Evaluating Vibration Environments Using the Shock Response Spectrum

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The shock response spectra computed directly from the time histories of simulated stationary vibration data are compared to the expected values for the maximum response peaks computed from the autospectra of the data. The data are assumed to be random with a Gaussian probability density function and the results reveal excellent agreement when that is truly the case. However, for the complex vibration environment generated by a repetitive shock machine, the shock response spectra computed directly from the time history data are higher than the expected values. This indicates that for the same autospectrum, the damage potential of a repetitive shock machine is greater than that for a truly random vibration. The same conclusion undoubtedly applies to many other complex but not random vibration environments such as those produced by reciprocating engines.

The shock response spectrum (SRS) is broadly defined as the peak response of a simple oscillator (single degree-of-freedom system) to an excitation as a function of the natural frequency of the oscillator.[1] It was originally introduced to evaluate the damage potential of mechanical transients, but can also be used to evaluate the damage potential of stationary random vibrations as measured by the peak value for the response of a simple oscillator exposed to the vibration over a finite duration.[2,3] This latter application of the SRS directly competes with the use of statistical procedures to predict the peak value for the response of a simple oscillator under the assumption that the excitation to the oscillator is a stationary random process.[2-4] Since the SRS does not require the excitation to be random, a direct comparison of SRS results to a statistical computation of the maximum value of the oscillator response can be used to evaluate the randomness of the excitation. Such a comparison has been used to detect transients in otherwise stationary random vibration signals.[5] Of interest here is the use of such a comparison to detect differences between the damage potential of random vibrations versus the complex (sometimes called quasi-random) vibrations produced by pneumatic hammer-type vibration test machines, commonly referred to as repetitive shock machines.

Statistical Computation of Shock Response Spectrum

The computation of the shock response spectrum (SRS) for a stationary random vibration involves the determination of the maximum value for the response of a lightly damped, linear oscillator to the excitation, as illustrated in Figure 1a. Assuming an acceleration excitation *x(t)* produces an acceleration response *y(t)*, the frequency response function of the simple oscillator is given by[6]

$$H_{xy}(f) = \sqrt{\frac{1 + j2\zeta f_n}{1 - \left(\frac{f}{f_n}\right)^2 + j2\zeta f_n}} \tag{1}$$

where f_n is the undamped natural frequency and ζ is the damping ratio of the oscillator. Assuming the acceleration excitation *x(t)* is random with an autospectrum $G_{xx}(f)$, the response *y(t)* of the oscillator will be random with a narrow bandwidth, as shown in Figure 1b, and will have a standard deviation approximated by[6]

$$\sigma_y = \sqrt{\int_0^\infty \left|H_{xy}(f)\right|^2 G_{xx}(f)\,df} \tag{2}$$

If it is further assumed the autospectrum of the excitation is relatively uniform at frequencies near the natural frequency f_n, then Equation 2 is closely approximated by[6]

$$\sigma_y = \sqrt{\frac{G_{xx}(f_n)\pi f_n\left[1 + 4\zeta^2\right]}{4\zeta}} \tag{3}$$

It should be mentioned that the standard deviation of the oscillator response is often computed in terms of velocity rather than acceleration,[2,3] because the stress produced by the resonant response of a structure is proportional to velocity.[7] For the application at hand, however, only comparisons of relative values are of interest, so acceleration units are used since they are more familiar to most shock and vibration engineers.

It is recommended that a conservative maximum value for the response of the oscillator to a stationary random excitation be estimated by[2,3]

$$Y_m\left[P(T)\right] = \sigma_y\sqrt{2\ln\left[\frac{f_n T}{P(T)}\right]};\ P(T) \ll 1 \tag{4}$$

where
- f_n = undamped natural frequency of the oscillator
- T = duration of the excitation
- σ_y = standard deviation of the oscillator response, as defined in Equation 2
- $P(T)$ = probability that the value Y_m will be exceeded during the exposure duration T

For design purposes, a probability of $P(T) = 0.05$ (5%) is commonly assumed in Equation 4.[2,3] However, to estimate an SRS, the expected value of Y_m is needed since it corresponds to the average value of the SRS as normally computed. The expected value and standard deviation for the maximum response of the oscillator is estimated by[4]

$$E[Y_m] = \sigma_y\left[\sqrt{2\ln(f_n T)} + \frac{0.5772}{\sqrt{2\ln(f_n T)}}\right];\ \ln(f_n T) \gg 1 \tag{5}$$

$$\sigma[Y_m] = \frac{1.28\sigma_y}{\sqrt{2\ln(f_n T)}};\ \ln(f_n T) \gg 1 \tag{6}$$

where all terms are as defined in Equation 4. It follows that the normalized random error (coefficient of variation) for an estimate of the maximum response of the oscillator is given by

$$\varepsilon_r\left[\hat{Y}_m\right] = \frac{\sigma[Y_m]}{E[Y_m]} = \frac{1.28}{2\ln(f_n T) + 0.5772} \tag{7}$$

where the hat (^) denotes "estimate of." Note from Equation 7 that the normalized random error for estimates of the maximum response value will be less than $\varepsilon_r < 0.10$ (10%) for $f_n T > 150$.

The results in Equations 4 through 7 involve two critical assumptions:

1. The response *y(t)* of the oscillator has a normal (Gaussian) probability density function. As long as the excitation *x(t)* is random and the oscillator response is linear, even if *x(t)* is not Gaussian, this assumption is often acceptable because the narrow bandwidth filtering of the oscillator suppresses deviations from the Gaussian form in the response *y(t)*.[8]
2. The peak values of the oscillator response are statistically independent. It is clear from the relatively smooth variations in the envelope for the peak values of the oscillator response in Figure 1b that the peak values are not statistically inde-

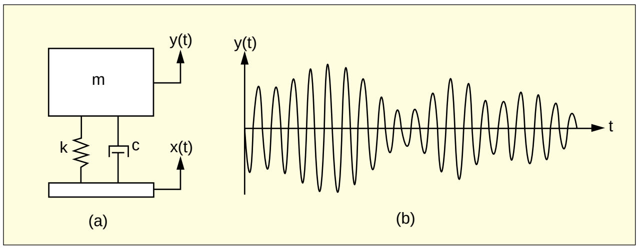

Figure 1. Illustration of simple oscillator: (a) schematic diagram. (b) time history response to random excitation.

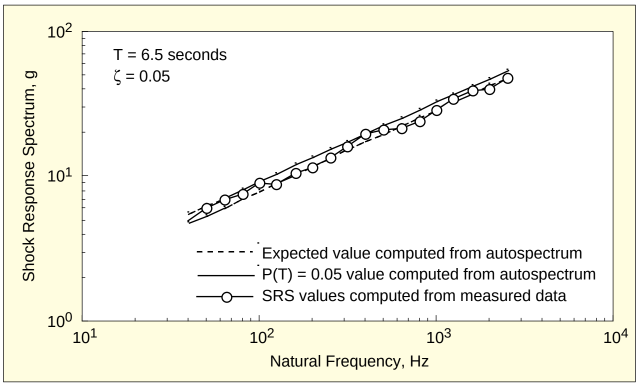

Figure 2. Predicted versus computed shock response spectrum for Gaussian random data.

pendent. However, computer simulation studies indicate the statistical independence assumption is acceptable for values of $Y_m/\sigma_y > 3.5$, which corresponds to $f_n T > 250$ for a Gaussian random process.[2]

Comparisons of Estimated and Computed Results

Computations were performed on data produced by two sources: (a) a computer generated random signal simulating a Gaussian random vibration, and (b) the signal from an accelerometer mounted on the table of a commercial repetitive shock (RS) machine. In both cases, an autospectrum (PSD) was computed using a conventional fast Fourier transform (FFT) based PSD analysis algorithm,[6] and a shock response spectrum (SRS) was computed using the "Ramp-Invariant Method."[9] All analyses were performed over T = 6.5 sec of data that were digitized using a sampling rate of 25,000 samples per sec. The lower frequency limit for the analyses was fixed to 40 Hz to comply with the second assumption after Equation 7. The upper frequency limit was fixed at 2500 Hz (10% of the sampling rate) to restrict the magnitude error in the SRS values to less than 5%[10] as well as to suppress an inherent bias error in the Ramp-Invariant Method.[11] Damping ratios of 5% (ζ = 0.05 corresponding to Q = 10) and 1% (ζ = 0.01 corresponding to Q = 50) were used for all standard deviation and SRS computations. The frequency resolution for the analyses was 10 Hz for the PSD values and 1/12 octave band for the SRS computations. However, all PSD and SRS results are presented at 1/3-octave band center frequencies for clarity.

Random Data. The simulated random vibration data were generated with a PSD of $G_{xx}(f)$ = 0.003 g^2/Hz over the frequency range from 40 to 2500 Hz. The directly computed SRS for the simulated random vibration data, the expected value for the maximum response given by Equation 5 and the $P(T)$ = 0.05 value given by Equation 4, all computed with 5% damping, are compared in Figure 2. Note in Figure 2 that the directly computed SRS values are in good agreement, on average, with the predicted values of Equation 5 and are just enveloped by the $P(T)$ = 0.05 values of Equation 4, as would be expected with 19 SRS values. It is clear from these results that Equations 3 and 4, using the standard deviation computed from Equation 2, provide accurate results for truly random data.

RS Machine Data. The probability density function for the table vibration produced by the repetitive shock (RS) machine used for these studies is shown in Figure 3. Note that the probability density function for the table motion in terms of acceleration values deviates substantially from the Gaussian form. These results are consistent with the findings in Henderson[12] for a different RS machine and, further, would be intuitively anticipated for a vibration response that is produced by a sequence of transients rather than a stationary random vibration. However, it should be mentioned that these particular RS machines are both of an early design. Studies of an RS machine of more recent design revealed a table motion that is much closer to the Gaussian form.

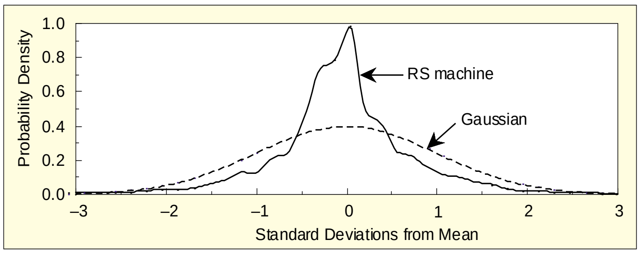

Figure 3. Probability density function for the table vibration of a repetitive shock machine.

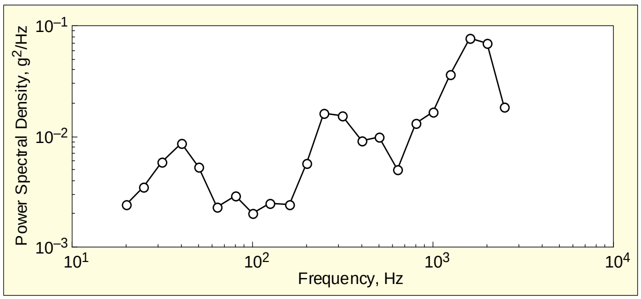

Figure 4. Power spectral density function for the table vibration of a repetitive shock machine.

The PSD values of the vibration generated by the RS machine at the 1/3-octave band center frequencies used for the computations in Equations 2 through 5 are shown in Figure 4. The directly computed SRS values for the RS machine vibration data, the expected value of the maximum response given by Equation 5 and the $P(T)$ = 0.05 value given by Equation 4, all computed with 5% damping, are compared in Figure 5. A similar comparison of the values computed with 1% damping are shown in Figure 6. From the results in Figure 5, it is seen that the directly computed SRS values with 5% damping exceed the predicted values of Equations 4 and 5 at most frequencies by margins of up to 3:1. It is clear that Equations 4 and 5, using the standard deviation computed from Equation 2, do not provide accurate results for this particular RS machine vibration. That is, a component exposed to the excitation of the RS machine would have a substantially higher peak response than predicted by either Equation 4 or 5. On the other hand, the results with 1% damping in Figure 6 also reveal higher computed SRS values than predicted by Equations 4 and 5 at most frequencies, but by smaller margins.

The results in Figures 5 and 6 might be explained as follows. Repetitive shock machines produce what is essentially a complex periodic vibration with natural and sometimes intentionally introduced random modulations of both magnitude and frequency. Hence, the vibration does have a limited random character. The half-power point bandwidth for a simple oscillator is approximated by[6] $B_{hp} \approx 2\zeta f_n$ where the bandwidth of the oscillator is directly proportional to the damping ratio. The narrow bandwidth filtering operation of the simple oscillators essentially invokes the Central Limit Theorem and thus suppresses deviations from the Gaussian form where the narrower the bandwidth, the greater the suppression of non-Gaussian characteristics.[8] It follows that the narrower bandwidth for the 1% damping produces a more Gaussian response that makes Equations 2 through 4 more accurate. This conclusion is con-

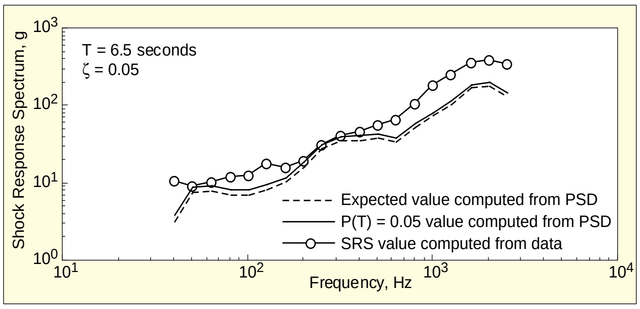

Figure 5. Predicted versus computed shock response spectrum for the table vibration of a repetitive shock machine. Assumed damping ratio = 5%.

sistent with findings that show the response of lightly damped cantilever beams exposed to the vibration produced by an RS machine is quite close to the Gaussian form, at least at natural frequencies above about 500 Hz.[13]

Conclusions

The shock response spectra computed directly from the time histories of vibration data were compared to the expected value for the maximum response peaks computed from the autospectra of the data, assuming the data are random with a Gaussian probability density function. The results reveal excellent agreement for simulated vibration data that are in fact random in character. However, for the complex vibration environment generated by a repetitive shock machine, the shock response spectrum values computed directly from the time history data with 5% damping (a common damping ratio for many structural assembles) are substantially higher (by up to 3:1) than the expected value. The discrepancies between the directly and indirectly computed shock response spectra are smaller when computed with 1% damping but the directly computed results still exceed the indirectly computed results at most frequencies. This indicates that for the same autospectrum, the damage potential of a repetitive shock machine is greater than that for a truly random vibration. This same conclusion undoubtedly applies to many other complex, but not truly random vibration environments, such as those produced by reciprocating engines.

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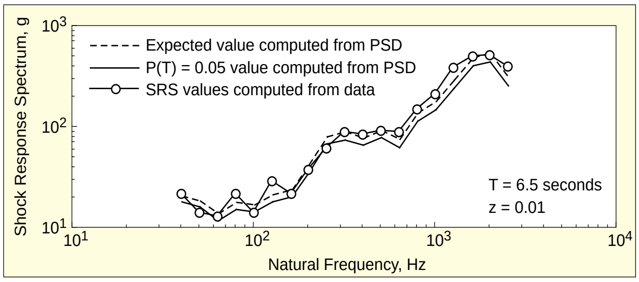

Figure 6. Predicted versus computed shock response spectrum for the table vibration of a repetitive shock machine. Assumed damping ratio = 1%.

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