Gabor Expansion for Order Tracking

Shie Qian, National Instruments Corporation, Austin, Texas

Automotive and machinery reliability engineers rely heavily on order analysis for examining rotating machinery. While many different techniques for order analysis have been developed, this article introduces the Gabor expansion-based order tracking approach. Compared to other existing order analysis methods, this technique is not only more intuitive and more powerful, but can also be used for applications where rotational speed information is not available.

Automobile and machinery engineers regard order analysis as one of the most powerful methods for analyzing rotating machinery (see front cover). In contrast to the time-invariant harmonic, which uses Fast Fourier Transforms (FFTs), engineers traditionally achieve order analysis by resampling or using an adaptive filter-based technique, such as the Vold-Kalman filter.[7] In this article, we introduce the Gabor expansion as an alternative to resampling or adaptive filter-based methods for order analysis problems. In contrast to the Vold-Kalman filter, the Gabor expansion is more intuitive, more powerful and can even be used when rotational speed information is not available.

This article begins with a discussion of some real world challenges faced by industries using order tracking analysis. Next, the limitations of some FFT-based methods are addressed and the Gabor expansion-based solution is presented. Applications from various industrial equipment manufacturers will illustrate the effectiveness of order analysis. Finally, the article concludes by comparing analyses performed by Gabor expansion and adaptive filter-based approaches.

Real World Challenges

Many industries rely on order analysis during the design, manufacturing and testing phases of product development. Before introducing the Gabor expansion, we will first briefly review the basics of classical time-invariant analysis and the challenge for time-varying harmonic analysis.

Harmonics usually refer to frequencies that are integer (or fractional) multiples of a fundamental frequency. A simple example encountered in everyday life is the sound generated by a running engine and its combination of various vibrations. Although the causes of vibrations are very different, some may be associated with the bearings and others may be associated with the cooling fan. The vibration frequencies are all functions of the fundamental frequency – engine rotation speed or RPM (revolutions per minute). For instance, the vibration frequency related to the bearing may be equal to the fundamental frequency multiplied by the number of balls inside the bearing housing. The vibration frequency related to the cooling fan may be equal to the fundamental frequency multiplied by the number of blades. Traditionally, automobile and machinery reliability engineers name the rotational speed (the fundamental frequency) as the first order and a related m^{th} harmonic as the m^{th} order. Accordingly, the resulting harmonic analysis is referred to as order analysis. By analyzing the amplitudes and phases of different orders, engineers can often determine whether an engine is running normally.

For an illustrative look at the uses of order analysis, consider the cover photograph, which depicts a factory acceptance test of a 3000 HP, 3600 RPM induction motor manufactured by TECO-Westinghouse. Using National Instruments PXI hardware and LabVIEW™ software, engineers created a test system capable of order tracking and analysis as well as balancing and electrical performance testing. To illustrate the need for order analysis, we conducted a similar test on a 4-pole electrical motor (a PC cooling fan).

The need for order analysis is apparent when performing factory acceptance testing of industrial motors, engines, compressors, generators, turbines and other industrial equipment. Equipment manufacturers such as TECO-Westinghouse Motor Co. design these factory acceptance tests to include run-up and coast-down tests where rotational speed changes over time. Here it is paramount to track phase and amplitude of the 1^{st}, 2^{nd} and other key orders for verification of machine performance. These measurements are also used to establish operating vibration benchmarks for setting alarms, operating speeds, etc. TECO-Westinghouse is one such company that performs extensive vibration testing on electric motors prior to customer shipment.

Another example comes from the automotive industry where order analysis is an important tool for examining the loudness of engine components. To serve customers in the automotive industry, Roush Industries provides testing services and products to analyze the rotational harmonics of automotive components. Their Gabriel application performs order domain analysis as part of their investigations into vibratory and acoustic responses of automotive mechanical systems.

An additional automotive example is Roush Industries' BrakeDAQ, which was designed to replace a manual system that required subjective driver ratings of brake noise. Results were completely subjective with no correlation of vehicle parameters and noise. As a result, Roush was challenged with correlating vehicle parameters and brake noise, managing and storing large amounts of data, conducting real-time brake noise analysis, and objectively determining noise ratings during event-based acquisition. Their LabVIEW-based solution allows users to design a custom graphical interface and define each channel monitored. The frequency and SPL of each brake noise event can be replayed, allowing easy correlation of noise with all vehicle parameters (speed, temperature, humidity, brake pressure, etc.). Thus, results can easily be viewed while in-vehicle and brake noise can be objectively quantified.

Case Study

A test was conducted using a 4-pole electrical motor with a tachometer pulse output (two pulses for each revolution), measured with a National Instruments data acquisition board and LabVIEW. The test equipment and setup are shown in Figure 1 and typical measurement results are shown in Figure 2. Since the motor contains four poles, we expect to observe 4^{th}, 8^{th} and 12^{th} orders – that is, harmonics with frequencies at 4, 8 and 12 times the rotation speed. In addition, the fan has 7 blades, so we also expect to observe 7^{th}, 14^{th} and 21^{st} orders – harmonics at 7, 14 and 21 times the rotation speed.

The data shown in Figure 2 were collected while the motor was running at a constant 3000 RPM (50 Hz). The bottom plot depicts the signal from an accelerometer mounted on the motor. The plot on the left shows the conventional FFT-based power spectrum. In the middle is a STFT (short-time Fourier transform or windowed Fourier transform) based spectrogram computed by applying a 2048 point Hanning window. While the x-axis indicates the time, the y-axis represents frequency. The signal's energy at particular time t and frequency f is characterized by the color map on the right of the STFT plot.

When the rotational speed is constant at 3000 RPM (50 Hz), we can clearly see a 4^{th} (at 200 Hz) and 7^{th} order (at 350 Hz) from both the FFT-based power spectrum and the STFT-based spectrogram. In this case, both FFT-based and STFT-based methods work well. Generally speaking, when the RPM (or fundamental frequency) does not change with time, the FFT can be effectively used to compute the phase and magnitude of each individual order of interest.

However, the frequency associated with the first order may

Figure 1. Top – The PXI-based data acquisition system contains 24 channels of 24-bit dynamic signal acquisition, multiple temperature, pressure, and tachometer inputs, and a 6 ½ digit DMM. Bottom – A 4-pole motor drives the unit under test, a seven-blade cooling fan.

change, such as when the electrical motor or engine speed changes. In that case, the FFT-based method will no longer be applicable. Figure 3 shows two synthetic linear chirp signals. The signal's frequency in Figure 3a increases over time, whereas the signal's frequency in Figure 3b does not. Comparing FFT-based spectra of these two linear chirp signals, we can see that the signal's frequency bandwidth in Figure 3b is wider than its counterpart in Figure 3a. It can be shown mathematically that the signal's frequency bandwidth is proportional to the rate at which the signal's frequency and amplitude change.[6] The faster the change of frequencies or amplitudes, the wider the corresponding frequency bandwidth. As the fundamental frequency bandwidth widens, the bandwidth of related harmonics will also become wider. Finally, as illustrated in Figure 4, different harmonics will overlap in the frequency domain. In this case, the conventional FFT-based power spectrum will no longer be able to distinguish the vibrations caused by different sources (see Figure 5). When the fundamental frequency evolves over time, the resulting harmonics are time varying. Based on the definition, the term 'order' – used by automobile and machinery reliability engineers – covers both time-invariant and time-varying harmonics. Therefore, order analysis can be considered a general harmonic analysis.

Consider the large plots in the middle of Figures 4a and Figure 5. It is interesting to note that regardless of the change in RPM (or fundamental frequency), we can always clearly identify different orders (or harmonics) through a STFT-based spectrogram. This suggests that STFT is a better tool for order analysis (or general harmonic analysis). In fact, STFT is not only good at characterizing time-invariant harmonics, but is also

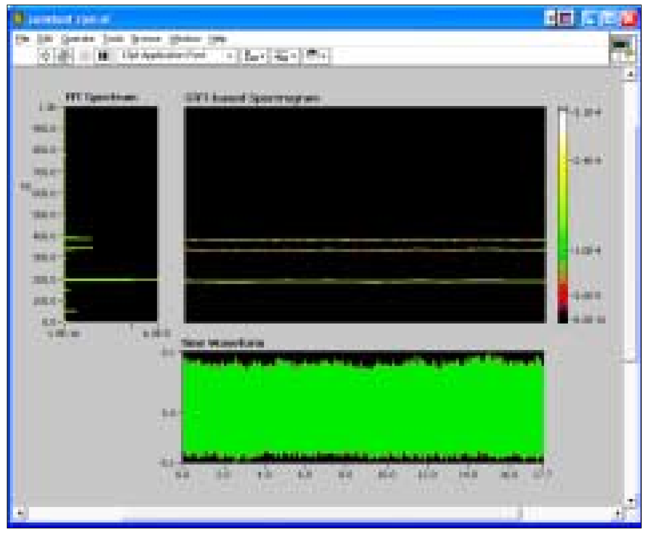

Figure 2. When the rotational speed of the fan is constant at 50 Hz (or 3000 RPM), from both the conventional FFT-based power spectrum and STFT-based spectrogram, we can clearly identify the orders caused by poles (at 200 Hz, 400 Hz) and blades (at 350 Hz).

suitable for time-varying harmonics. Because a hidden fault is usually much easier to discover at engine run-up/run-down than at a constant RPM, the technique of order analysis (or general harmonic analysis) is very important for engine diagnostics, as well as many other structural dynamics.

As shown in Figures 3, 4 and 5, STFT offers more insight into the physical process than the FFT, but it is generally not invertible. In some applications, engineers do need time waveforms of particular orders. Having corresponding time waveforms allows further analysis, such as cross-correlation. Order tracking refers to the process of recovering the time waveform of a particular order. How do we perform time-varying harmonic analysis? Can we extract time waveforms of desired orders from a STFT spectrogram directly?

Solving the Challenges

Due to the limitation of the FFT mentioned in the previous section, engineers commonly use the short-time Fourier transform (STFT) for visualizing orders of rotating machinery. In contrast to the classical FFT that describes signals in the time or frequency domain separately, the STFT characterizes signal magnitude and phase in the time and frequency domain simultaneously. Although it does offer insight into the physical process, STFT is usually not invertible. In other words, from a given STFT, one cannot extract a time waveform of particular orders. However, we can recover a time waveform from the modified STFT by the Gabor expansion with special care. Without going into mathematical detail, consider the following example. For a more in-depth mathematical analysis, consult Reference 6.

The so-called Gabor expansion was first introduced in 1946 by Dennis Gabor, a winner of the 1971 Nobel physics prize for his contributions to the principles underlying the science of holography. However, it was not until recently that the Gabor expansion has been used for processing signals whose frequency contents evolve with time, such as order analysis.

For a given set of discrete time data samples $s[k]$, a modified STFT is computed by

$$c_{m,n} = \sum_{k=0}^{L-1} \tilde{s}[k]\gamma^*[k - m\Delta M]e^{-j2\pi nk/N}, \; 0 \le m < M, \; 0 \le n < N \qquad (1)$$

where $\tilde{s}[k]$ denotes preprocessed data samples $s[k]$ (i.e., periodic extension). The parameters ΔM and N represent the discrete time sampling interval and the total number of frequency bins, respectively.

From a filter bank point of view, N is nothing more than the number of channels and ΔM is the decimation factor. We can roughly consider L as the length of the sequence $s[k]$. The in-

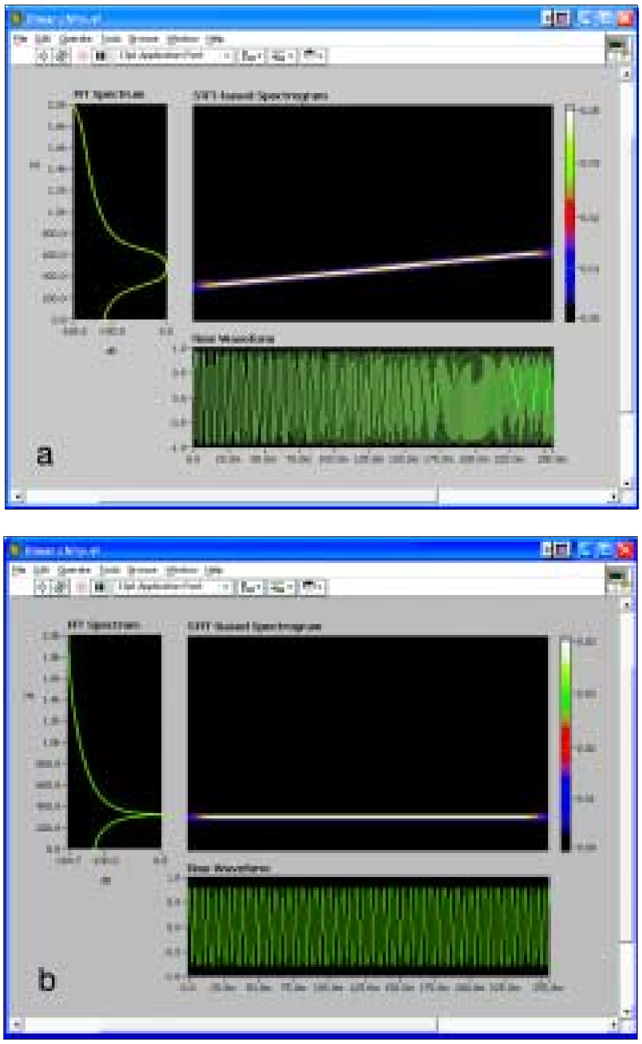

Figure 3. (a) The signal's frequency bandwidth is proportional to the rate of the signal's frequency change. The faster the frequency changes, the wider the corresponding frequency bandwidth. (b) While the frequency change rate of the linear chirp signal is equal to zero, the corresponding frequency bandwidth is minimized.

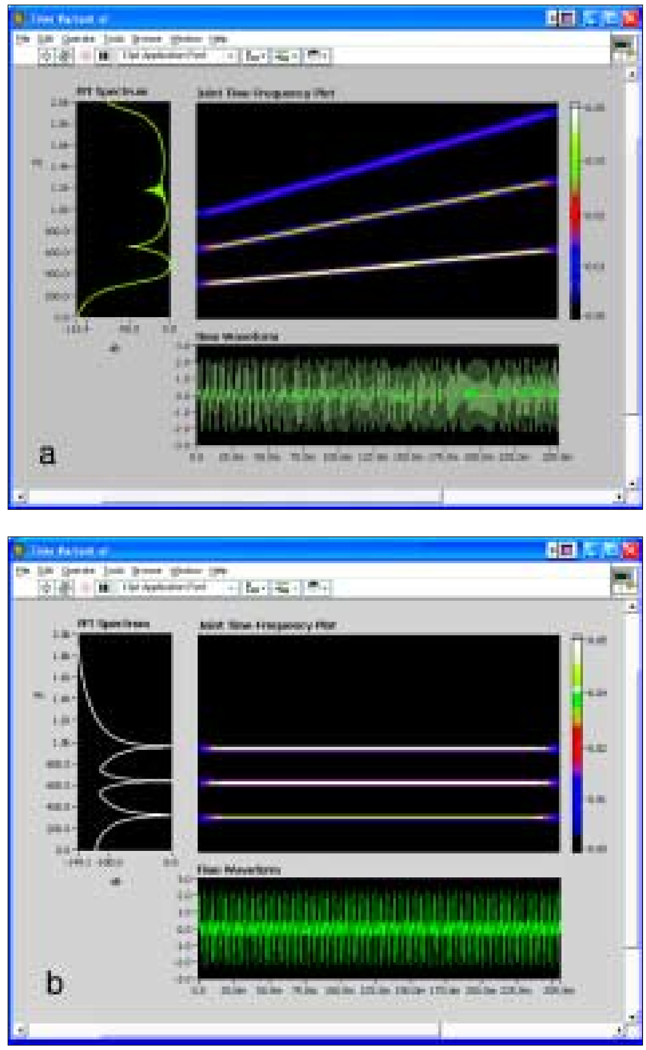

Figure 4. (a) As the fundamental frequency bandwidth becomes wide, the bandwidth of related harmonics will become wide, too. Finally, different harmonics will overlap in the FFT-based spectrum. (b) For constant RPM (or constant frequency), both the FFT-based power spectrum and the joint time-frequency plot clearly identify three distinct harmonics.

teger M in Eq. 1 indicates the total number of time points, that is, $M = L/\Delta M$. The total number of coefficients $c_{m,n}$ is equal to the product MN. In many references, the modified STFT in Eq. 1 is also referred to as the *discrete Gabor transform*. Accordingly, $c_{m,n}$ is named the *Gabor coefficient*.

The ratio $N/\Delta M$ determines the Gabor sampling rate. For numerical stability, the Gabor sampling rate must be greater than or equal to one. Critical sampling occurs when N is equal to ΔM, while $N/\Delta M > 1$ is known as oversampling. For oversampling, the number of Gabor coefficients $c_{m,n}$ is more than the number of original data samples $s[k]$. In this case, the transform in Eq. 1 contains redundancy from a mathematical point of view.

If all requirements mentioned above are satisfied, we can recover the original data samples by

$$s[k] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} c_{m,n} h[k - m\Delta M] e^{j2\pi nk/N} \qquad (2)$$

that is, the summation of a group of weighted time-shifted and frequency modulated window functions $h[k]$. Because it was first proposed by Dennis Gabor, Eq. 2 is traditionally known as the Gabor expansion.

The key issue of implementing the Gabor expansion is the selection of window functions $h[k]$ in Eq. 2 and $\gamma[k]$ in Eq. 1. Many different implementation schemes for the discrete Gabor expansion have been proposed over the years.[2] The one used in this article is an extension of the method originally proposed by Wexler and Raz.[8] This method guarantees the lengths of the analysis and synthesis window functions to be the same, which is a very useful property for DSP implementation. The reader can find a comprehensive treatment of window design from Reference 6. Note that the positions of the window functions $h[k]$ and $\gamma[k]$ can be interchanged. That is, either function can be used as the synthesis or analysis window function. As a result, $h[k]$ and $\gamma[k]$ are referred to as *dual functions*.

At first glance, the pair consisting of the discrete Gabor transform in Eq. 1 and the discrete Gabor expansion in Eq. 2 seems to provide a feasible vehicle for converting an arbitrary signal from the time domain into the joint time-frequency domain or vice versa. As a matter of fact, it is only true for $\Delta M = N$ (critical sampling). For oversampling (as is the case for most applications), the Gabor coefficients are the subset of entire two-dimensional functions. In other words, for an arbitrary two-dimensional function, there may be no corresponding time waveform.

However, it can be shown[6] that if the functions $h[k]$ and $\gamma[k]$ are identical, then Gabor coefficients of the resulting time waveform will be optimally close, in terms of the least square error, to the modified Gabor coefficients $w_{m,n}c_{m,n}$, where $w_{m,n}$ denote the user defined weighting function. Note that the above conclusion holds regardless of the selection of the weighting function $w_{m,n}$. If $w_{m,n}$ is limited to binary values (that is, either one or zero), then it behaves as a mask, preserving $c_{m,n}$ when $w_{m,n} = 1$ and removing $c_{m,n}$ when $w_{m,n} = 0$. Under certain conditions of this case, we can achieve a time waveform

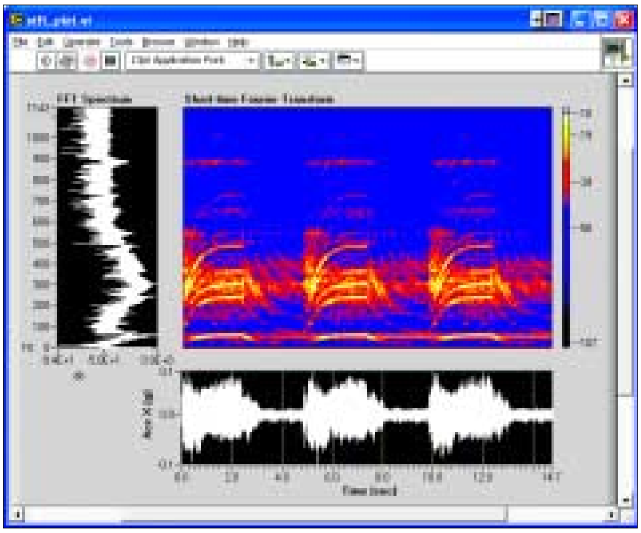

Figure 5. When different orders overlap in the frequency domain, the conventional FFT-based power spectrum will no longer be able to distinguish the vibrations caused by different sources, whereas the STFT can. Hence, the STFT is a better tool for order analysis.

whose Gabor coefficients are exactly inside the area defined by the mask function $w_{m,n}$. Readers who are interested in this topic can consult Reference 9.

Finally, making the analysis and synthesis window functions identical ($\gamma[k]$ and $h[k]$) normally requires significant oversampling. For the sake of memory consumption and computation speed, we usually seek the orthogonal-like Gabor transform.[6] In this case, we can often have a very similar pair of dual functions with a relatively low oversampling rate.

Results

This section considers several examples showing the effectiveness of the Gabor expansion method. The testing illustrated in Figure 6 used a 4-pole electrical motor. The analysis function is a 2048-point Hanning window. The number of frequency bins N is equal to the window length L_w. The oversampling rate is four, which is a 75% overlap. In this case, the difference between the analysis and synthesis windows is negligible. Consequently, the distance between the masked Gabor coefficients and that of the extracted time waveform, in the sense of the mean square error (MSCE), is minimal because the analysis and synthesis window are almost identical. By limiting the weighting function $w_{m,n}$ to binary values, the resulting weighting function behaves as a mask, preserving $c_{m,n}$ when $w_{m,n} = 1$ and removing $c_{m,n}$ when $w_{m,n} = 0$. The center frequency of the selected order is automatically computed from the rotational speed, whereas the bandwidth of the weighting function is manually selected.[5]

The results of the Gabor expansion-based order tracking analysis of a 4-pole electrical motor are shown in Figures 6a and 6b. This shows how we extract the 4th and 7th orders: the 7th order directly associates with the number of fan blades while the 4th order relates to the number of poles. It is interesting to note that from Figure 6b, the vibration caused by the cooling fan blades will get substantially larger when the motor speed passes the resonant frequencies. Generally speaking, a strong first order usually indicates unbalance while the second order often signifies misalignment.

When using Gabor expansion-based order tracking, the length of the window functions and the lower boundary of the rotational speed determine the extracting resolution. In Figures 6a and 6b, the sampling frequency is 4000 Hz, and the window length is 2048 points, so each frequency bin corresponds to 4000/2048 Hz. Because the lower boundary of the rotation speed is about 19.5 Hz – approximately 10 frequency bins – we can comfortably distinguish between components with a resolution of 0.1 orders. The longer the window function, the better the order resolution. On the other hand, the longer the window function, the poorer the time resolution. Therefore, we cannot arbitrarily increase the window length for a good order resolution.

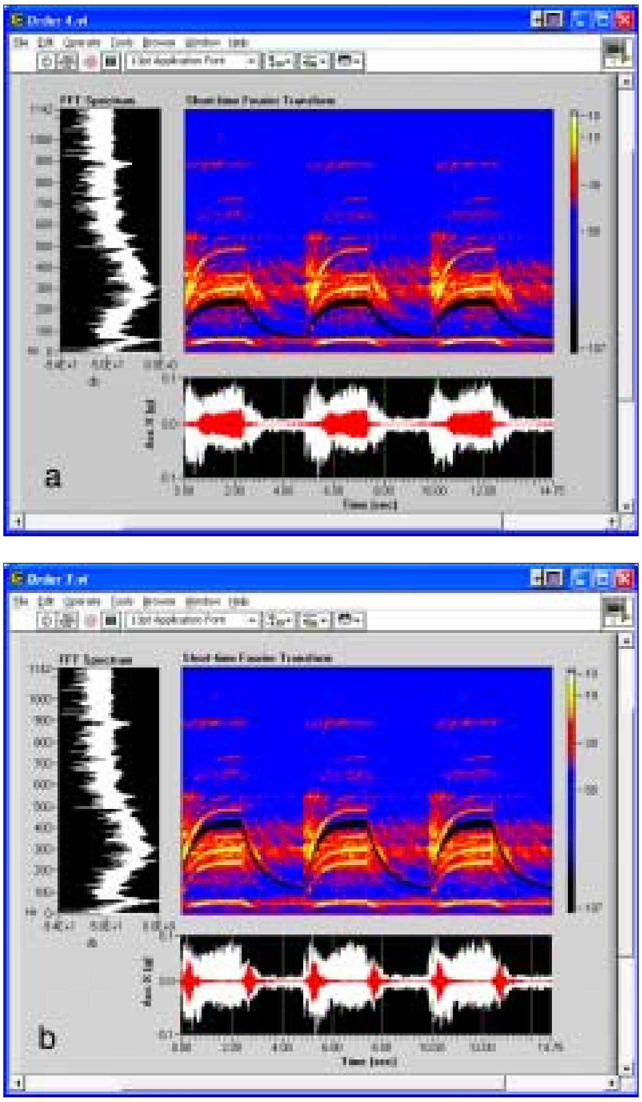

Figure 6. The fourth and seventh orders extracted by the Gabor expansion-based method. While the fourth order in (a) relates to the number of poles, the seventh order in (b) is directly associated with the number of fan blades. Figure 6b shows that the vibration caused by the cooling fan blades will get substantially larger when the fan speed changes (passing the resonant frequency).

In addition to the Gabor expansion-based approach introduced in this article, adaptive filters such as the Vold-Kalman have also been successfully used for order tracking.[7] Figure 7 presents the results of an experiment for comparing the Vold-Kalman filter shape to the Gabor expansion-based approach. A long sequence of uniformly distributed random noise was passed through a constant frequency Vold-Kalman filter utilizing a series of HC (harmonic confidence) factors to sharpen the filter. Note that bigger HCs require more computational time. The spectra of these filtered sequences are illustrated at the left of Figure 7. Next, the identical data were subjected to a Gabor expansion-based order extraction by using two different mask widths both centered on 1 kHz. The spectra of these sequences are shown at the right of Figure 7. These illustrations show that the Gabor expansion-based approach is far more selective than even the sharpest of the Vold-Kalman filters, not to mention computationally faster. This is mainly because the Gabor expansion-based time-varying filter is a non-causal system, whereas the Vold-Kalman filter is causal.

In the insets of Figure 7, the spectra of the extracted signal components are compared more closely to the spectrum of the original signal. An inspection shows that the Gabor expansion-

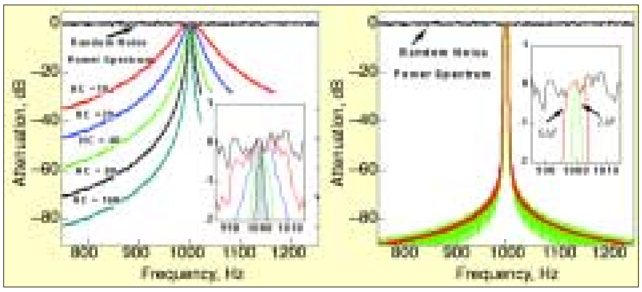

Figure 7. The left graph shows spectra of a broadband random noise sequence and selected extractions by a 1 kHz Vold-Kalman Filter. The right graph shows spectra of the same sequence and constant 1 kHz Gabor extractions with different mask widths. Obviously, the Gabor expansion-based approach is far more selective than even the sharpest of the Vold-Kalman filters.

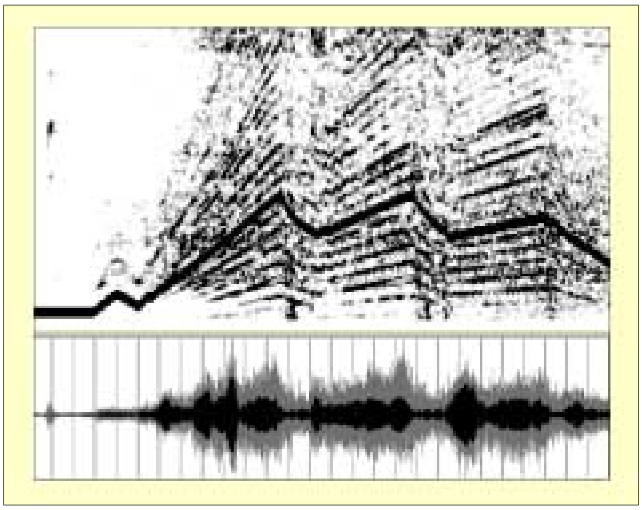

Figure 8. The upper graph is vehicle interior sound pressure for a wide open throttle sweep with a 5-bin wide mask at 4th order. The lower graph is the parent signal and 4th order extraction by Gabor order tracking.

based order extractions reproduce the spectrum of the original signal exactly inside the mask, while the Vold-Kalman filters closely approximate it but do not reproduce it exactly. The reader can find a more comprehensive comparison in Reference 1.

Although Gabor expansion-based order tracking is clearly more selective than the Vold-Kalman filter in terms of rejecting out-of-band/mask energy, there is a finite width restriction of the mask in that all spectral artifacts of smearing and leakage error should be captured to accurately reproduce the signal. Using higher over-sampling rates in the time/frequency domain is one way to sharpen the image and effectively narrow the mask width requirement. In Vold-Kalman filtering, the analogous choice is between filter bandwidth and filter response time. The limitation here is that a wide filter may not be well suited to isolate closely spaced orders from each other.

Consider the example of a full throttle run-up of a vehicle on the road, including shifts. Order analysis of a signal of this type defies all traditional methods that are ill equipped to handle the rapid transient that occurs during the shift. However, it is a perfect application for both the Vold-Kalman filter and the Gabor expansion-based method.

The STFT-based spectrogram of the run-up shown in Figure 8 clearly indicates three regions separated by gear changes. A 5-bin wide mask positioned over the fourth order throughout the sweep is represented as a dark line running throughout the graph. Figure 9 compares the 4th order component extracted from the sweep by the Gabor expansion approach to the identical 4th order component extracted with a Vold-Kalman filter. Figure 9 also superimposes the envelope of both of these signal components, which is augmented by two insets to reveal details. The close comparison of the two methods is obvious. In fact, the Gabor expansion-based approach duplicates each

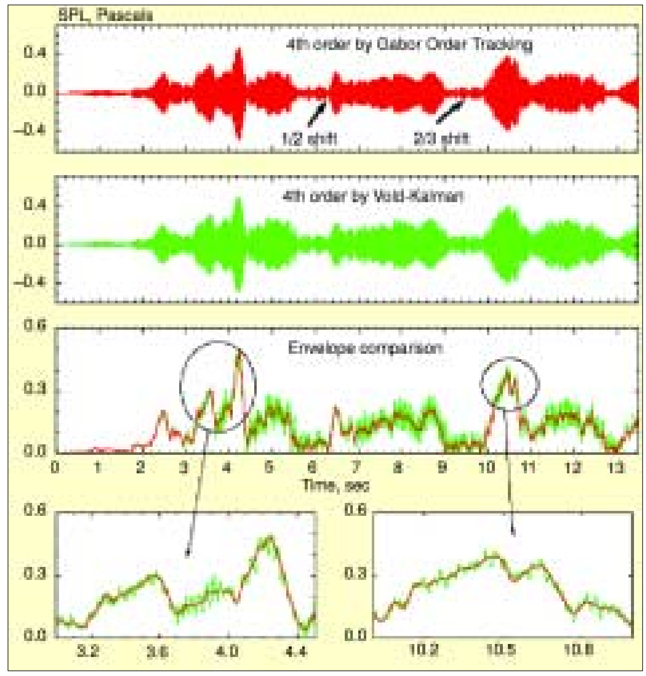

Figure 9. 4th order extractions from a wide open throttle sweep by both Vold-Kalman filter and Gabor order tracking. The Gabor expansion-based approach duplicates each detail that is present in the Vold-Kalman extractions. However, it produces a smoother result.

detail that is present in the Vold-Kalman extractions, but it produces a smoother result. This is a direct consequence of the superior stop-band selectivity of the non-causal Gabor expansion-based approach. Additional optimization of harmonic confidence may produce a better Vold-Kalman extraction for this example. However, the fundamental equivalence of the two methods is clear.

Conclusions

This article introduced a discrete Gabor expansion for order tracking – a popular application in the automobile industry, as well as many other areas in which harmonics are driven by rotating components. The powerful method presented here gives more insight into the underlying physical process. Because it is particularly well-suited for interactive analysis, we can perform order tracking without a tachometer signal, drawing the mask by hand as long as the order is visible and sufficiently resolved.

Acknowledgment

The author would like to express his deep appreciation to Michael Albright for Figures 7, 8 and 9 and valuable comments and suggestions.

References

1. M. Albright, and S. Qian, "A Comparison of the Newly Proposed Gabor Order Tracking Technique vs. Other Order Tracking Methods," SAE Paper Number 2001-01-1471, 2001.
2. H. Feichtinger, and T. Strohmer (editors), *Gabor Analysis and Algorithms*, Birkhauser, 1998.
3. D. Gabor, "Theory of communication," J. IEE (London), Vol. 93, No. III, pp. 429-457, November 1946.
4. S. Hui, J. Wei, and S. Qian, "Discrete Gabor Expansion for Order Tracking," will appear in IEEE Trans. of Instrumentation and Measurements, June 2003.
5. Order Tracking Toolset User's Manual, National Instruments, 2001.
6. S. Qian, *Introduction to Time-Frequency and Wavelet Transforms*, Prentice Hall, 2002.
7. H. Vold, and J. Leuridan, "High Resolution Order Tracking and Extreme Slew Rates, Using Kalman Tracking Filters," SAE Paper 931288, SAE Noise and Vibration Conference, Traverse City, MI, May 1993.
8. J. Wexler, and S. Raz, "Discrete Gabor Expansions," *Signal Processing*, Vol. 21, No. 3, pp. 207-221, November 1990.
9. X. Xia, and S. Qian, "Gabor expansion based time-variant filter," IEEE Trans. Signal Processing, Vol. 47, No. 10, pp. 2894-2899. S/V

The author can be contacted at: shie.qian@ni.com.