EDITORIAL

Using the Velocity Shock Spectrum to Predict Shock Damage

Howard A. Gaberson, *Oxnard, California*

The shock spectrum plots an analysis of a shock motion (e.g., transient motions due to explosions, earthquakes, package drops, railroad cars coupling, vehicle collisions, etc.) that calculates the maximum response of many different frequency damped, single degree of freedom systems exposed to the motion. The plot is a graph of maximum response versus frequency. Surprisingly, pseudo velocity – the maximum relative displacement times frequency in radians – is the best response quantity for indicating severity or capacity to cause damage.

The best way to plot these spectra is on four coordinate paper. (Abbreviated here 4CP, although the academics call it 'tripartite' paper.) 4CP is a logarithmic graph paper that has four sets of lines relating frequency, displacement, velocity and acceleration of sinusoidal motions. Plotting pseudo velocity shock spectra on 4CP for a shock that begins and ends with zero velocity shows three important regions. On the left is the maximum deflection or required rattle space. In the center a flattened region shows the frequency range where the maximum velocities are generated – the severe frequency content of the shock. On the right the spectrum slopes downward along the maximum acceleration line. These three regions show the needed rattle space, the frequency range for which the shock is severe, and the peak acceleration.

Many don't see a need for predicting shock damage by plotting the pseudo velocity shock spectrum on 4cp. In this editorial, I will explain my reasons for encouraging this type of analysis. I want to increase the acceptance and appreciation of the pseudo velocity shock spectrum plotted on 4CP. It should be used along side, if not in place of, the acceleration shock spectrum to make sure that shock severity aspects are not overlooked. I have been using this procedure for 20 years and the more data I process in this way, the more I am convinced it clarifies our understanding of shock motions.

Dick Chalmers and I wrote a paper in 1969 that pointed out maximum stress was proportional to the maximum modal velocity in vibrating rods and beams.[1] For a cantilever beam, the first mode is a flapping motion; the maximum velocity is out at the tip and the maximum stress is at the root. Put an accelerometer on the tip of a steel beam when it is vibrating steadily in the first mode, integrate that signal to velocity in ips (inches per second), and the stress in psi at the root will be 253 times that peak velocity. Size of the beam and frequency do not matter. This leads to the conclusion that there are limits to the modal velocity a structure can tolerate. When shock spectra are plotted in terms of velocity, there is no need to consider a wide range of velocity values, which simplifies the presentation. Virtually nothing can withstand a shock motion that generates shock spectrum velocities of 1000 ips, and even a shock with spectrum velocities of 200 ips is a very severe shock.

Unknown to us at the time, both Fred Hunt of Harvard[2] and Eric Ungar of BBN[3] had previously published that maximum strain (we used stress) was proportional to maximum modal velocity. We were actually the first to point out that this result applied to shock severity, but rather than argue for pseudo velocity spectra plotted on 4CP, we explained how to pick off the severe velocities on acceleration shock spectra. In hindsight, I wish we had emphasized the reasons for plotting the shock spectrum in terms of velocity, which was what the civil structures community was doing all along and still does.[4] The paper had virtually no effect. If anything, the plotting of shock spectra with acceleration as the ordinate increased.

Chalmers encouraged me to beat that horse some more, so in 1995 we did another more blunt paper[5] with nothing but reasons for plotting the shock spectrum with pseudo velocity as the ordinate. Dick hauled out his examples of why it made sense from a dynamic range point of view, and I converted my Fortran shock spectrum program to MATLAB® and calculated the relative velocity and pseudo velocity shock spectra of five shocks showing that relative velocity gives poor results at low frequencies. We presented a profound proof (that I have seen elsewhere) that pseudo velocity is the square root of twice the maximum energy per unit mass stored in the single degree of freedom spring by the shock. Pseudo velocity indicates the maximum elastic energy developed in the single degree of freedom spring by the shock motion. We also discussed Fung's proof credited to Housner that pseudo velocity squared bounds the energy delivered to a structure during shock,[6] indicating that the shock spectrum applies to composite structures.

I have often tried to figure out how the acceleration shock spectrum became so popular. It is presented in almost all mechanical engineering vibration texts and handbooks that mention shock spectrum at all. Many books and articles go into great detail plotting complicated theoretical acceleration shock spectra of so-called classical pulses like terminal peak saw tooth, trapezoidal and half sines. They are all similar when plotted as pseudo velocity shock spectra on 4CP. I discussed this with a technical historian friend who explained: $F = ma$, and F, the force, breaks things; since acceleration is pro-

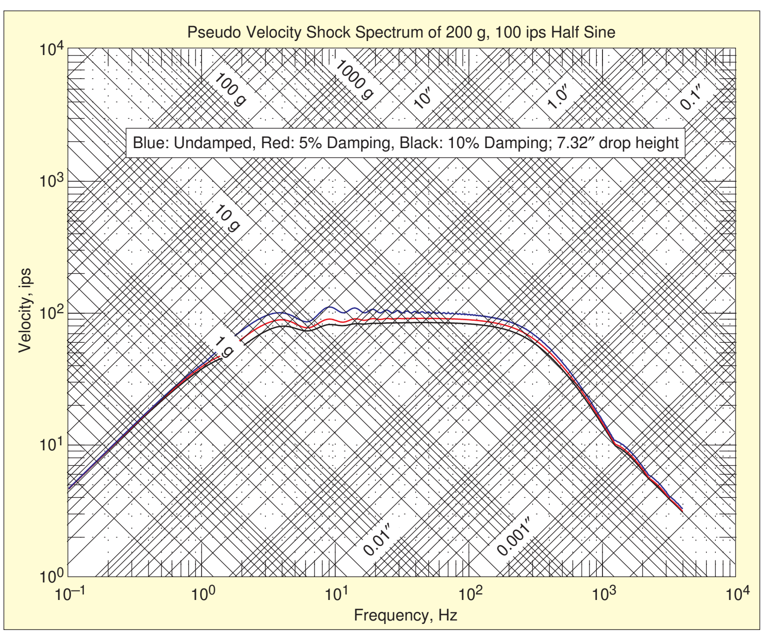

Figure 1. This is an example spectrum for illustrative purposes, showing the PVSS plotted on 4CP of a drop table half sine shock. The drop, the half sine and the rebound are included in the analysis. I adjusted the parameters to have a 100 ips velocity change with a 200 g maximum acceleration. I had it drop onto a pad with a 0.33 coefficient of restitution. This requires a 7.32 in. drop height and a rebound height of 0.797 in. where it is gently caught. The impact duration comes out to be 2.03 ms. The figure shows the undamped, 5% damped and 10% damped overall PVSS shock spectra. On the left side of the curve we see the peak displacement drop height of 7.32 in. In the flattened center region we see the 100 ips velocity change. However notice that the severe velocity is only developed in SDOFs with frequencies from about 200 Hz down to 2.5 Hz. On the right side of the curve notice the 200 g asymptote. That is a great deal of information about that shock.

portional to *F*, acceleration is it. Maybe no one ever gave it a second thought. All of you have heard g's used to describe a severe shock. That's wrong – g's don't matter! About four coordinate paper, another friend surmised that it became unpopular because it was kind of an unsophisticated nonmathematical nomogram.

I am convinced that the missing piece was the study of why things break from shock – critically breaking things with shock and looking for an analysis that relates severe shock motions. I sold this task in the early 70s, and my team did it, with Chalmers advising and helping. We tested many fans with six different shocks, increasing the level of each until the fans broke. I analyzed these data five ways and found that pseudo velocity plotted on 4CP best related the shocks that could fail the fans. The Shock and Vibration Symposium rejected that paper in 1973. Chalmers felt terrible, but he continued encouraging me to get it published. He would suggest we publish it here or there, but I was totally defeated. However, the 1995 paper rejuvenated me, and I started reassembling that rejected paper from 1973. Dick died in 1998, but he was aware of my renewed efforts. In 1999 the paper was presented and published at the Shock and Vibration Symposium (the original rejecters were gone). From there, I also published the paper at MFPT and IMAC. The IMAC publication attracted the attention of this fine magazine, which published it in the May 2000 *Sound and Vibration*. That did it. That was the exposure we needed. Chalmers died before he could see it, but I bet he was smiling. I received many encouraging letters about that article, and so here I am again, pushing pseudo velocity and 4CP three years later.

The pseudo velocity shock spectrum analysis plotted on 4CP is not a difficult process. MATLAB draws that complicated looking 4CP just beautifully – if you would like my 'fourcp' m-file or my shock spectrum program m-file, just send me an e-mail. Run my ss program, which log-log plots the spectrum; and then say 'hold on' and 'fourcp.' That lays the 4CP on the log-log plot. I wrote a lengthy (60 page), detailed, but easy (I think) derivation and discussion on shock spectrum calculation. If you are going to write a shock spectrum program, want to see a development of the computing details or the original Fortran program, you won't need a masters in EE or Math to read it. That original Navy Technical Note[7] is still available for $7 from DTIC ($31 from NTIS if you are not government or a government contractor, www.ntis.gov). Go to www.dtic.mil and search stinet for 'shock spectrum' or 'ADA097162.' By the way, search on anything else of interest – it is probably a treasure trove of the DoD's R&D.

I hope this will generate discussion and additional testing to corroborate or disprove the facts that convince me of this. There are two morals to this story: The first and most important is to analyze shock motions with pseudo velocity shock spectra plotted on four coordinate paper. And the second – persist.

1. Gaberson, H. A., and Chalmers, R. H., "Modal Velocity as a Criterion of Shock Severity," *Shock and Vibration Bulletin*, v 40, Part 2, pp. 31-49, Dec 1969.
2. Hunt, F. V., "Stress and Strain Limits on the Attainable Velocity in Mechanical Vibration," *J. Acoustical Soc. Am.*, v 32, n 9, pp 1123-1128.
3. Ungar, E. E., "Maximum Stresses in Beams and Plates Vibrating at Resonance," Trans ASME, *J. Engrg. Ind.*, v 32, n 1, pp 149-155, Feb 1962.
4. Eubanks, R. A., and Juskie, B. R., "Shock Hardening of Equipment," *Shock and Vibration Bulletin*, n 32, part III, p 23-73, 1963.
5. Gaberson, H. A., and Chalmers, R. H., "Reasons for Presenting Shock Spectra with Velocity as the Ordinate," Proceedings of the 66th Shock and Vibration Symposium, v 2, 1996.
6. Fung, Y. C., "Shock Loading and Response Spectra," from *Shock and Structural Response*, Ed. by M. V. Barton, pp 1-17, 1960.
7. Gaberson, H. A., "Shock Spectrum Calculation from Acceleration Time Histories," Technical Note, TN N-1590, Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, CA 93043, 1980, ADA 097162. (Available from Defense Technical Information Center, www.dtic. mil; or National Technical Information Service, www.ntis.gov.) SV

The author may be contacted at: hagaberson@att.net.