The Use of Wavelets for Analyzing Transient Machinery Vibration

Howard A. Gaberson, Oxnard, California

Transient events are sometimes buried in continuous machinery vibration data. Conditions causing these transient events include: a bearing ball rolling over a defect, a pit or chip on the face of a gear tooth, clearance in a bearing that allows a repetitive pounding, engine rod or main bearing knock and piston slap. Detection of the magnitude and timing of these events can be valuable for diagnostics. These can be identified with time-frequency methods that show frequency content and time of occurrence on a two-dimension contour plot. Wavelets can also be used to detect magnitude and timing. The orthogonal wavelet is inexpensive to compute and has the potential to display something new. The code decomposes the signal into wavelets, the location and magnitude of which identify the transient. Partial inverse transformation also shows regions of the signal where different wavelet levels build up to recreate the transient in the signal. The recent availability of orthogonal wavelets and MATLAB® wavelet code has made such analyses convenient. This article explains these methods and the signal processing calculations involved.

Many problems in machinery diagnostics are characterized by transient or impulsive events in the vibration signal that cause the frequency content to vary considerably and regularly with time. Several methods are available for analyzing transient events. Signal analyzers average individual spectra to a single spectrum, but they convert the time signal to a frequency signal and cannot show any time-frequency variations. The Short Time Fourier Transform (STFT) or spectrogram is used to display time-frequency variations in speech analysis but does not provide sufficient resolution for some machinery diagnostics problems. The Wigner distribution increases resolution in time-frequency distributions. It can locate the angular position of an impact or the discontinuity associated with individual gear tooth faults.[1,2] However, the Wigner distribution has severe interference (cross) terms that confuse the interpretation and require additional efforts to resolve. The Reduced Interference Distributions (RIDs) mitigate the Wigner cross terms while preserving sharp resolution. The Choi and Williams distribution[3] yields impressive detail and a significant structure in the time-frequency plane with recognizable impact. Finally, the wavelet transform has recently become available and offers an additional approach.[4] With some development, this tool could become very helpful in machinery diagnostics.

Short Time Fourier Transform

We begin by considering the Fast Fourier Transform (FFT) that is used in most data collectors and signal analyzers. Performing a FFT results in a complex function of frequency (complex because each frequency component has a magnitude and a phase). Normally the magnitude is retained and presented as the spectrum. The spectrum of a vibration signal is a graph that indicates amplitude or content as a function of frequency. Usually spectra are averaged, windowed, discrete Fourier transform magnitudes of vibration time histories.[5] The FFT is actually a Fourier series of a segment of a vibration signal, thus the FFT will indicate frequency content as the actual sine wave components that make up the signal (the original signal is the summation of all these components). Unfortunately for this application, the FFT assumes everything in the segment is periodic. This causes errors when evaluating components in the segment that do not have an integral number of periods. It also implies a discontinuity where the assumed periodic segments join. To diminish these "leakage errors" the time segments are 'windowed' with bell shaped functions that approach zero at their ends (e.g., Hanning, Kaiser-Bessel, flattop, etc.). Each term of each segment value is multiplied by the window function, causing a reduction in the signal amplitude. To compensate, the spectra values are multiplied by a window factor. Unfortunately the FFT is not useful for some diagnostic purposes because it cannot show where impulsive-like events occur in time. However, the FFT can still be used in obtaining some time versus frequency data.

The first step in considering a time-frequency representation is to evaluate the windowed FFT magnitude of adjacent overlapping segments of the time signal and observe how the frequency content of those segments changes. This is called the Short Time Fourier Transform (STFT) and is illustrated in Figure 1. The signal to be analyzed is in the upper left portion of the figure and below it are five 50% overlapped segments. In the next column above time = 0.05 the segments have been windowed to reduce leakage. Next to the windowed segments are the spectra. To obtain the full STFT, we skip a suitable number of points, move to a new time and repeat the process. Each FFT is the frequency content of a windowed segment of the function centered about time *t*. Each blip within that window contributes to the frequency content at time *t*. Shortening the window allows for more accurate measurements in the time domain, but restricts the lowest frequency you can detect (one entire wavelength must be within the window). Shortening the window also reduces the number of lines or frequency spacing. The STFT is used on signal analyzers as a 'waterfall' plot or spectrum.

The Wigner and Reduced Interference Distributions

The STFT does not have the time or frequency resolution of the Wigner distribution. In this autocorrelation-based approach, the signal is analyzed as a list of numbers plotted on a graph. Imagine that there are two copies of the list side by side. The autocorrelation function (ACF) involves multiplying the signal (i.e., the list) by itself, number by number, and adding up the values.[5] Next we shift the lists by one value, remultiply and add. The shift is called a lag and the values of the calculations for each shift are the ACF. The FFT of the ACF turns out to be the spectrum squared, which is nothing new. But consider a symmetrical ACF: Imagine now we have a single copy of the signal (the long list of numbers). As an example, start 128 numbers down from the top; take that number and square it. Now take one number above and below it and multiply those together; then move one number above and below those and multiply them together. Continue until we have covered 128 numbers in each direction. We now have a list of 128 values for the multiplications, with the squared value of our center point at the top. We then take the values below the first value, flip them end for end, and put them on top of the list, making the list 255 numbers long. That is the symmetrical autocorrelation function for that center point. Take the FFT, skip a few points and do it again. This is a rough approximation to the Wigner distribution. The actual Wigner requires windowing the 255 values before you FFT them and converting the signal to a complex function. Even a rough approximation of the Wigner distribution can highlight discontinuities in the signal and indicate the frequency content better than the STFT.[1,2] Unfortunately the Wigner distribution generates artificial cross terms at times and frequencies where there is nothing happening. It finds the actual discontinuities but also adds some fictional discontinuities.[6]

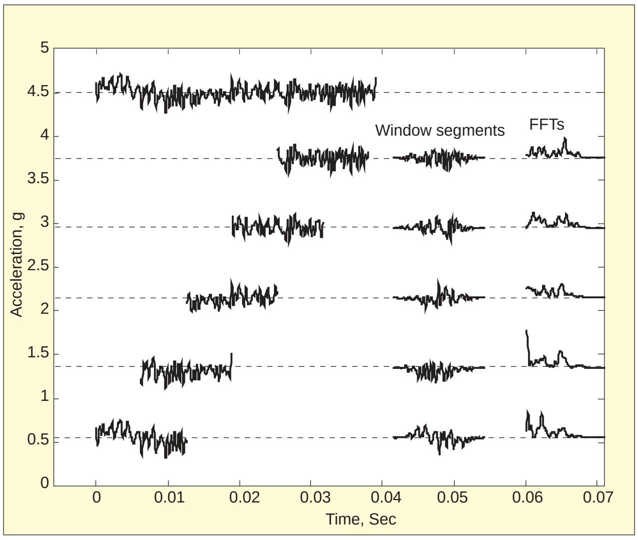

Figure 1. Five time segments from a vibration signal, 50% overlapped running from 0.01 to 0.04 sec. From 0.04 to 0.55 sec, the segments are multiplied term-by-term by a Hanning window. At 0.06 sec, the FFT is applied to each segment. The sequence of FFTs form the STFT of the signal.

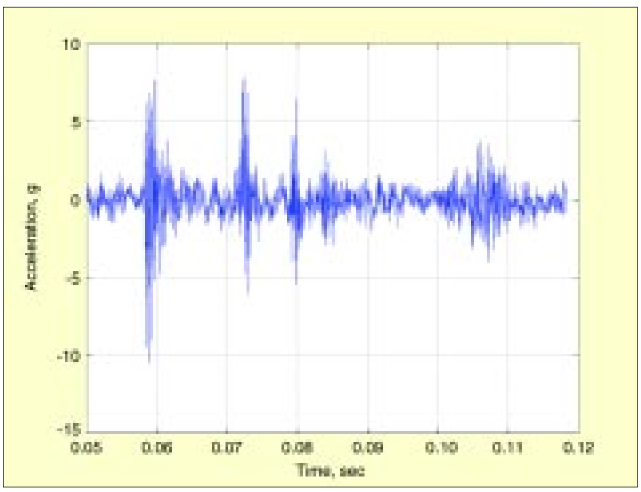

Figure 2. Acceleration from an intake valve cap on a large reciprocating compressor. The burst of acceleration at 59 msec is from the intake valve channels slamming open. The burst at 72 msec is when they close.

The reduced interference distributions (RIDs) were invented to eliminate the Wigner's unwanted cross terms. Approximately 10 RIDs exist, including the Choi-Williams distribution.[3,6] Like a waterfall plot, these time-frequency distributions can be displayed as a terrain map with time and frequency axes and elevation indicating content. The RIDs perform specific averaging of adjacent time and frequency values on a time/frequency map. The cross terms fluctuate rapidly and the averaging smooths them out, more or less. With the Choi-Williams, I have noticed that amplitudes are affected and cross terms are almost completely eliminated.

Machinery Vibration Analysis Examples

To illustrate what you can expect from these methods, consider Figure 2. It shows some accelerometer data taken from the intake valve cap on the head of the high pressure cylinder of a large reciprocating compressor. The time interval shown is about one revolution. The burst of acceleration at 59 msec is caused by the intake valve channels slamming open. The burst at 72 msec is when they close. Other impact events include the opening and closing of the intake and exhaust valves from the three cylinders.

Figure 3a shows a contour plot of an STFT of the data. In order to display the impact events somewhat sharply in time, I used a very short 64 point FFT (equivalent to a 32 line display).

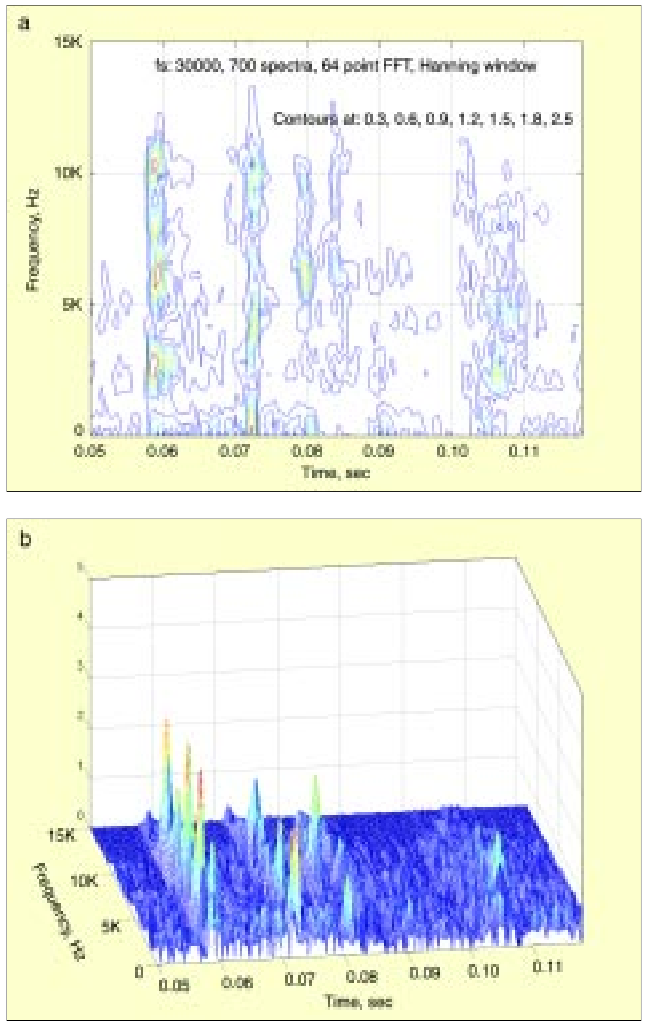

Figure 3. a) Contour plot of the STFT of the compressor data. b) Mesh plot of the STFT of the compressor data.

With a normal length FFT virtually all time detail would be lost, because the STFT responds to the impact anywhere within its window. Figure 3b is a three dimensional mesh plot of this analysis that shows the complexity of the time/frequency content. The contour plot precisely shows the location of amplitude peaks in the time/frequency plane whereas the mesh plot shows the "big picture," overall arrangement.

Figure 4a shows a contour plot of a Wigner analysis of these same data. Depending on the number of contours plotted, it can be quite precise with respect to time. However, notice in the corresponding mesh plot (Figure 4b) that several more peaks occur in the impact. Compare the region prior to $t = 0.06$ between 0 and 7500 Hz to the STFT of Figure 3b. Many of these peaks are fictitious "cross terms," the biggest drawback to the Wigner method.

Figure 5a is a contour plot of the same data using the Choi-Williams RID. It has reduced the interference caused by the cross terms and appears to be the most precise of the three. The three-dimensional mesh plot for these data is shown in Figure 5b. Although beautiful, the mesh plot contains too much information to grasp by itself; the contour plot remains a good map to the location of the high amplitude regions. Examination of contour plots using trial and error contour levels alongside the mesh plot provides the most revealing information. In the following section, these three methods are reviewed for comparison with wavelet analysis.

Wavelet Analysis

Wavelet analysis of machinery vibration data is a different form of time-frequency analysis. There are two major categories of wavelet transforms: continuous and orthogonal; continu-

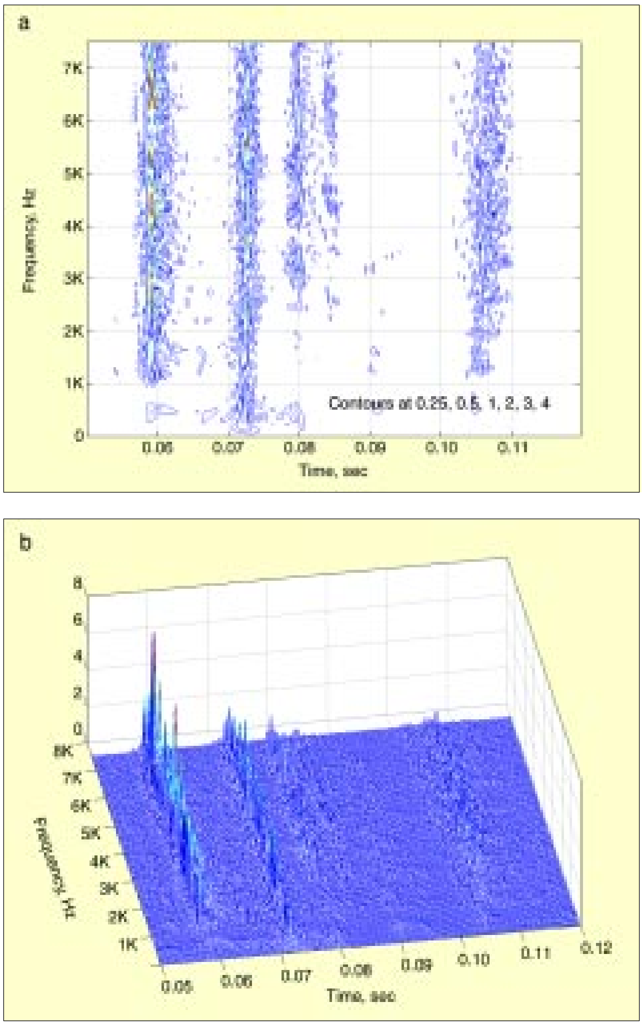

Figure 4. a) Contour plot of a Wigner analysis of the compressor data. b) Mesh plot of the Wigner analysis of the compressor data.

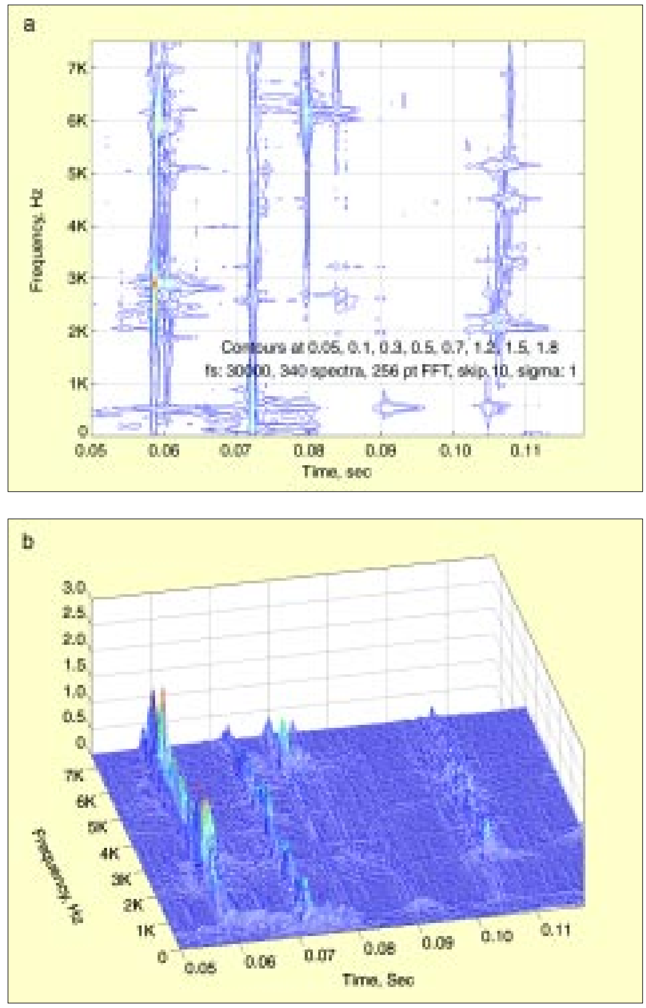

Figure 5. a) Contour plot of a Choi-Williams analysis of the compressor data. b) Mesh plot of a Choi-Williams analysis of the compressor data.

ous is easier to describe.[8] The wavelet transform, or distribution, is a short wavy function that is stretched or compressed and placed at many positions on the signal to be analyzed. The wavelet is then term-by-term multiplied with the signal; the sum of those products is the wavelet value. The amount that the wavelet is stretched or compressed is called its scale and it is called translation when the wavelet is moved from position to position.

Figure 6 illustrates a continuous wavelet transform. Figure 6a shows an antisymmetrical wavelet (the imaginary part of the Morlet wavelet) centered at $t = 0.003$ sec. In Figure 6b, the wavelet has been flipped left to right. Notice that the wavelet is only nonzero for a small region. Figure 6c shows the signal we want to analyze, a portion of acceleration data from an air handler. Figure 6d shows the result of a term-by-term multiplication of the second and third figures. Adding up the values of Figure 6d would give the wavelet distribution value for this scale (frequency) and position of the wavelet. We would hold the frequency or scale constant, advance the wavelet slightly and calculate a new coefficient. After we had advanced the wavelet at this frequency across our signal, we would select a new frequency or scale and repeat the process.

Figure 7a presents a contour plot of the same compressor data using my Morlet continuous wavelet MATLAB analysis. Notice its ability to pick out the discontinuities. The corresponding mesh plot for this analysis is shown in Figure 7b. Although I have no proof, I prefer the Choi-Williams analysis to the Morlet. All of these must be compared to the STFT, which is a known understandable analysis. After establishing this background, we will now examine the unusual orthogonal wavelets.

Orthogonal Daubechies Wavelets, a Transform

The orthogonal wavelet is a true transform that converts a length of data equal to a power of two (e.g., 512, 1024, 2048, etc.) data points into a transform of the same length;[9] the inverse transform returns the original data set. It is interesting to note that the wavelet transform decomposes the signal into transients (the wavelets). The type of wavelet used determines the wavelet shape; each of the values of the wavelet transform is the magnitude of an individual wavelet. The position or placing of each transform in the list of values gives the location and duration of that wavelet. Whereas the FFT tells us how to build up the signal from the same number of sines and cosines as there are data points, the wavelet transform tells how to build up the signal from the same number of wavelets as there are data points.

The transform looks quite different. Partial inverse transformation shows regions of the signal where the different wavelet levels build up to recreate the transient in the signal. I have experimented with 10 of these wavelets, the nine Daubechies and Newland's harmonic.[10] It takes many pages of detailed math to prove the transforms will work, but this can be done much faster using MATLAB programs to perform the forward and inverse transformations. Daubechies[11] discovered her nine different wavelet transforms in the late 1980s and computed the coefficients needed to implement them. The Daubechies-4 wavelet transform uses 4 coefficients and the Daubechies-6 uses 6 coefficients. The others are the Daubechies 8, 10, 12, 14, 16, 18 and 20 transforms. Newland[10] gives the MATLAB programs[12] and the numerical values of the required coefficients to perform the forward and inverse transforms. He also presents a harmonic wavelet transform and the MATLAB programs for

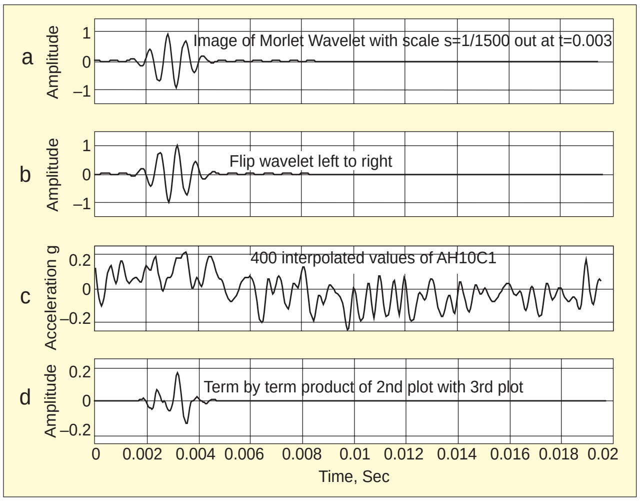

Figure 6. An illustration of the first four steps in the calculation of a continuous wavelet transform coefficient. The wavelet is centered at t = 0.003. The sum of the values in the fourth plot would be the wavelet coefficient for this frequency (scale) and position.

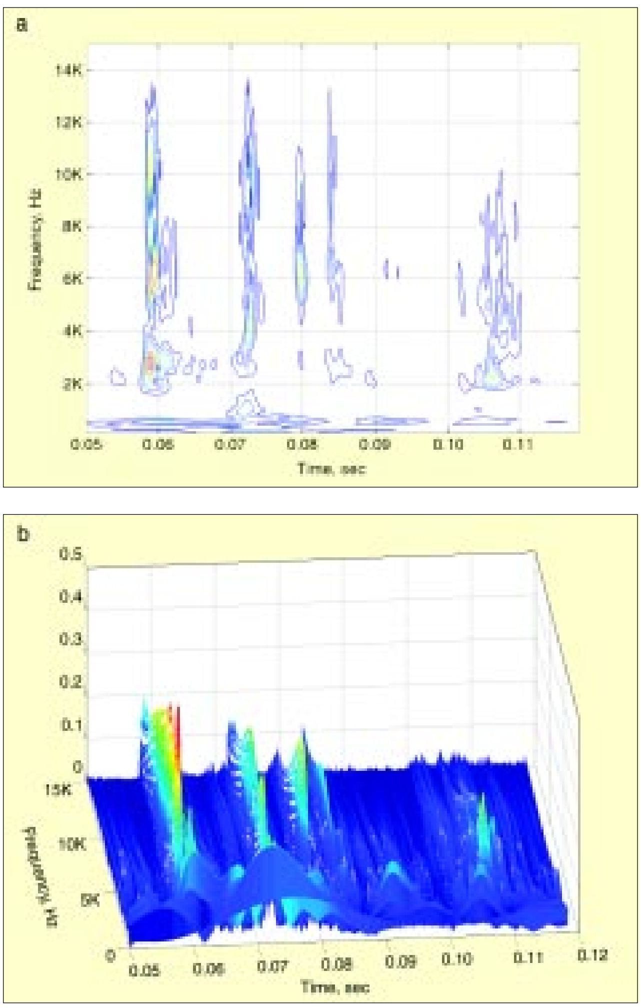

Figure 7. a) Contour plot of a Morlet continuous wavelet analysis of the compressor data. b) Mesh plot of the Morlet continuous analysis of the compressor data.

using it. The availability of orthogonal wavelet software[10] for high level signal processing programs has recently made such analyses convenient and easy to use. These programs are copyrighted so they are not presented in this article; you must obtain your own copy. The Daubechies Wavelet coefficients are used to keep transforming the signal into half smooths and half details. The detail portion is saved each time while the smooth portions are repeatedly transformed into more smooths and details until we have all details and only two smooth values. We get 11 levels for 1024 points; so in effect we are analyzing into 11 bins as opposed to our normal minimum of 400.

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 & & & & & & & \\ c_3 & -c_2 & c_1 & -c_0 & & & & & & & \\ & & c_0 & c_1 & c_2 & c_3 & & & & & \\ & & c_3 & -c_2 & c_1 & -c_0 & & & & & \\ & & & & & & & & & & \\ & & & & & & & c_0 & c_1 & c_2 & c_3 \\ & & & & & & & c_3 & -c_2 & c_1 & -c_0 \\ c_2 & c_3 & & & & & & & & c_0 & c_1 \\ c_1 & -c_0 & & & & & & & & c_3 & -c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix} = \begin{bmatrix} s_1 \\ d_1 \\ s_2 \\ d_2 \\ s_3 \\ d_3 \\ s_4 \\ d_4 \\ s_5 \\ d_5 \\ s_6 \end{bmatrix} \quad (1)$$

Numerical Recipes[13] explains the calculating and programing of the orthogonal wavelet transform in an understandable manner. Matrix (1) illustrates the process for a 4 coefficient Daubechies wavelet, where the cs are the coefficients.

Rearrange the output column with the smooths in the top half and the details on the bottom. Next, apply Equation (1) to the top half (the smooths) and again break those into half smooths and half details. You will end up with $N/2$ details for the first level, $N/4$ details for the second level, $N/8$ details for the third level, etc. The process is finished when you are down to just two details and two smooths.

To perform the inverse transform, the procedure is reversed with the transpose of matrix (1) or:

$$\begin{bmatrix} c_0 & c_3 & & & & & & & c_2 & c_1 \\ c_1 & -c_2 & & & & & & & c_3 & -c_0 \\ c_2 & c_1 & c_0 & c_3 & & & & & & \\ c_3 & -c_0 & c_1 & -c_2 & & & & & & \\ & & c_2 & c_1 & & & & & & \\ & & c_3 & -c_0 & & & c_0 & c_3 & & \\ & & & & & & c_1 & -c_2 & & \\ & & & & & & c_2 & c_1 & c_0 & c3 \\ & & & & & & c_3 & -c_0 & c_3 & -c_2 \end{bmatrix} \begin{bmatrix} s_1 \\ d_1 \\ s_2 \\ d_2 \\ s_3 \\ d_3 \\ s_4 \\ d_4 \\ s_5 \\ d_5 \\ s_6 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix} \quad (2)$$

Figure 8 is a plot of the 2048 Daubechies-4 wavelet transform values of the compressor data just as they are returned by the "wavedn.m" program.[10] The values plotted are the sets of details and the final two smooths. The right half of the plot is the 10th level of 1024 details, the finest detail of the decomposition representing the shortest and narrowest wavelets. Going from right to left, the next 512 points are the details from the 9th level. Again moving to the left, the next 256 values are the 8th level details. Newland calls the final two values (the two leftmost values of Figure 9) the –1th (–'one'th) and the 0th ('zero'th) level values.

The transform tells how to construct the signal from the wavelets. The positive or negative height of the wavelet is given by the value of the transform; where the wavelet value is placed in the transform shows the interval of time over which the wavelet applies. A shorter time interval means a narrower wavelet. The convention is as follows (now starting from the left):

The 1st value, called the –1th level, applies to the whole time interval. This is the DC level.

The 2nd value, called the 0th level, also applies to the whole time interval. It has part of the wavelet shape.

The 3rd and 4th values are the first level and apply to each half of the time interval respectively.

The 5th through the 8th values are the 2nd level and each applies to 1/4 of the time interval.

The 9th through the 16th values are the 3rd level and each applies to 1/8 of the time interval.

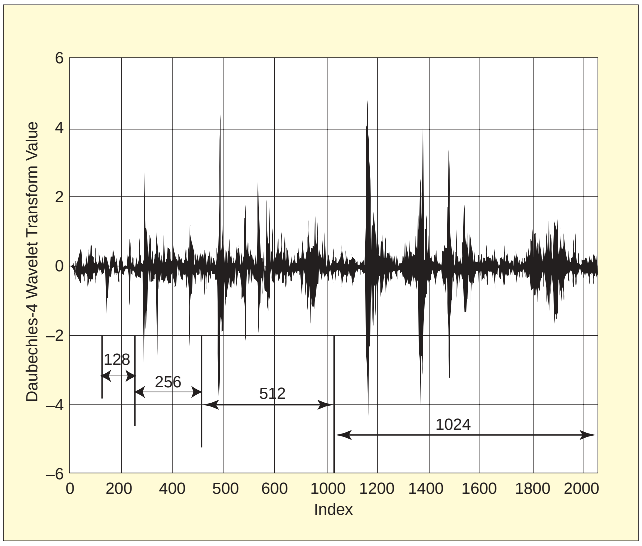

Figure 8. The Daubechies-4 wavelet transform of the compressor data (2048 values).

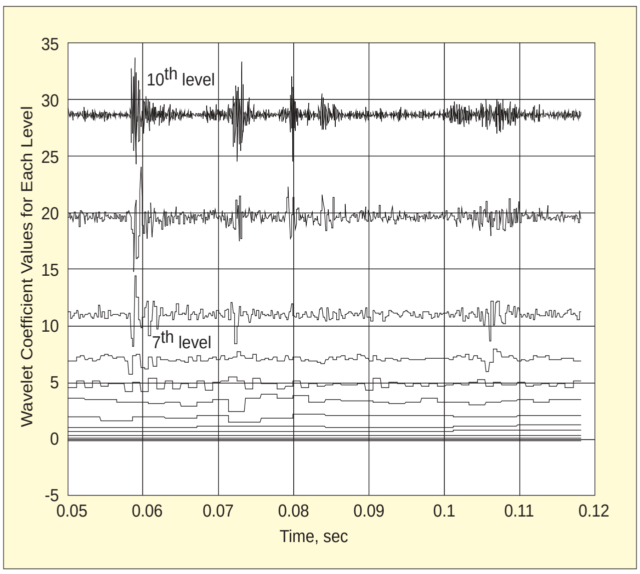

Figure 9. Daubechies-4 wavelet coefficients for each level of the wavelet analysis of the compressor data. The higher levels show the discontinuities.

The 17th through the 32nd values make up the 4th level and each applies to 1/16 of the time.

The 33rd through the 64th are the 5th level and each applies to 1/32 of the time.

The 65th through the 128th are the 6th level and each applies to 1/64 of the time.

The 129th through the 256th are the 7th level and each applies to 1/128 of the time.

The 257th through the 512th are the 8th level and each applies to 1/256 of the time.

The 513th through the 1024th are the 9th level and each applies to 1/512 of the time.

The 1025th through the 2048th are the 10th level and each applies to 1/1024 of the time.

And so on.

For example, the 304th value of a transform is in level 8 and occupies the 48th slot in the 256 time intervals.

To view the wavelet transform values graphically in a meaningful way, we should plot each level as a constant during the time values to which it pertains. Such a plot is presented in Figure 9 for the 2048 point compressor data. Consider the list of values plotted in Figure 8, a vector 'a.' The left most value (of Figure 8), a(1), is the –1th level and it applies to all the time values. It is the DC level of the transient and is shown as the bottom line on Figure 9. The second value, a(2), (again of Figure 8) the 0th level, also applies to all time values but has a portion of the wavelet shape. It is the second line from the bottom of Figure 9. The first level is the third line up from the bottom and has two values: a(3) and a(4). They are so close in value that the line appears to have a constant value. The four values of the second level (fourth line from bottom) can be barely seen; its fourth value is a little higher than the other three and shows as a step in the last quarter of the line. The eight values of the third level are plotted as the fifth line up from the bottom. Toward the right side of the figure a slight staircase can be seen with stairs 1/8 of the line length. The sixth line up from the bottom or the fourth level has 16 values that are more visible. The 32 values of the fifth level are even clearer. In this case I analyzed 2048 data values. This results in 1024 values in level 10, 512 in level 9, 256 in level 8, 128 in level 7 and so on down to two values in level 1. The top three levels pick out the discontinuities. Again, each of the values is the size or height of the transient. Where the value is placed in the 2048-value transform tells where the wavelet is located in time and how wide or narrow it is.

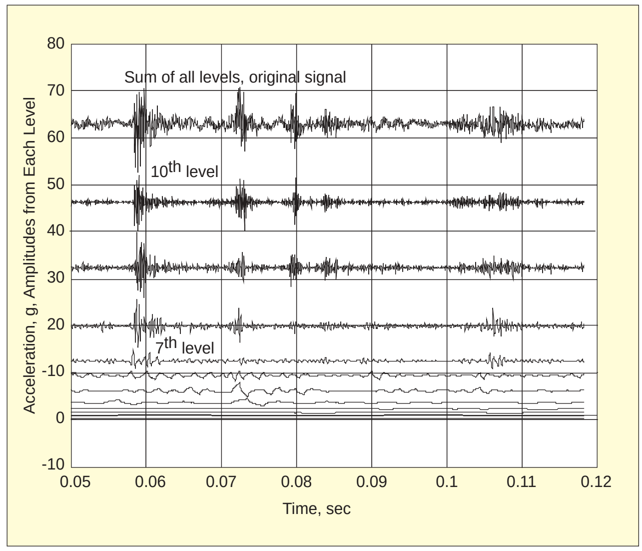

Figure 10. Daubechies-4 wavelet reconstruction of the original signal. The levels were separately inverse transformed to display the contribution of each level to the composition of the original signal.

Figure 10 shows the components of the discontinuities better and may be the best use of orthogonal wavelets for diagnostics. It shows the inverse transforms of the wavelet analysis level by level, so that you can see which part of the signal is built up by each level. The sum of the 12 levels exactly adds up to the original signal shown at the top of the figure.

Some Additional Insight

Suppose the WT is the program that does the wavelet transform on a vibration signal and the IWT is the program that does the inverse. A set of values, such as those plotted in Figure 8, is what the IWT program expects to receive. If we give the IWT program a transform of all zeros, it will transform them into a signal of all zeros. One way to understand what the WT accomplishes is to put single nonzero values in various positions of an all-zeros transform and then perform the IWT. Think of a signal that consists of 512 values. It has a transform 512 numbers long that we will call 'b.' If we make 'b' all zeros except for a '1' in the first position (the –1th level position) and perform IWT, we will get back 512 '1s.' The –1th level or first value is the DC value of our signal. If we make 'b' all zeros, except for a '1' in the sixth position (2nd level, 2nd position) and run IWT, we get back 512 values that are plotted in Figure 11 showing the Daubechies-4 wavelet. This procedure can be used to examine any of the wavelets.

Hubbard's 'elementary' book[9] states that the automatic fingerprint classification problem was solved at least in part by

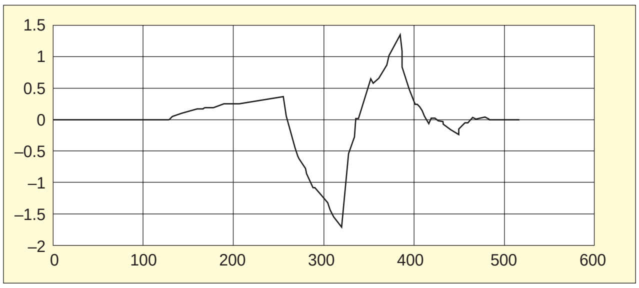

Figure 11. The Daubechies-4 wavelet in position 6, with a value of 1. It was formed by inverse transforming the 511 zeros and a single one in position 6.

wavelets. On page 105, she shows a plot of the time history of the microphone signal of the spoken word 'greasy,' sampled at 8000 times per sec. Below that plot is a matching pursuit analysis of the signal. ". . . Most of the signal is characterized by a few time-frequency atoms: although the original signal has 3000 samples, it can be reconstructed from only 250 atoms, with a signal to noise ratio of 40 dB . . ."

Hubbard's writing caused me to think of the FFT of a 45-minute microphone signal of some wonderful symphony, sampled at 44,000 Hz. This would be 118,800,000 samples. The next higher power of two is 27, giving us 2^{26} lines or 67,108,865 frequency values, amplitudes and phases. (When you FFT N even values, you get back $N/2+1$ distinct values, two of which are real and $N/2–1$ are complex. Thus N values are returned, but they only apply to $N/2+1$ frequencies.) If we perform an inverse FFT on them, we would get back the exact symphony. Somewhere in the symphony there would be a symbol crash. It would be very difficult to find in the FFT, because the FFT would just be 67 million amplitudes and phases. A wavelet approach would find the symbol crash and would even find a bearing signal or chipped tooth as well.

Other wavelet approaches veer away from the idea of a short, variable length wavelet convolved with the signal to be analyzed at various time locations. There are the wavelet packets and matching pursuits analysis. One analysis used these and proposed a dictionary of families of informative wavelets to and a procedure that would test different wavelet families on a machinery vibration signal to see which provided the most information.[14]

Conclusion

The Choi-Williams distribution seems to display time/frequency characteristics with the most precision. However, the short time Fourier transform provides a more accurate calculation of the amplitudes and might be best for slowly varying time/frequency characteristics. The Wigner distribution can locate time events and is quick to compute, but it does introduce spurious cross terms. The Morlet wavelet is a very logical analysis to perform and seems like it should produce an analysis as good as the STFT. In my testing of it, however, it seems to form suspicious vertical constant time ridges that suggest impacts not found in the other analyses. The Daubechies orthogonal wavelets can quickly accomplish time/frequency analyses. With further development, I expect they will become very useful. All of the analyses can be computed on a PC with high level signal processing software such as MATLAB. I made a few modifications to the programs of Reference 10 and would be happy to provide the m-files that were used to prepare Figures 9 and 10.

References

1. Forrester, B. D., "Time-Frequency Analysis in Machine Fault Detection," Chapter 18 of *Time-Frequency Signal Analysis Methods and Applications*, 1992, Ed. by Boualem Boashash, Wiley, New York, pp 406-423.
2. McFadden, P. D., and Wang, W. J., 1992, "Analysis of Gear Vibration Signatures by the Weighted Wigner-Ville Distribution," Institute of Mechanical Engineers, C432/134; pp 387-393.
3. Gaberson, H. A., "Application of Choi-Williams Reduced Interference Time-Frequency Distribution to Machinery Diagnostics," *Shock and Vibration*, Vol. 2, No. 6, pp 437-444, 1995.
4. Wang, W. J., and McFadden, P. D., "Application of Orthogonal Wavelets to Early Gear Detection Damage," *Mechanical Systems and Signal Processing*, Vol. 9(5), 1995, pp 497-507.
5. McConnell, K. G., *Vibration Testing; Theory and Practice*, John Wiley & Sons, Inc., 1995.
6. Choi, H., and Williams, W. J., "Improved Time-Frequency Representation of Multicomponent Signals Using Exponential Kernels," IEEE Trans. in Acoustics, Speech, and Signal Processing, Vol. 37, No. 6, pp 862-871, 1989.
7. Gaberson, H. A., "Impulsive Event Detection in Machinery Vibration Signals by Time-Frequency and Wavelet Methods;" Joint Oil Analysis Program International Condition Monitoring Meeting; Mobile, AL, April 2000.
8. Torrance, C., and Compo, G. P., "Practical Guide to Wavelet Analysis," Bulletin of the American Meteorological Society; Vol. 79, No. 1, Jan 1998. (This article is downloadable as a PDF file and their MATLAB software is available on their web site, http://paos.colorado.edu/research/wavelets/.)
9. Hubbard, B. B., *The World According to Wavelets*, 2nd Edition, A. K. Peters. Ltd., Natick, MA, 1988.
10. Newland, D. E., *An Introduction to Random Vibrations, Spectral and Wavelet Analysis*, 3rd Edition, Longman Scientific & Technical. Copublished in the United States by John Wiley & Sons, NY, 1993; pp 295-370.
11. Daubechies, Ingrid, "Orthonormal Bases of Compactly Supported Wavelets," *Communications on Pure and Applied Mathematics*, Vol. XLI 909-996, John Wiley & Sons, Inc., 1988.
12. Matlab for Windows, Version 4.2, 1994, High Performance Numeric Computation and Visualization Software, The MathWorks, Inc., Natick, MA.
13. Press, W. H., Teukolsky, W. T., Vetterling, W., and Flannery, B. T., *Numerical Recipes in Fortran*, 2nd Edition, Cambridge University Press, pp 584-599, 1992.
14. Liu, B., and Ling, S. F., "On the Selection of Informative Wavelets for Machinery Diagnosis," *Mechanical Systems and Signal Processing*, pp 145-162, Jan 1999.
15. Gaberson, H. A., "Machinery Diagnostic Application of the Morlet Wavelet Distribution," SPIE, The International Society for Optical Engineering, Aerosense Conference, Orlando, FL, April 2001.

The author can be contacted at: hagaberson@att.net.