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A Brief History of Early Rotor Dynamics

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Rotor dynamics has a remarkable history, largely due to the interplay between its theory and its practice. Indeed, one could argue that rotor dynamics has been driven more by its practice than by its theory. This statement is particularly relevant to the early history of rotor dynamics and its explication is the central theme of this brief review.

First a caveat – reconstructing the origins of a methodology is a challenging task. Readers will have their own ex post facto interpretations of the historical literature. So be aware that the following contains some of mine. In several cases, I have followed the interpretations of Neville Rieger.[1]

Rankine to Kerr. W. J. Macquorn Rankine performed the first analysis of a spinning shaft in 1869.[2] He chose an unfortunate model (see Figure 1) and predicted that beyond a certain spin speed ". . . the shaft is considerably bent and whirls around in this bent form." He defined this certain speed as the "whirling speed" of the shaft. In fact, it can be shown that beyond this whirling speed the radial deflection of Rankine's model increases without limit.[3] Today, this speed would be called the threshold speed for divergent instability. However, Rankine did add the term 'whirling' to the rotor dynamics vocabulary.

In 1895, Stanley Dunkerley published a study of the vibration of shafts loaded by pulleys.[4] The first sentence of his paper reads, "It is well known that every shaft, however nearly balanced, when driven at a particular speed, bends, and, unless the amount of deflection be limited, might even break, although at higher speeds the shaft again runs true. This particular speed or 'critical speed' depends on the manner in which the shaft is supported, its size and modulus of elasticity, and the sizes, weights, and positions of any pulleys it carries." As far as I know, this is the first use of the term "critical speed."

It is regrettable that what Dunkerley regarded as well known was actually little known. For example, few – if any – practitioners of that day were aware of the 1895 analysis by the German civil engineer August Föppl who showed that an alternate rotor model exhibited a stable solution above Rankine's whirling speed.[5] We cannot blame them too much since Föppl published his analysis in *Der Civilingenieur*, a journal that was probably not well known by contemporary rotor dynamicists. More telling was the apparent indifference to the practical work of the Swedish engineer, Carl G. P. De Laval, who in 1889 ran a single stage steam turbine at a supercritical speed.

One can speculate that engineers of the day labored under a confusion of concepts – equating Rankine's whirling speed with Dunkerley's critical speed. This was particularly unfortunate since Rankine was far more eminent than Dunkerley and, as a result, his dire predictions were widely accepted and became responsible for discouraging the development of high speed rotors for almost 50 years.

It was in England in 1916 that things came to a head. W. Kerr published experimental evidence that a second "critical speed" existed,[6] and it was obvious to all that a second critical speed could only be attained by the safe traversal of the first critical speed. The Royal Society of London then commissioned Henry H. Jeffcott to resolve this conflict between Rankine's theory and the practice of Kerr and De Laval.

Jeffcott to Prohl. Jeffcott published his classic paper in 1919,[7] in a place (*Philosophical Magazine*) where it is was more likely to be read by those interested in rotor dynamics. His rotor model is depicted in Figure 2.

Jeffcott confirmed Föppl's prediction that a stable supercritical solution existed and he extended Föppl's analysis by including external damping (i.e., damping to ground). There is no evidence that Jeffcott was aware of Föppl's prior work; in fact, Jeffcott's paper does not contain a single reference. To this day, at least in the USA and the UK, a rotor model consisting of rigid disks on a compact, flexible shaft is called a Jeffcott rotor.

In 1924, Aurel B. Stodola showed that these supercritical solutions were stabilized by Coriolis accelerations.[8] The unwitting constraint of these accelerations was the defect in Rankine's model. It is interesting to note that Rankine's model is a sensible one for a rotor whose stiffness in one direction is much greater than its stiffness in the quadrature direction. Indeed, it is now well known that such a rotor will have regions of divergent instability. It is less well known that Ludwig Prandtl was the first to study a Jeffcott rotor with a non-circular cross-section, publishing this work in 1918.[9]

However, the rotor dynamicist's respite from worrying about instability was brief. In the early 1920s a supercritical instability in built-up rotors was encountered and, shortly thereafter, shown by A. L. Kimball to be a manifestation of rotor internal damping (i.e., damping between rotor components).[10] Kimball's explanation of this counter-intuitive phenomenon is quite convoluted and more recent explanations are to be preferred. Then, B. L. Newkirk and H. D. Taylor described an instability caused by the nonlinear action of the oil wedge in a journal bearing, which was dubbed "oil whip."[11] The phenomenon of oil whip and its precursor, oil whirl, are still imperfectly understood despite the subsequent efforts of numerous investigators. Other instabilities have since been discovered. Prominent among these are those due to cross-coupling stiffnesses in bearings and seals and steam whirl, which can also occur in gas turbines.

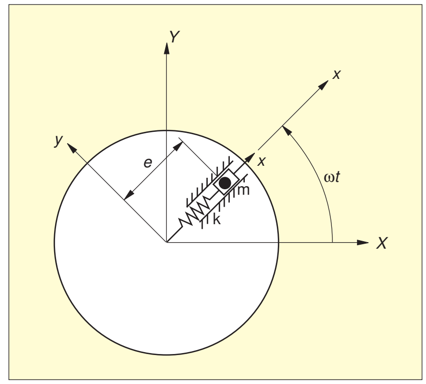

Figure 1. A lumped constant model that is dynamically equivalent to Rankine's shaft model.

In Jeffcott's analytical model the disk did not wobble. As a result, the angular velocity vector and the angular momentum vector were colinear and no gyroscopic moments were generated. This restriction was removed by Stodola in 1924.[12]

In 1933 David M. Smith obtained surprisingly simple formulas that predicted how the threshold spin speed for supercritical instability varied with bearing stiffness and with the ratio of external to internal viscous damping.[13] To quote from Smith's paper " . . . [the] increase of dissymmetry of the bearing stiffness and in the intensity of [external] damping relative to [internal] damping raises the [threshold] speed . . . and [this threshold] speed is always higher than either critical speed." The formula for damping was obtained independently by Stephen Crandall some 40 years later.

Gradually, the Jeffcott rotor model, in its many variations, came closer to the practical needs of the rotor dynamicists of the day. But, not close enough. Many practical rotors, especially those being designed for aircraft gas turbines, were not suitable for a Jeffcott model. For one thing, the distinction between disk and shaft is blurred in the typical aircraft gas turbine. A more general modeling technique was needed. This was supplied by Melvin Prohl in the late 1930s and published in 1945.[14] It is similar to the method published about the same time by N. O. Myklestad for the dynamic analysis of aircraft wings but was developed

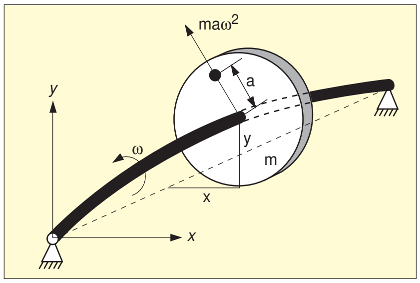

Figure 2. Jeffcott's 1919 model: a rigid disk centrally located on a circular, flexible shaft supported on rigid bearings.

independently. Of course, Prohl's method was designed for a rotating structure rather than a stationary one. Together, Prohl's and Myklestad's work led to a broader method, now called the Transfer Matrix Method (TMM). The TMM for rotors remains viable; indeed, it seems still to be the method of choice for most industrial rotor dynamic analyses.

The Modern Era. I consider World War II as the demarcation between the early stages of rotor dynamics and what might be called modern rotor dynamics. I believe this is the consequence of two factors. First, there was a growing awareness of the contributions of rotor dynamicists from non-English speaking countries, e.g., F. M. Dimentberg in Russia, Alex Tondl in Czechoslovakia, Erwin Kramer in Germany, and Toshio Yamamoto in Japan, among many others. Clearly, after WWII, rotor dynamics had become an international endeavor, a fact that was recognized by the founding of the Rotor Dynamics Committee of the International Federation of the Theory of Machines and Mechanisms (IFToMM). Beginning in 1982, international conferences have been organized by this committee in Rome, Tokyo, Lyon, Chicago, Darmstadt and Sydney.

Second, there was a revolution in solution capability; a transition from somewhat simplified models to almost actual geometry. In the 1960s there was a coalescence of numerical methods applied to structural dynamics and of digital computer capacity that fostered the development of a series of general purpose computer codes. The initial application of these codes to rotor dynamics was based on the TMM method but in the 1970s another underlying algorithm, the Finite Element Method (FEM), became available for the solution of the prevailing beam-based models. Now, in the beginning of the 21st century, rotor dynamicists are combining the FEM and solids modeling techniques to generate simulations that accommodate the coupled behavior of flexible disks, flexible shafts and flexible support structures into a single, massive, multidimensional model. We are now a long way from the approaches of Jeffcott and Prohl, a journey that deserves its own history sometime.

Conclusion. The underlying theme of this brief review has been that an understanding of the behavior of rotating machinery is best achieved through an interplay between theory and practice. Rotor dynamics are not unique in this regard but do offer an unusual number of vivid examples. Lately, the balance between analysis and experience has been affected by the emergence of modern computational tools. Lest a fascination with these tools replace the lessons of practice, we would be wise to heed the words of Dara Childs,[15] "the quality of predictions from a computer code has more to do with the soundness of the basic model and the skill and physical insight of the rotor dynamicist than the particular algorithm used. Good engineers get good results from good models, leading to sound engineering judgments with a variety of algorithms or computer codes. Superior algorithms or computer codes will not cure bad models or a lack of engineering judgment."

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