Helpful Guidelines for Single-Axis Shaker Testing

Roman Vinokur, Westlake Village, California

This article presents common-sense guidelines for single-axis sweep sine and random vibration testing. Several actual case histories are included as well to demonstrate these techniques.

Single-axis shakers (mostly electrodynamic[1]) are widely employed for modal analysis, failure simulation and environmental testing in many research and design projects. In particular they are used for automotive and aerospace applications and (in combination with laser-vibrometers) for testing miniature high-technology MEMS sensors. A test engineer should have a good theoretical background to help prevent new designs from encountering long and costly feedbacks for design correction actions. Demonstration skills and teamwork with design and manufacturing engineers are also desirable. Many theoretical and experimental 'details' on shaker testing are not clearly covered, although they save time and assure high quality. This article presents helpful guidelines for sweep sine and random vibration testing, general relationships between natural and resonant frequencies and fatigue failure testing. Simple (but useful) equations are derived and applied to predict or interpret test parameters and results. The practical examples provided here are based on real case histories with names and project/test details changed.

Maximum Shaker Acceleration

Phil, a young dynamic testing engineer, was engaged in measuring the fundamental resonant frequencies of two aluminum condensers for an automotive air conditioning system. He bolted the condensers, with no rubber vibration isolators, symmetrically to an aluminum fixture as shown in Figure 1. He installed accelerometers on the top of each specimen and started a sweep sine test in the vertical direction at an excitation of 4 g pk and sweep rate of 0.5 octave/min. The sweep sine frequency range of 80-160 Hz was recommended by Jay, a finite element modeling engineer, who estimated the condenser's natural frequency to be 113 Hz. The acceleration seemed high for an experimental modal analysis but Phil did it on purpose to later employ the results on a fatigue test at an acceleration of 4 g pk. From an equation found in a manual, Phil calculated the maximum shaker acceleration

$$g_{\max} = \frac{\text{Shaker Force Rating}}{\text{Total Accelerated Mass}} = \frac{2000 \text{ lbf}}{24 + (2 \times 8) = 40 \text{ lb}} = 50 \text{ g pk} \qquad (1)$$

where the combined mass of the armature and fixture was 24 lb and the mass of one condenser was 8 lb. The manual stated, however, that the dynamic response of the fixture and test item may reduce the calculated rating but did not provide details. To play it safe, Phil introduced a "safety factor" of 5. But even in this case, the calculated value (50 g pk/5 = 10 g pk) was well above 4 g pk; the test setup seemed right. Nevertheless, the sweep sine test was automatically stopped within 1 min by the vibration controller because of an overload. It happened repeatedly and Phil addressed the problem to his supervisor Robert who restarted the test at an acceleration of 1 g pk and obtained experimental data for both condensers. The resonant frequencies measured 102 and 104 Hz (below the predicted value by approximately 10%) but the peak transmissibilities proved very high – 48 and 49. To explain what happened, Robert sketched a single-degree-of-freedom model[2] incorporating a mass, base and parallel spring and dashpot as shown in Figure 2. The base played the role of the shaker and fixture; the mass, spring and

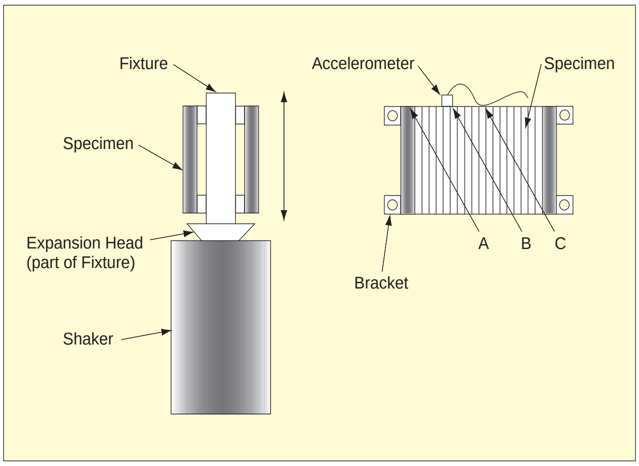

Figure 1. Test setup for 1-DOF shaker test on two condensers symmetrically installed on the fixture.

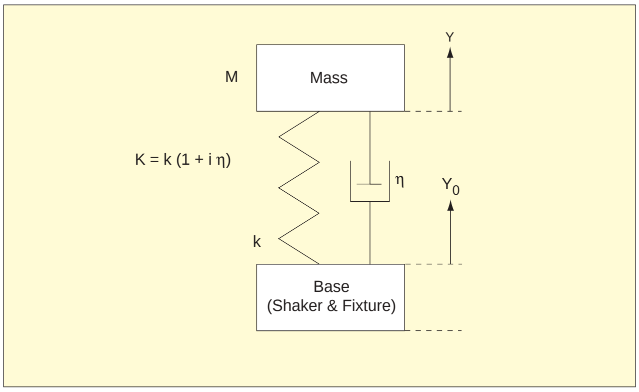

Figure 2. Single-degree-of-freedom vibration system. The damping element is a dashpot but in Equation (2), a more general case of damping is considered.

dashpot simulated the test unit parameters. Assuming that the base moved harmonically with a displacement $Y_0 = y_0 e^{i\omega t}$, so the differential equation for the displacement $Y = ye^{i\omega t}$ of the mass took the form:

$$M\ddot{Y} + K(Y - Y_0) = 0 \qquad (2)$$

where $\omega = 2\pi f$ was the angular frequency of vibration and $K = k\,(1+i\eta)$ was the complex spring constant combining the spring constant k and loss factor η. In the ideal case of viscous damping (Figure 3), the loss factor equals

$$\eta = 2\frac{c}{c_c}\frac{\omega}{\omega_n} \qquad (3)$$

where the quantities c and $c_c = 2\sqrt{Mk}$ are the coefficients of viscous damping and the critical damping, respectively; $\omega_n = 2\pi f_n = \sqrt{k/M}$ is the angular natural frequency and f_n is the natural frequency.

In general, the loss factor may be governed by multiple energy dissipation mechanisms. For the all-metal structures, the loss factor includes two main components: (1) the internal loss factor that appears in most solid materials with alternating stresses due to hysteresis and (2) the so-called structural loss factor depending on vibration energy absorption at junctures, edges and adjacent structures. It is also true for building elements: walls, window glazing, ceilings, etc.[3,4] The internal friction component is typically constant and the structural component normally decreases with frequency so the total loss factor tends to reduce with frequency (Figure 3). It is noteworthy in the literature, that damping in vibratory systems is commonly considered viscous for mathematical simplification and all other damping mechanisms (hysteresis, etc.) are modeled by an equivalent damping component obtained from equal energy considerations. It must be emphasized again that the damping component in Equation (2) may simulate multiple dissipation mechanisms. The solution of Equation (2) is

$$y = y_0 \frac{1+i\eta}{1-\left(\frac{f}{f_n}\right)^2 + i\eta} \tag{4}$$

From Equation (4), one can find the transmissibility

$$T = \left|\frac{y}{y_0}\right| = \left|\frac{a}{a_0}\right| = \sqrt{\frac{1+\eta^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2 + \eta^2}} \tag{5}$$

where $a = -y\omega^2$ and $a_0 = -y_0\omega^2$ are the accelerations of the mass and base, respectively. The transmissibility (Figure 4) is the modulus of the frequency response function defined as the Fourier transform of the output divided by the Fourier transform of the input. The frequency response function should not be confused with the transfer function defined as the Laplace transform of the output divided by the Laplace transform of the input. These terms are often used interchangeably and are sometimes a source of misunderstanding. The other important result following from Equation (4) is the amplitude of the force developed by the spring

$$F = \left|K(y-y_0)\right| = \left|\frac{k(1+i\eta)\omega^2 y_0}{\omega_n^2\left[1-\left(\frac{f}{f_n}\right)^2 + i\eta\right]}\right| = Ma_0\sqrt{\frac{1+\eta^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\eta^2}} = Ma_0 T \tag{6}$$

From Equation (6), the maximum spring force equals

$$F_{\max} = Ma_0 T_{peak} \tag{7}$$

(9) where $T_{\text{peak}} = T(f_r)$ if the resonant frequency f_r belongs to the test frequency range ($f_r \approx f_n$ provided that $\eta << 1$) and $T_{\text{peak}} \approx 1$ if the resonant frequency is well above the test frequencies. Using Equation (7), the maximum shaker acceleration limit can be expressed as

$$g_{\max} = \frac{\text{Shaker Force Rating}}{\sum_j M_j T_{\text{peak}\, j}} \tag{8}$$

where M_j and $T_{\text{peak}\, j}$ are respectively the mass and peak transmissibility of the single-degree-of-freedom structural components moved by the shaker (j = 1, . . .). Considering the armature and fixture like the systems with $T_{\text{peak}} \approx 1$ and substituting the numeric data into Equation (8), Robert calculated the maximum shaker acceleration as

$$g_{\max} = \frac{2000 \text{ lbf}}{[(24\times 1)+(8\times 48)+(8\times 49)]\text{lb}} \approx 2.5 \text{ g pk} < 4 \text{ g pk}$$

Thus, the shaker acceleration on Phil's test notably exceeded the maximum shaker acceleration. Indeed, test specimens and fixtures are not ideal single-degree-of-freedom structures with the same transmissibility at all points. Thus, the average transmissibilities should be utilized in Equation (8). For Phil's test specimen (Figure 1), the transmissibility is maximum at point C (in the central position) and close to 1 at point A (near one of the corner brackets bolted to the fixture). The transmissibility at point B (between points A and C) should be close to the average transmissibility used in Equation (8). In any case, Equation (8) is more accurate than Equation (1) if the transmissibilities can be obtained through the preceding shaker test at a lower acceleration or evaluated. It is noteworthy that Phil never measured such a high transmissibility in his prior experience. He commonly attached the specimens to the fixture through special rubber isolators with loss factors of 0.15-0.25 (peak transmissibilities ranged between 4 and 7). However, for all-aluminum or all-steel structures at relatively high frequencies, the loss factors may be as low as 0.001-0.010 in order of magnitude. For this test, the average peak transmissibility was 48.5 and according to Equation (5), that corresponds to a loss factor of 0.02. Structures with high peak transmissibilities may endure severe internal stresses at their resonant frequencies that can result in fatigue failure. It makes sense to increase vibration energy absorption in the test fixtures to attenuate inherent high-frequency resonances. Fixtures made of magnesium are superior because the internal loss factor for magnesium is notably higher than aluminum or steel.[5] However, in practice this is useful only at frequencies where the hysteresis outperforms the structural loss component.

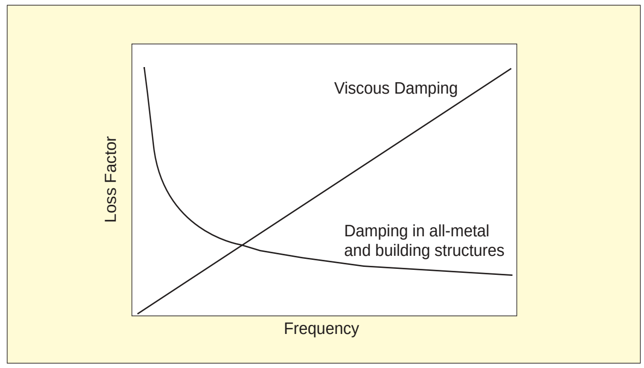

Figure 3. The loss factor dependence on frequency for two different damping mechanisms.

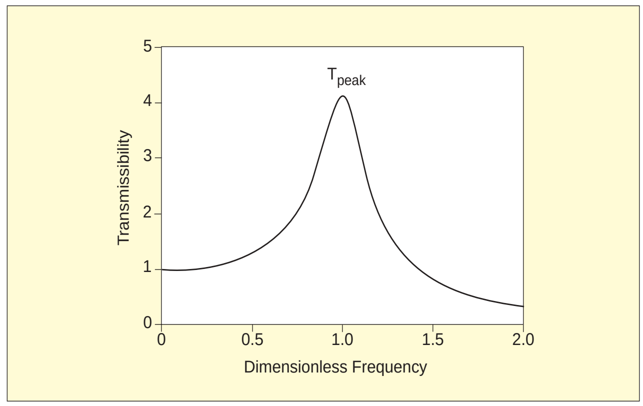

Figure 4. The transmissibility of the 1-DOF vibration system from Figure 2. The dimensionless frequency equals f/f_r.

Natural and Resonant Frequencies

The transmissibility of a real vibration system occurs not at its undamped natural frequency but at a somewhat different frequency called the resonant frequency or the frequency of maximum forced amplitude.[2] It is noteworthy that in books on vibration theory, the relationship between the two is analyzed only for the case of ideal viscous damping. For small values of damping, the frequencies are very close together, but from time to time they become a matter of dispute. It happened again when Jay learned that his prediction fell 10% above the average resonant frequency measured. In Jay's opinion, it was just a normal discrepancy between the natural (computed) and resonant (measured) frequencies rather than a deficiency of his computer model. To explain the real tendencies, Robert suggested that in Equation (5),

$$\frac{f}{f_n} = 1 + \varepsilon$$

$|\varepsilon| << 1$, and $\eta << 1$, and transformed Equation (5) to a simpler form

$$T \approx \sqrt{\frac{1}{4\varepsilon^2 + \eta^2}}$$

As follows from this equation, the peak transmissibility should be attained at a minimum of the function $z(\varepsilon) = 4\varepsilon^2 + \eta^2$. The necessary condition for the minimum is

$$\frac{dz}{d\varepsilon} = 8\varepsilon + 2\eta \frac{d\eta}{d\varepsilon} = 0$$

with the only solution

$$\varepsilon = \varepsilon_r = -\frac{\eta \frac{d\eta}{d\varepsilon}}{4}$$

Thus, the ratio of the resonant and natural frequencies is

$$\frac{f_r}{f_n} \approx 1 - \frac{\eta \frac{d\eta}{d(f/f_n)}}{4}$$

where both loss factor η and its derivative are calculated at the frequency $f = f_n$. Several important conclusions result from Equation (9):

1. If the loss factor, at frequencies equal or close to the natural frequency, is constant

   $$\frac{d\eta}{d(f/f_n)} = 0$$

   then the resonant frequency coincides with the natural frequency ($f_r = f_n$).
2. If, in the range mentioned, the loss factor grows with frequency

   $$\frac{d\eta}{d(f/f_n)} > 0$$

   then the resonant frequency is below the natural frequency ($f_r < f_n$).
3. If, in the range mentioned, the loss factor decreases with frequency

   $$\frac{d\eta}{d(f/f_n)} < 0$$

   then the resonant frequency exceeds the natural frequency ($f_r > f_n$).
4. In case of ideal viscous damping, the loss factor grows with frequency. Using Equations (3) and (9) one obtains

   $$\frac{f_r}{f_n} = 1 - \frac{\eta^2}{4}$$

   This equation clearly indicates that the relative difference between the resonant and natural frequencies is very low for small values of damping (for instance at $\eta = 0.1$, it comes to only 0.25%).
5. As mentioned previously for a number of solid structures, the loss factor tends to reduce with frequency (Figure 3). In this case, the resonant frequency is expected to exceed the natural frequency.

Jay checked the calculation, agreed and got back to his FEA model. For product development, computer modeling is mostly a good guess until the experiment makes the product speak for itself. Without a thorough theoretical analysis, neither experimental nor computer modeling can be fully successful.

Random Excitation Test

Phil's new project was a single-axis vibration fatigue test on the small auxiliary radiators (2 lb each) for trucks. The radiators were designed to be bolted to the truck's frame through two aluminum brackets with no rubber isolators. Two small radiators were attached to the shaker similar to their "in-vehicle" installation. As shown in Figure 5, the shaker was arranged in the "slip-table" mode to excite the radiators in their most vulnerable state. First of all, Phil performed a sweep sine test to measure the resonant frequency (31 Hz) and peak transmissibility (15). The fatigue failure test was to be performed with random excitation to closely simulate real environmental conditions. The project leader provided Phil with the averaged auto spectral density (ASD) of the acceleration measured on the truck frame during a special road test. The curve looked cumbersome, but in a rough approximation it can be depicted like a horizontal segment at a level of 0.01 g^2/Hz between 5 and 41 Hz and zero at all other frequencies (Figure 6). In this case, the total acceleration calculated was

$$a_{0rms} = \sqrt{0.01\text{g}^2/\text{Hz} \times (41-5)\text{Hz}} = 0.6 \text{ g rms}$$

For random vibration, the rms (root-mean-square) acceleration is indeed the standard deviation of the acceleration. Peak values may numerically exceed the rms value at least by a factor of 3. The peak acceleration from the road test report ranged up to 3 g. To implement accelerated life testing, shaker acceleration should be increased to appropriately excite stresses in the specimen. Phil reasonably suggested that the most significant stresses develop at the resonant frequency (31 Hz) and calculated the total numbers of cycles for the product lifetime (10 years) and accelerated test duration (24 hours). Using a fatigue strength dependence on the total number of cycles,[6] he calculated an increased excitation factor of 3. So, the auto spectral density of the shaker acceleration should be increased to S^2 = 0.09 g^2/Hz in the same frequency range (5-41 Hz). The calculated total acceleration in this case was 1.8 g rms. The next important step was to check that the shaker total displacement did not exceed the maximum peak-to-peak (pk-pk) displacement. The shaker was designed for a maximum displacement of 2 in. pk-pk, but it was recommended not to exceed 75% of that value for long tests. The maximum permissible displace-

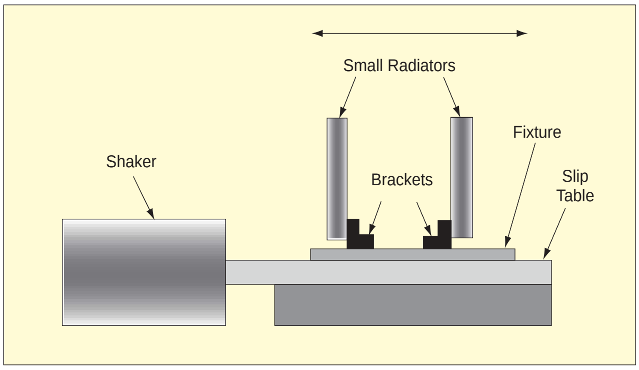

Figure 5. Test setup in the "slip-table" mode.

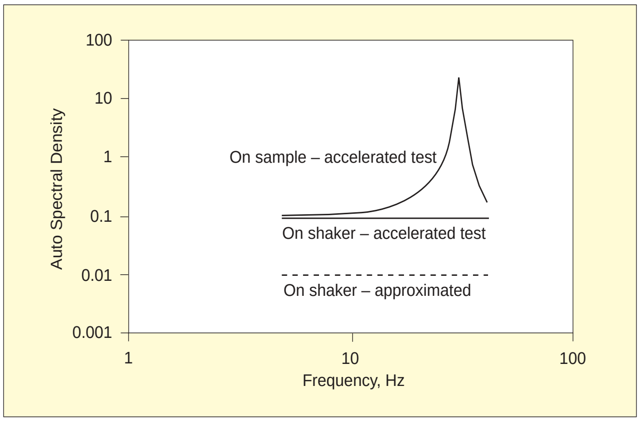

Figure 6. The auto spectral density g^2/Hz of the acceleration of the shaker and sample.

ment was considered to equal 1.5 in. or 38 mm pk-pk. Phil knew a simple equation

$$y_0 = -\frac{a_0}{(2\pi f)^2}$$

interrelating the peak displacement and acceleration on sine shaker tests. But what if the acceleration and displacement are random functions?

Robert helped again. As follows from the previous equation, the auto spectral density for the displacement should be

$$S_y^2 = \left| -\frac{1}{(2\pi f)^2} \right|^2 S_a^2$$

where $S_a{}^2$ is the auto spectral density for the acceleration. After that, the standard deviation for the displacement was calculated as

$$D = \sqrt{\int_0^\infty S_y^2(f)df} = \frac{S}{4\pi^2}\sqrt{\int_L^U \frac{df}{f^4}} = \frac{S}{4\sqrt{3}\pi^2}\sqrt{\frac{1}{L^3} - \frac{1}{U^3}} =$$
$$\frac{0.3}{4\sqrt{3}\pi^2}\sqrt{\frac{1}{5^3} - \frac{1}{41^3}} \text{ g rms/Hz}^2 \approx 3.9 \text{ mm rms}$$

where L and U equals 5 and 41 Hz, respectively. The pk-pk value may numerically exceed the rms value more than 6 times but even with a factor of 9, the shaker pk-pk displacement was below the permissible value of 38 mm. As for the maximum shaker acceleration limit in random mode, it should not be a problem for this test based on Robert's experience.

Design Improvement

Phil started the vibration fatigue test in random mode. It proved relatively short. Within 25 minutes, all the specimens suffered the same critical failure – broken brackets. Phil bolted new specimens to the fixture through the rubber washers for vibration energy absorption but with no significant improvement. For effective vibration energy isolation, the rubber isolators should be somewhat loose. When compressed, they are not very effective. A design modification seemed unavoidable to fix the problem. It was not good news for the company because the small radiators should have already been at the production stage and the manufacturing engineers were waiting for the shaker test results as the final 'go' condition. Fortunately, Phil found the no-delay and no-cost engineering solution.

During the test, Phil observed the auto spectral density of the signals measured by the accelerometers on the test units. Due to a notable peak caused by the resonance (Figure 6), the rms acceleration on the units proved as high as 10 g rms. It seemed beneficial to move the resonant frequency beyond the effective frequency range of 5-41 Hz. At an engineering meeting, he suggested that steel brackets of the same shape be used in place of the aluminum ones. It proved very convenient because there was no need to change the existing tooling equipment. However, Jay had a concern. The ultimate tensile strength of the cheap steel recommended by Phil was just twice that of the aluminum material in the existing brackets. It may not be sufficient to resist fatigue failure and would have to be experimentally confirmed. However, Phil was positive that the selection of the material was right because: (1) available tooling may fail bending a stronger steel; (2) it should be a no-cost modification in order to save the company's money; and (3) most importantly with the steel brackets, the natural frequency is well outside the effective frequency range. Since the brackets play the role of springs, the natural frequency of the radiator with the steel brackets was evaluated as

$$f_{n\ \mathrm{Steel}} = f_{n\ \mathrm{Al}}\sqrt{\frac{E_{\mathrm{Steel}}}{E_{\mathrm{Al}}}} \approx 31 \text{ Hz}\sqrt{\frac{200 \text{ GPa}}{70 \text{ GPa}}} \approx 52 \text{ Hz} \qquad (10)$$

where $f_{n\ \mathrm{Al}}$ is the measured natural frequency of the radiator with the aluminum brackets, E_{Al} is the Young's modulus of aluminum, and E_{Steel} is the Young's modulus of steel. The project leader approved the proposal and it was implemented with no delay. The shaker test on the new samples was promptly completed, the results proved very satisfactory, and mass production moved forward.

Summary

This article, written in the form of interrelated case stories based on the author's experience, presents some feasible techniques for single-axis shaker testing, an important and sometimes critical part of research, design and manufacturing projects. The simple (but effective) equations can help vibration engineers in creating sweep sine and random tests, interpreting the experimental data and developing improvement proposals. The relationship between resonant and natural frequencies, commonly described only in the case of viscous damping, may be of more general interest. In addition to experimental and theoretical facts, this article illustrates the benefits of creative engineering teamwork.

References

1. Lang, G. F., and Snyder, D., "Understanding the Physics of Electrodynamic Shaker Performance," *Sound and Vibration*, Oct. 2001, pp.24 - 33.
2. Den Hartog, J. P., *Mechanical Vibrations*, Dover Publications, Inc. edition, 1985.
3. Vinokur, R., "Influence of the Edge Conditions on the Sound Insulation of a Thin Finite Panel," *Soviet Physics – Acoustics* (USA), 1981, Vol. 26, No. 1, pp. 72-73.
4. Craik, R., "Damping of Building Structures," *Applied Acoustics*, 1981, Vol. 14, No. 5, pp. 347-359.
5. Vibration Test Equipment, Fixture Design Notes by John Raymond IV, Unholtz-Dickie Corporation.
6. Linderberg, Michael R., *Mechanical Engineering Reference Manual*, 9th edition, recommended by the National Society of Professional Engineers, 1994.

The author can be reached at rvinokr@aol.com.