dfrac{\omega_a}{\omega_b}\right)^2 + 1}{\left(\dfrac{\omega_a}{\omega_b}\right)^2 - 1}$$

where:

ω_a = the frequency value above resonance at the top of the slope, approximately 35,000 CPM in this case
ω_b = frequency value below resonance at the bottom of the slope, approximately 37,100 CPM

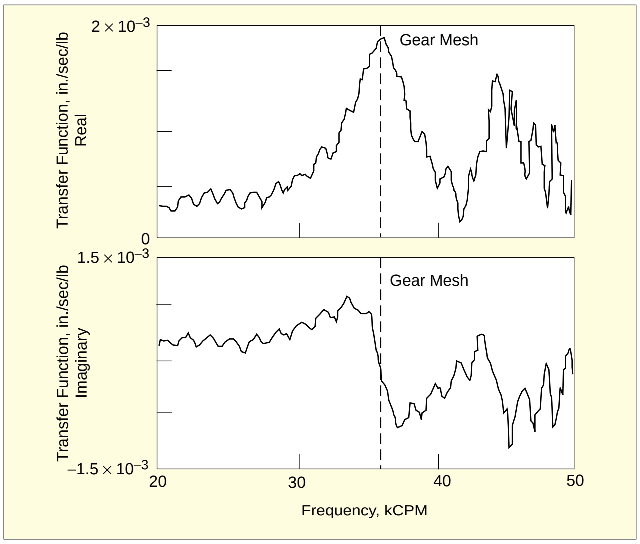

Figure 5. Bump test of the gearbox in the area of the input shaft bearing cap in the axial orientation. The top trace is the total transfer function, and the lower trace is the imaginary portion. Note the two resonances at approximately 36,000 and 44,000 CPM. Also, note that there appears to be an anti-resonance at approximately 41,500 CPM. In the top trace, the resonance around gear mesh, lower half power and higher half power frequencies are 36,011, 33,800, and 37,400 CPM, respectively. In the lower trace, the top and bottom frequencies of the slope around gear meshing frequency are 35,000 and 37,100 CPM respectively.

Thus,

$$Q = \frac{\left(\frac{37{,}100}{35{,}000}\right)^2 + 1}{\left(\frac{37{,}100}{35{,}000}\right)^2 - 1} = 17.1$$

and:

$$\xi = \frac{1}{2(17.1)} = 0.029$$

The two methods gave somewhat different values; however, it is often difficult to obtain a precise value for damping. In order to reduce the magnitude of error that could result from relying on a single test, the two derived values will be averaged:

$$\xi_{\text{Avg}} = 0.039$$

Now that a value has been obtained for the critical damping ratio ξ the dynamic amplification factor can be determined:

$$\frac{XK}{F_0} = \frac{1}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left[2\xi(\omega/\omega_n)\right]^2}} = \frac{1}{\left[1-1^2\right] + \left[2\times 0.039\times 1\right]^2} = 12.8$$

From the dynamic amplification factor and the dynamic spring constant, the static spring constant can be determined:

$$K = (12.8)(1.04\times 10^6) = 1.33\times 10^7 \text{ lb/in.}$$

By rearranging terms in the resonant frequency equation, the effective mass can be determined:

$$M = K/\omega_n^2$$

where ω_n is the resonant frequency:

$$\omega_n = 36{,}000\frac{\text{cycles}}{\text{min}}\times 2\pi\frac{\text{radians}}{\text{cycle}}\times\frac{1\,\text{min}}{60\,\text{sec}} = 3770 \text{ radians/sec}$$

and:

$$M = \frac{1.33\times 10^7\,\text{lb/in.}}{(3770/\text{sec})^2} = 0.933\frac{\text{lb}-\text{sec}^2}{\text{in.}}$$

This mass can be converted to a weight by multiplying by gravitational acceleration, 386 in./sec^2:

$$\text{Weight} = 0.933\frac{\text{lb}-\text{sec}^2}{\text{in.}}\times 386\frac{\text{in.}}{\text{sec}^2} = 360 \text{ lb}$$

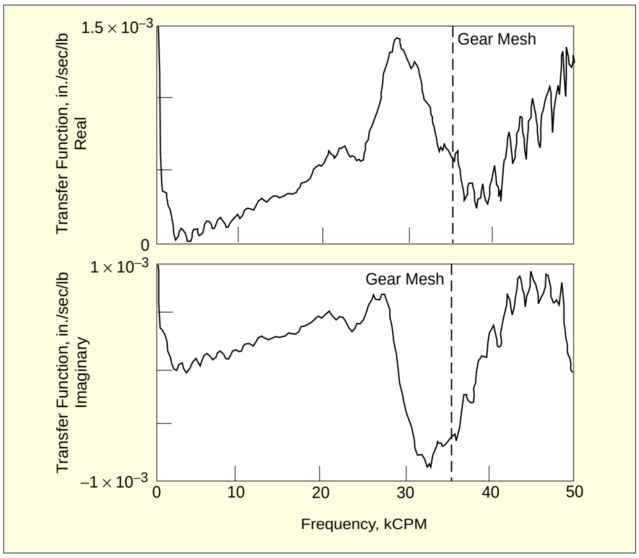

Figure 6. Bump test after an inertial mass had been added to the input flange. Note that the resonance has been shifted downward to approximately 29,000 CPM and that at the gear meshing frequency, approximately 36,000 CPM, the amplification factor is minimal.

This is a good example of how a relatively small part of the total system can be involved in the resonance. In this case, the gearbox weighs approximately 8000 lbs, but the effective mass of the resonant system is only 360 lbs.

Now, all of the parameters of the resonant system are known. So, what do we want to do about it? If we refer back to Figure 5, the frequency range from approximately 38,000 to 42,000 CPM has a relatively small amplification factor, and there appears to be an anti-resonance at a frequency of approximately 41,500 CPM. If the system can be retuned so that this curve is shifted to the left such that the gear meshing frequency falls into the trough between 38,000 and 42,000 CPM, the level of vibration should go down significantly. Care must be taken not to lower the resonance too much, or we could move the resonance that currently resides at 44,000 down into the range of the gear meshing frequency.

We will try to move the anti-resonance, currently at 41,500 CPM, down to the gear meshing frequency, 36,000 CPM. In order to do this, the resonance will be lowered approximately 41,500 – 36,000 or 5500 CPM. This can be done by increasing the effective mass of the system. The new resonant frequency for which the system will be tuned will be:

$$36{,}000 - 5500 = 30{,}500 \text{ CPM}$$

or 3180 radians/sec. Going back to the resonant frequency equation:

$$M = \frac{K}{\omega_n^2} = \frac{1.33\times 10^7\,\text{lb/in.}}{(3180/\text{sec})^2} = 1.32\frac{\text{lb}-\text{sec}^2}{\text{in.}} \quad \text{or} \quad \text{Weight} = 508 \text{ lb}$$

This is the weight of the new effective mass of the system. In order to implement a change in the effective mass, a concentrated inertial mass the weight of which was equal to the new, target effective mass minus the weight of the current effective mass should be bolted to the input shaft flange:

$$\text{Concentrated Weight} = 508 - 360 = 148 \text{ lb}$$

The inertial mass was fabricated and installed, and another bump test was performed. The transfer function indicated that the resonance had indeed been shifted to approximately the target frequency and out of the frequency range where the gears would excite a resonant frequency as shown in Figure 6.

A vibration survey taken immediately after the inertial mass had been installed revealed that the level of vibration had been reduced by approximately a factor of four. Another survey taken after the gearbox had been in service for approximately one week showed an even more dramatic reduction in the level of vibration as shown in Figure7.

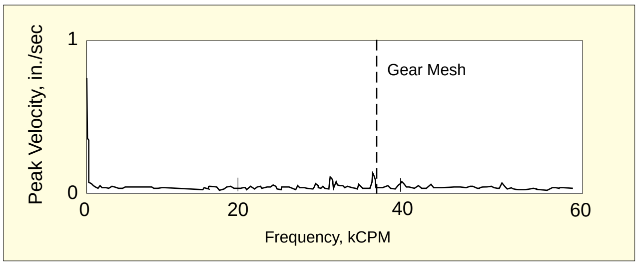

Figure 7. Vibration spectrum of the gearbox after installing the inertial mass and running for approximately one week. Compare the amplitude of vibration at the fugitive frequency in this spectra with that in Figure 4. The amplitude scales in both Figures 4 and 7 are the same, and both surveys were conducted under similar operating conditions.

In this case history, it was possible to realize a very significant reduction in the level of vibration with a relatively simple fix. The cost of fabricating and installing the inertial mass was minuscule compared to the cost of the gearbox.

Conclusions

Because the resonant frequency is a function of the square root of both the effective mass and stiffness, it is often not practical to cause a significant change in the resonance by changing either of these parameters when the entire structure is part of the vibration system. Such was the case with the motor mentioned earlier. In such a case, it would be necessary to add a much larger mass relative to the mass of the primary system. In the case of the motor, it was not practical to change the resonance by adding mass to the system, and other methods were employed to control the vibration. However, in the case of the gearbox, the effective mass of the vibrating system was a relatively small part of the entire system, and adding an inertial mass was very effective in controlling the vibration. SV