Recent System Developments for Multi-Actuator Vibration Control

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This article discusses recent developments in multi-input multi-output (MIMO) vibration control systems and their applications to vibration testing. Highlights of some of the issues discussed are: 1) the proper control of singularities that exist in the system-under-test's frequency response matrix (FRM); 2) the benefits provided by a patented adaptive-control method; and 3) the advantages of characterizing the system-under-test with the simultaneous excitation from multiple actuators. These advanced methods provide the control-system user a better understanding of the system-under-test's dynamic characteristics and more realism during the test process. To illustrate this point, test results – those obtained during MIMO random, swept-sine and transient waveform control – will be presented. The test results will also be used to illustrate how the discussed advancements solve many of the MIMO-control problems associated with multi-actuator testing.

A vast majority of vibration testing today is performed with a single vibration excitation system. Here, the system-under-test by the controller typically consists of the shaker, amplifier, fixture, test article, transducers and cables. However, some conditions exist where a single shaker is not appropriate to achieve the results desired from the test. Examples of this include:

- Design and Qualification testing of products such as long missiles where a single attachment point or point of force input might damage the test article in order to achieve desirable test levels throughout the structure.
- Testing of structures where, because of the shape of the test article, use of a single exciter would precipitate the design and fabrication of one or more prohibitively expensive test fixtures.
- Multi-axis tests which require simultaneous force inputs in several axes in order to meet the test criteria. These could include earthquake simulation and many types of automobile road simulations.
- Testing of large structures which require more force to be input than is available from a single shaker.
- Sophisticated testing in which several different waveforms need to be simulated at different locations on a large structure at the same time.
- A subset of this would include System Identification or Characterization of a large structure which requires the simultaneous excitation by multiple uncorrelated inputs in order to measure a set of poly-reference frequency response functions.
- A variation of the above is the System Identification of structures which require simultaneous excitation by multiple sinewave inputs, having a pre-specified phase relationship, in order to excite the particular modes associated with the structure. The simplest case of this is normal mode testing.
- A more general type of the System Identification of structures that requires the use of simultaneous excitation with multiple inputs that use a pre-specified coherence and phase relationship between random inputs for this purpose.

Impedance Matrix Approach to Multi-Shaker Control

An important beginning concept when embarking on the possible use of multi-shaker control systems is that not every shaker/table/fixture/UUT (Unit-Under-Test) may be fully controllable. Often, the use of multiple shakers involves the generation of large amounts of force, sometimes in the hundreds of thousands of pounds. To control forces this large requires a sophisticated physical design, including a very large isolation mass to prevent unwanted transmission of shaker forces into the surrounding environment and sophisticated provisions to allow for independence between the exciters and their associated fixturing. Multiple exciters typically also require an assortment of large tables and fixtures – depending on how many axes must be excited, the size of the UUT and other factors. If any element of this complex 'laboratory' design is less than excellent, an uncontrollable situation may ensue. Test prescribed test levels and phases, as well as the control transducer and their placement and also the excitation direction and their point placement may contribute to the success of the test as much as the actual control software. The locations chosen for the placement of the response transducer and its associated actuator may also cause near singular behavior in the system-under-test's frequency response matrix and compromise the associated control process.

Many multi-shaker vibration tests are performed on the basis of having one phase/amplitude control transducer per shaker. This is termed square control and assumes n control points for n shakers. Additional channels may also be available for the limit control of amplitudes, without affecting the phase parameters. For a four shaker test, for example, a control description of the system under test would look like:

Definition of Terms

MISO (Multiple Input, Single Output) – Typical single shaker system with multiple control transducers for Average or Peak Limit control.

MIMO (Multiple Input, Multiple Output) – Multiple Drive system for more than one shaker system attached to a single structure with multiple feedback control transducers.

MESA (Multiple Exciter, Single Axis) – A special form of MIMO control in which all exciters are set to provide force in a single direction. The control system is then, typically, asked to control 'Platform' motion, with all control points moving in Phase.

MEMA (Multiple Exciter, Multiple Axis) – The more general case of MIMO control that uses multiple exciters to create a controlled environment of 3 or more degrees of freedom.

Real-Time Adaptive MIMO Control – Closed loop control which updates the System Impedance Matrix (SIM) every loop to assure true control of non-linear responses rather than only updating the SIM at the beginning of the test.

System Frequency Response Matrix – The frequency response matrix that exists between the drive-vector that the MIMO controller uses to control the Unit Under Test (UUT) and the control-response vector formed by the control transducers.

System Impedance Matrix – The inverse of the system frequency response matrix.

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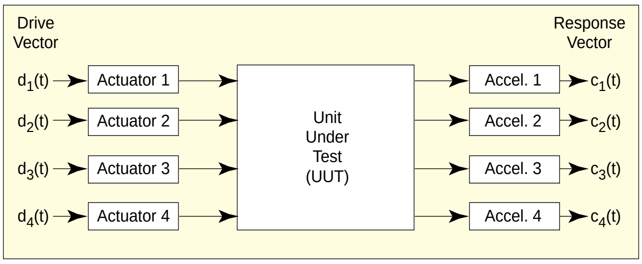

Figure 1. Block diagram of a multi-shaker vibration test setup.

$$\begin{Bmatrix} c_1(f) \\ c_2(f) \\ c_3(f) \\ c_4(f) \end{Bmatrix} = \begin{bmatrix} h_{11}(f) & h_{12}(f) & h_{13}(f) & h_{14}(f) \\ h_{21}(f) & h_{22}(f) & h_{23}(f) & h_{24}(f) \\ h_{31}(f) & h_{32}(f) & h_{33}(f) & h_{34}(f) \\ h_{41}(f) & h_{42}(f) & h_{43}(f) & h_{44}(f) \end{bmatrix} \begin{Bmatrix} d_1(f) \\ d_2(f) \\ d_3(f) \\ d_4(f) \end{Bmatrix} \tag{1}$$

where **{C(f)}** and **{D(f)}** are the vectors of spectra of the respective control and drive signals and **[H(f)]** is the system-under-test's frequency response matrix, i.e., the matrix of frequency response functions that exists between the various drive and control channels. This system can be described in block form as shown in Figure 1. The system can also be described in a more compact matrix-vector notation form as:

$$\{\mathbf{C(f)}\} = [\mathbf{H(f)}]\{\mathbf{D(f)}\} \tag{2}$$

The MIMO-random and MIMO-waveform replication control system measures the system frequency response matrix **[H(f)]** by exciting the system with low-level, spatially-uncorrelated, random drive signals during system characterization. It computes the associated spectral density matrix (SDM) of the drive-vector. It also measures the associated cross-spectral density matrix (CSDM) between the control-response and drive vectors. It then performs a matrix division of the resultant CSDM by its associated SDM matrix to obtain an initial estimate of the system-under-test's frequency response matrix (FRM). This estimate is further refined by the aforementioned and patented MIMO-adaptive control process. In MIMO-sine, the system's FRM is obtained initially and refined during the closed loop process by the same adaptive control method.

MIMO-Random Control

For MIMO-random control, the control vectors **{C(f)}** become a spectral density matrix (SDM) which we call $[\mathbf{G_{cc}(f)}]$ and the drive spectral vector becomes the drive SDM which we call $[\mathbf{G_{dd}(f)}]$. They are related by the system-under-test's frequency response matrix (FRM) **[H(f)]** as follows:

$$[\mathbf{G_{cc}(f)}] = [\mathbf{H(f)}][\mathbf{G_{dd}(f)}][\mathbf{H(f)}]^{*} \tag{3}$$

In order to accomplish MIMO control, these and other unit-under-test response equations need to be solved using an iterative manner so that the drive vector will cause, for example, a 4 shaker system to vibrate in such a way that the associated 4 control accelerometers will have the reference spectral density matrix **[R(f)]** as their response SDM within some acceptable margin. In other words, we need the MIMO drive signals to excite the 4 shakers such that all the control channel's SDM $[\mathbf{G_{cc}(f)}]$ vectors will agree within some tolerance with the specified reference SDM **[R(f)]**. These control equations are solved by the use of a proprietary MIMO-random control method that is based on the cited adaptive control methods. A detailed review of this process is outside the scope of our discussion here.

If this example were a MESA test, we would want: 1) the control-responses to be in phase, 2) the coherence between each of the control channels to be nearly one and 3) that their response PSDs (Power Spectral Densities) agree with the specified reference PSD – all within an acceptable error margin.

To solve the MIMO control problem, we need to solve the previously discussed control equations (2) and (3). This means that the initial drive needs to be given by:

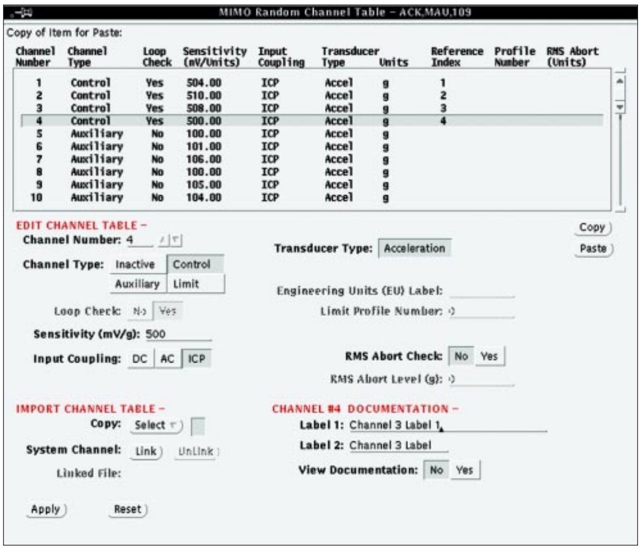

Figure 2. Typical system channel table for four control channels.

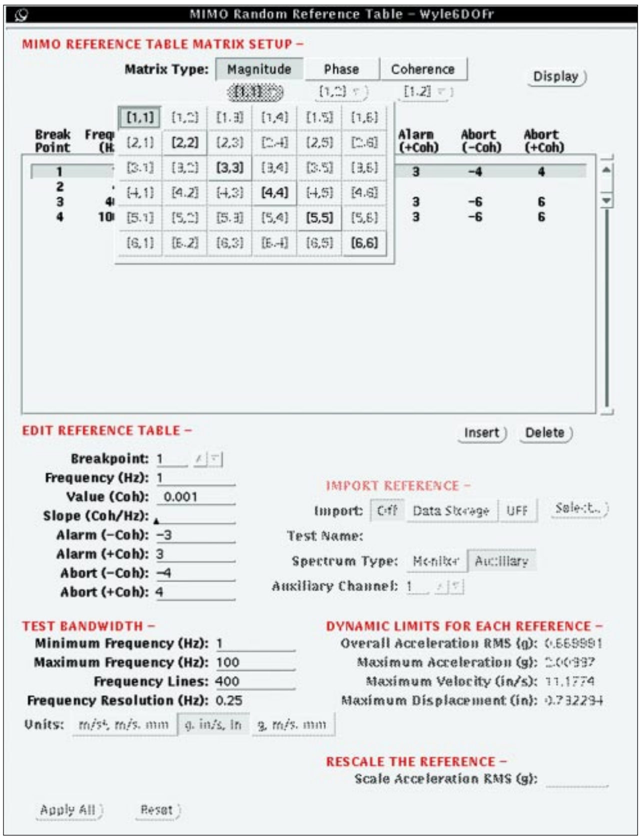

Figure 3. A typical four breakpoint random reference table as part of a 6 × 6 Spectral Density Matrix.

Figure 4. Special MIMO control parameters for singularities and impedance.

$$[\mathbf{G_{dd}(f)}] = [\mathbf{H(f)}]^{-1}[\mathbf{R(f)}][\mathbf{H(f)}]^{-*} \tag{4}$$

Thus, MIMO-Random needs to use the matrix-inverse of the measured frequency response matrix **[H(f)]** to create the initial drive. This matrix, which is the matrix inverse of **[H(f)]**, is called the system impedance matrix **[Z(f)]** of the system under control. Thus the system impedance matrix is given by:

$$[\mathbf{Z(f)}] = [\mathbf{H(f)}]^{-1} \tag{5}$$

Because there is a possibility that **[H(f)]** is singular or nearly so, a pseudo-inverse is actually calculated. The pseudo-inverse operation inspects the ratio of the largest eigenvalue and smallest eigenvalue of the matrix **[H(f)]*[H(f)]**, as a function of frequency. This ratio is used to decide whether **[H(f)]** is singular or nearly singular to appropriately evaluate the matrix-inverse operation. These eigenvalues are called the singular values of the matrix **[H(f)]**. If the ratio of the smallest to the largest singular value falls below a certain singularity threshold, the matrix is declared singular and a pseudo-inverse calculation is performed to obtain what we call the pseudo-impedance. The use of the pseudo-impedance mitigates the tendency for the actuators to 'fight' and also limits the magnitude of the drives used for control. Many such pseudo-inverses are possible. The Moore-Penrose pseudo-inverse is used since it's optimal in many regards. Test experience indicates that this method works very well. In fact it is much better than older methods used to reduce the matrix order of **[H(f)]** that assume rigid body relationships between the various responses as electrohydraulic multi-exciter vendors have done for years in the road and seismic simulation arena. These ideas are also quite useful for controlling rectangular systems where the number of control responses is greater than the number of drives. Again, a complete discussion of this issue is outside the scope of this article.

System Setup and Operation

Before continuing with the theoretical aspects of MIMO Random control, let's take a look at the user interface which is used to set up and control a MIMO Random test. You will note that it is very similar to that of a single shaker MISO control setup. Several new elements have been added to accommodate the need of defining the parameters associated with multiple drives. For the sake of example, the tests described here will assume the same PSD profile is prescribed at all 4 shakers and that all responses at the 4 control points are in phase. In an actual test, the reference PSD profiles may differ in shape and the control points may have different phase relationships but still have the same frequency range and same number of control lines.

For the case of a MIMO Random test some of the parameters which must be established prior to testing include: the Channel Table; the Random Reference Table; the Random Control Parameters; the Random Safety Parameters; the Transfer Function *H*(*f*) Table; and the special MIMO Parameters. One of these, a channel setup with 4 independent control channels, is illustrated in Figure 2.

Since multi-shaker phase control depends on accurate transducer signal handling, care should be taken to assure that the accelerometer signal conditioning does not create any unwanted or unknown phase shifts, amplitude gain or amplitude attenuations. If possible, transducers with built-in amplifiers can be used such as those using an ICP® or Isotron® technique. The accelerometers can then be attached directly to the JAGUAR and powered by the JAGUAR. Due to the excellent channel-to-channel phase match of the JAGUAR, this technique has proven quite popular.

Figure 3 shows how the PSD for each control channel is specified. Note how similar it is to the way the PSD is setup for a MISO test, even though it is now part of a 6 × 6 Spectral Density Matrix in this example. The design philosophy has been to maximize this similarity to MISO testing and setup to ease the operator's learning curve.

For random vibration MIMO tests, the reference spectrum for each shaker may be the same or different. If a different reference spectrum is defined for different shakers controlling the same structure, certain parameters such as frequency range and number of control lines must be common to all shaker control spectra. Other than that, different shapes, levels and number of breakpoints can be defined for each control point.

Random control parameters for MIMO are generally the same as those for MISO, except for some special parameters which have been added. In addition to controlling PSD shapes and levels, selection is now available for controlling the phase as well as the coherence between control locations. In order to accommodate all possible conditions in large structural testing, including both symmetrical and unsymmetrical structural conditions, several other parameters are needed. These include a method for adjusting the adaptive gain for updating the Impedance Matrix as well as the very important singularity threshold parameter. This is used by the pseudo-inverse operation to decide when the frequency response matrix is nearly singular as earlier discussed. An example of the user specifi-

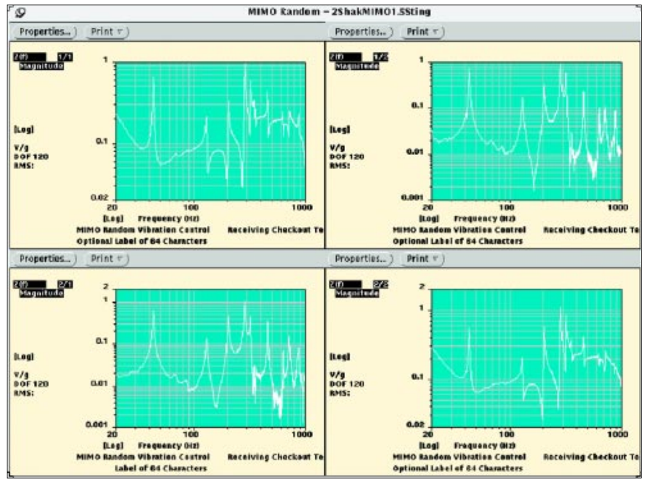

Figure 5. Typical 2 × 2 impedance matrix shown in magnitude form to 1,000 Hz.

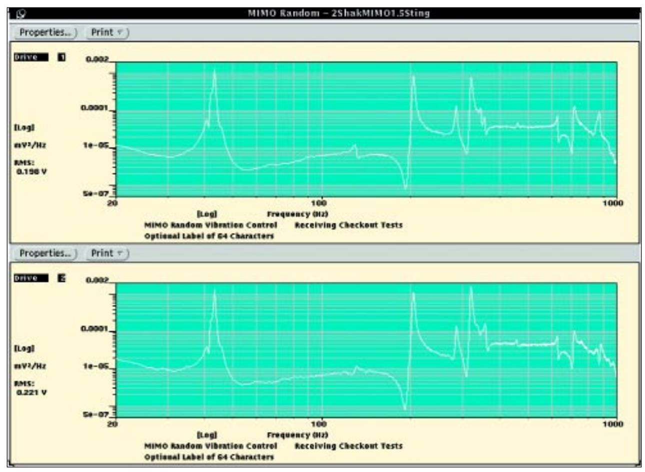

Figure 6. Random drive signals required for shakers 1 and 2 to achieve control of Figure 7.

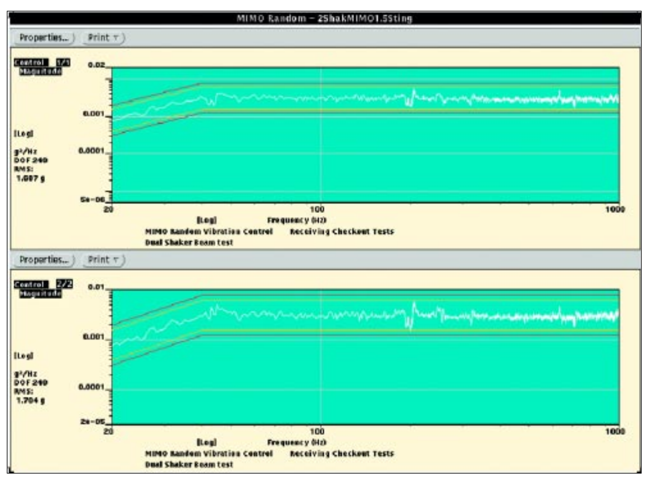

Figure 7. Two shaker control PSDs shown for 1,600 lines and 240 DOF.

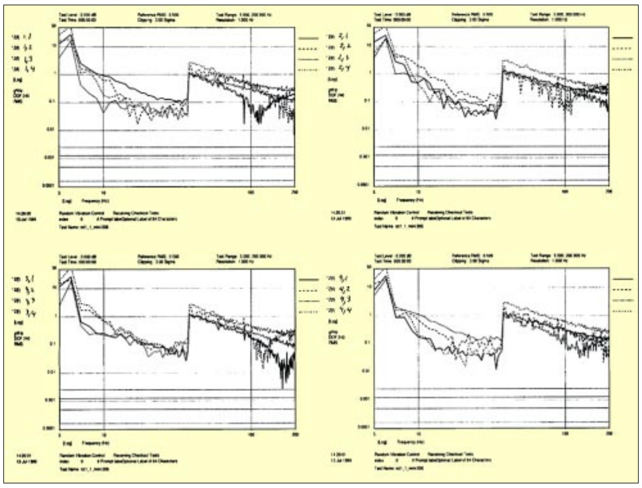

Figure 8. 4 × 4 impedance matrix showing singular behavior.

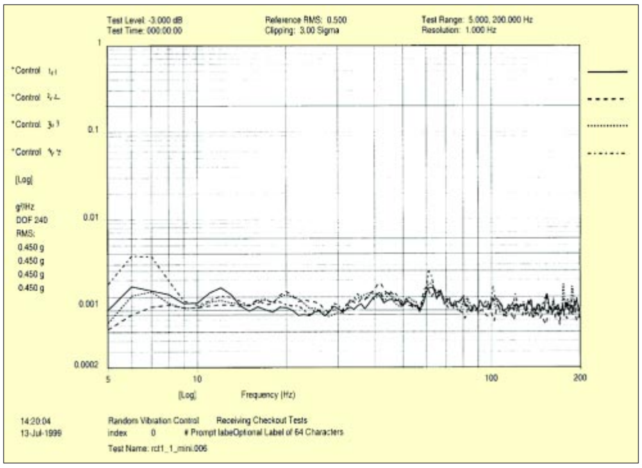

Figure 9. Four shaker control PSDs for undamped aluminum plate.

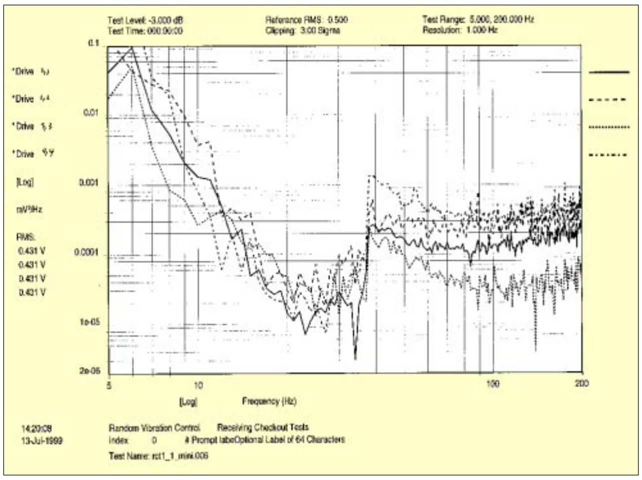

Figure 10. Four shaker drives required to achieve control of Figure 9.

cation panel is shown in Figure 4.

Test Results from MIMO-Random

In a typical multi-shaker vibration test with n shakers and n control points, the System Frequency Response Function Matrix becomes an $n \times n$ complex-matrix of FRFs. To calculate the necessary Drive signals for the n shakers, the FRF matrix must be inverted to form the Impedance Matrix as shown in Equation 5. The Magnitude of a typical 2×2 Impedance Matrix is shown in Figure 5. The associated drive signals obtained with the previous impedance matrix are shown in Figure 6. Note the similarity of the peaks in the Impedance Matrix Magnitude of 1/1 and 2/2. In particular, the peaks present in the drive signals for the 2 shakers – peaks at 43, 202, 285 and 315 Hz are clearly visible in both Z and D with the 135 Hz peak also common but less apparent in the drive spectra. The resulting control PSD magnitude, for each of the 2 control points is shown in Figure 7.

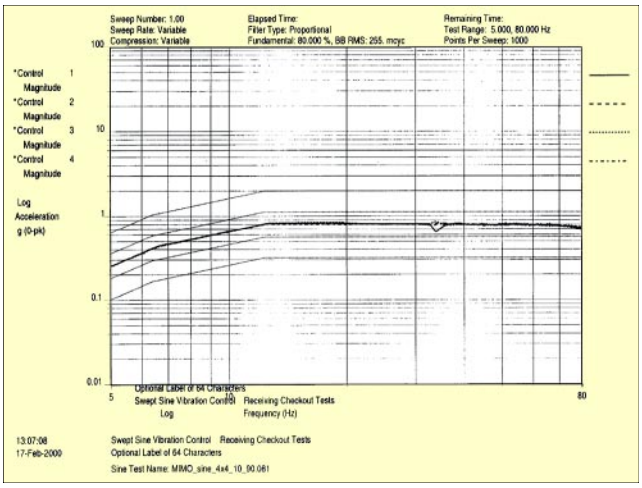

Figure 11. Four shaker control amplitudes during sine sweep.

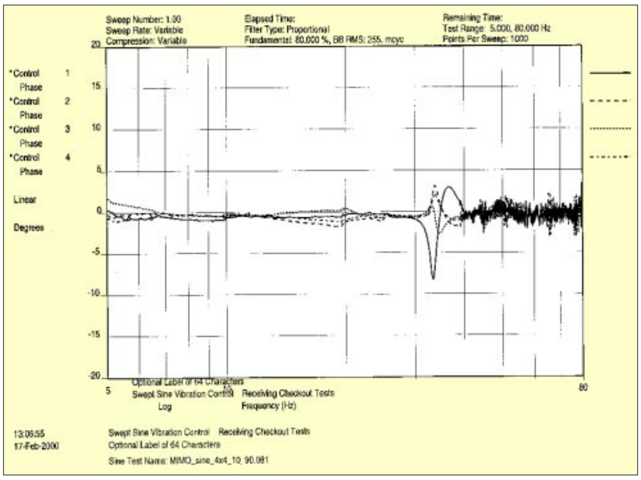

Figure 12. Four shaker control phases during sine sweep.

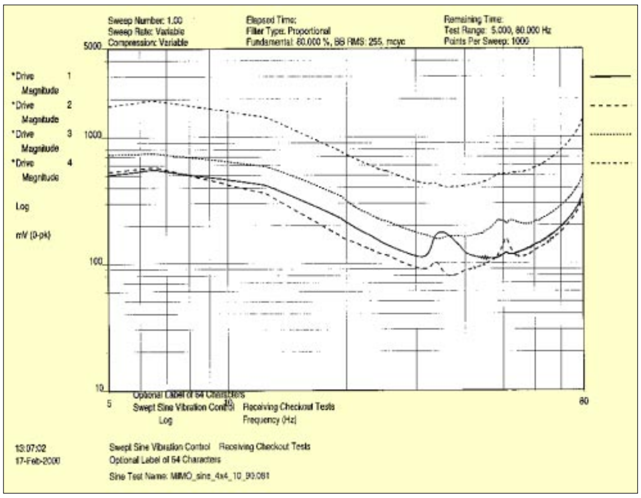

Figure 13. Four shaker drive magnitudes during sine sweep.

Figure 8 shows the impedance matrix, with singular behavior in the frequency range between about 6 Hz to almost 40 Hz. This was obtained from a relatively stiff aluminum plate, approximately 2 ft square, excited in the vertical direction, with four mini electrodynamic shakers. Figure 9 shows the PSDs at the four control points associated with using the previously shown impedance matrix. Figure 10 shows the drive PSDs as-

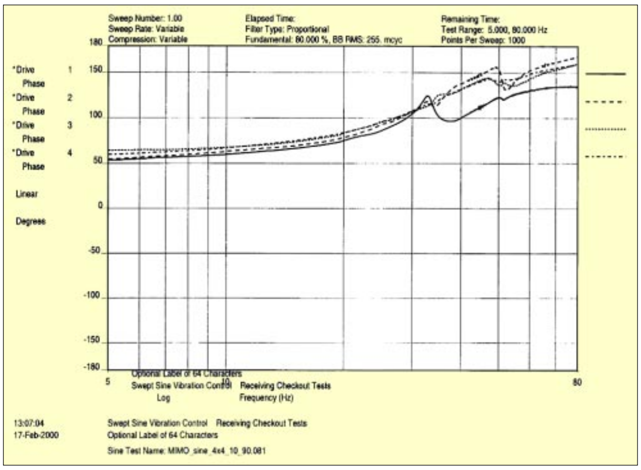

Figure 14. Four shaker drive phases during sine sweep.

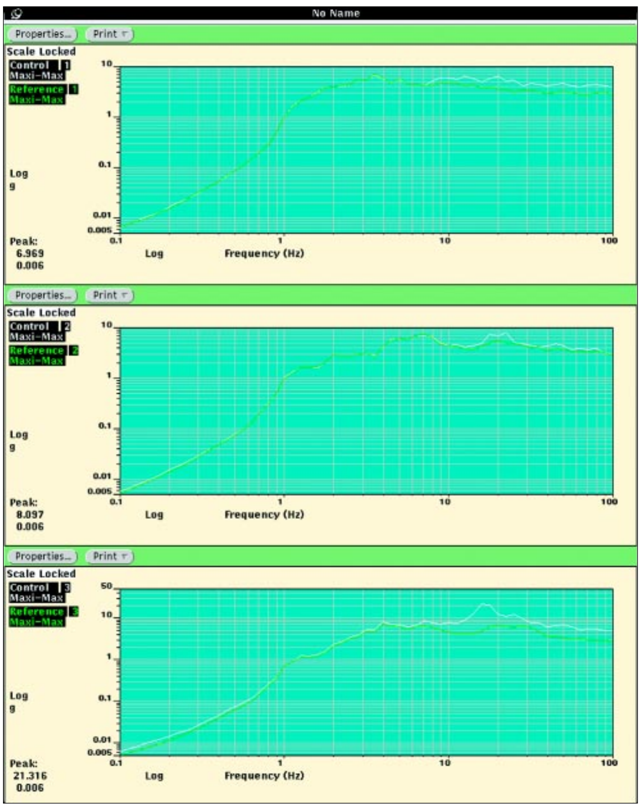

Figure 15. Reference Response Spectra (RRS) and Control Maxi-Max SRS for X,Y and Z axes.

sociated with the drive signals used to achieve these control responses. The control coherences and phases, not shown, during this test were also close to 1 and 0 degrees, respectively.

MIMO-Sine Test Results

We also ran some MIMO-sine tests with four shakers. In this case the aluminum plate was not as rigidly attached to the shakers. Otherwise the configuration was similar to what was presented previously for MIMO-Random. In this case, Figure 11 shows the control response achieved at the four control points from 5 to 80 Hz. Figure 12 shows the phase, with respect to the digital tracking filter's heterodyne signal, of the four response points. Figure 13 shows the magnitude of the drive signals used to achieve these control responses. Figure 14 shows the phase of the drive signals, again with respect to the tracking-filter's heterodyne signal, to achieve these responses.

MIMO-Transient Waveform Replication Control Test Results

The following test results were obtained at Wyle's seismic simulation facility. In this case, there are four electrohydraulic exciters, where two are used for the front-to-back and side-to-side horizontal axes. The other two actuators work together to drive the vertical axis. These two vertical actuators are con-

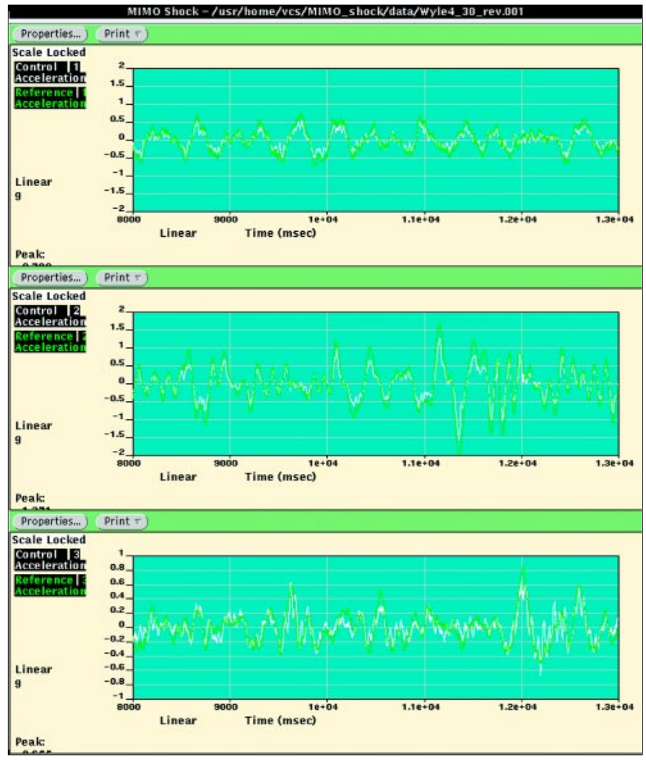

Figure 16. Expanded reference and control acceleration waveforms used to create desired SRS.

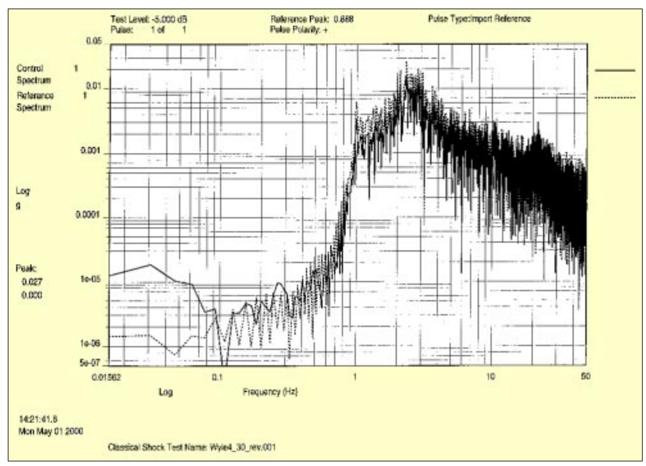

Figure 17. Fourier magnitude match, control and reference, X-axis.

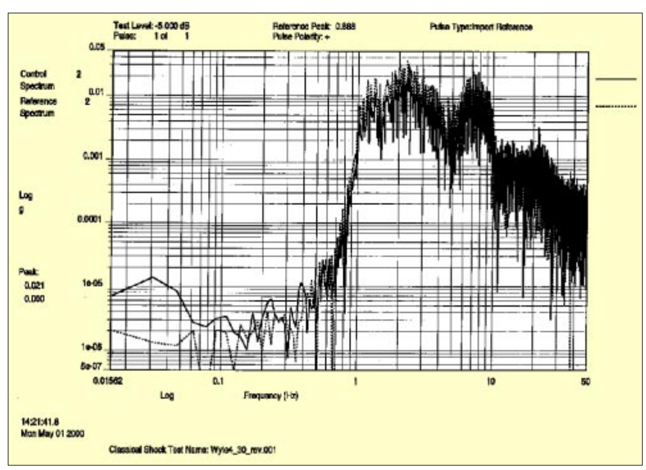

Figure 18. Fourier magnitude match, control and reference, Y-axis.

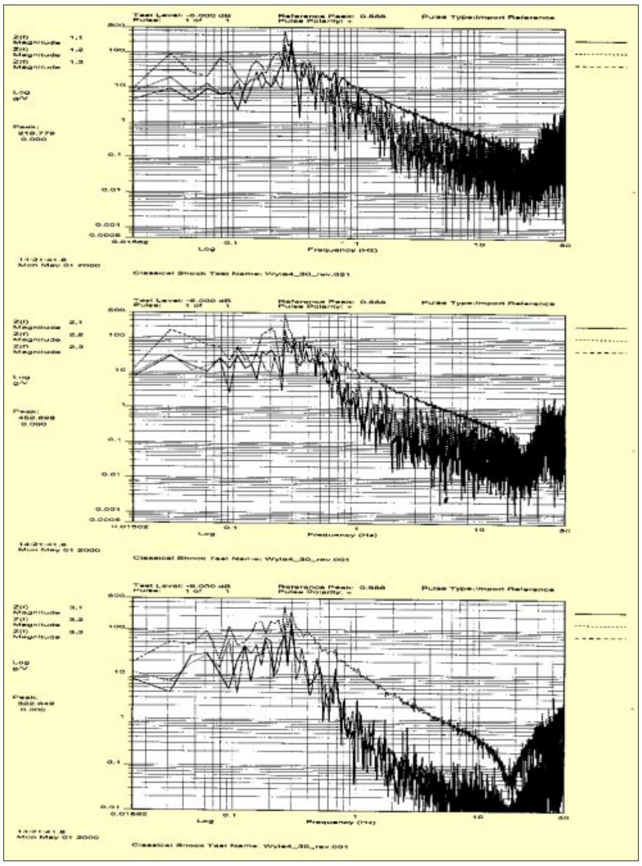

Figure 19. 3 × 3 impedance matrix measured prior to seismic qualification test.

trolled with a single drive signal. So in this case, the control system is 3-by-3.

For this case, the objective was to synthesize 3 broadband burst-random pulses, of 32 sec duration, which achieve or exceed a given Shock Response Spectrum. These are called the Required Response Spectra (RRS) by the seismic community.

Figure 15 shows the respective control and reference SRS for each control point. It shows that the system was able to match the given multi-axis RRS very well up to about 6 Hz and exceeded them for higher frequencies achieving successful test response performance. These tests were near the 40 in./sec capabilities of the excitation system. The reference pulses, not shown, had peak levels of over 3 g and displacements of about 4 in. peak.

Figure 16 shows a typical time waveform matching that which was achieved. The high frequencies present in the response and not in the reference were due largely to the oil-column resonance in the hydraulic excitation system. All in all, the waveform matching performance was very good in spite of this.

Figures 17 and 18 show typical Fourier spectra matching, of the respective control and reference pulses, for control channels 1 and 2. Again this shows good spectral matching from about 0.9 Hz onward.

Figure 19 shows the magnitude of the impedance matrix that was present during the test. The top graph shows the first row, the middle graph shows the second row and the bottom graph shows the third row of the impedance matrix. In this case, there are no noticeable singularities present.

Summary and Conclusions

We have discussed a new generation of control system technology that implements advanced MIMO-control for Random, Swept-Sine and Waveform replication. This technology allows for control of both MESA and MEMA applications. Excellent results have been achieved in the three discussed domains. The control system, in addition to employing a state-of-the art electrical design, embodies advanced control methods based on patented technology. There are provisions for updating the control model during the test as well as accommodating singularities that may be present in the actuation and instrumentation systems.

How the JAGUAR Performs Multi-Shaker Testing

The JAGUAR Acquisition and Control Peripheral (ACP) is completely designed and produced by Spectral Dynamics and works in conjunction with a Sun Microsystems workstation to create an expandable control and analysis system. Input and output modules can be combined on 10 channel boards to create a system which is optimized for a particular configuration, but which can also expand to suit future requirements. Up to 12 drive outputs can be configured in a JAGUAR and up to 98 Input/Control channels in a single chassis. Phase-match between channels is preserved to better than ±2° to 20 kHz to assist in accurate phase control. Channel-to-channel measured phase-match is typically better than ±0.05° to 2 kHz.

The system currently supports MIMO-random, MIMO-swept sine, MIMO-transient waveform control and MIMO-long term waveform replication. These developments are based on several patents including U.S. Patent #5,299,459 (EU Patent #0521853) and U.S. Patent #5,517,426. These have to do with multi-exciter control using swept sine and adaptive control for shock testing. These patents also cover the extensions to multifrequency testing which include MIMO-random and MIMO-waveform replication control. The JAGUAR is also based on other proprietary control technologies and other U.S. patents for random control, digital signal synthesis and computer based calibration methods for adjusting gain and phase. This creates an extremely solid analytical and technical foundation for the extremely challenging development of multi-shaker control systems. The staff working on these developments includes four Ph.D.s and several Masters Degreed Engineers from the disciplines of electronics, control systems, structural dynamics and computer sciences.

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