A New Procedure for Modal Parameter Estimation

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This article looks at the new LMS PolyMAX method for Experimental Modal Analysis (EMA). PolyMAX is compared with other EMA methods and two examples are provided. A theoretical summary of the method is included as well.

Experimental Modal Analysis (EMA) is currently one of the key technologies used in structural dynamics analysis. Based on the fundamentals of system identification, it has evolved into a 'standard' approach in mechanical product development. Modal analysis has, from the start, focused on solving specific problems related to the testing and modeling of large industrial structures. The merit of each new method or approach has always been evaluated for the added value it brought to help application engineers derive better models. EMA is now considered a 'commodity' tool with a continuously expanding application base.

Current Limitations of Experimental Modal Analysis

While the range of EMA applications is continuously expanding, the complexity of tested structures is increasing as well. In the past, isolated structures with low damping were analyzed. Modal analysis is now being used on complex structures with high damping such as trimmed car bodies. EMA has also evolved as a standard tool for Finite Element Model (FEM) updating and in combination with numerical technologies for hybrid engineering.

These recent evolutions have highlighted current limitations of the EMA process:

- The task of selecting the correct modal order and discriminating between spurious and structural system poles is quite complex, particularly in the case of high order and/or highly damped structures. This results in high operator dependance and numerous iterations of the analysis procedure.
- Whereas the quality of current modal parameter estimation technologies is acceptable for undamped or slightly damped structures, there is an increasing need for better modal parameter estimations of highly damped structures.
- Instead of a variety of parameter estimation techniques, each optimized for a specific test situation, there is a need for a single reliable and robust method that can be used in a wide variety of applications.

In an area where many critics claim no substantial advances were to be expected, the new LMS PolyMAX method brings a revolutionary modal parameter estimation technique that is easy to use, quick to perform, substantially reduces operator-dependant judgment and delivers high quality modal parameter estimations, even on complex data.

In this article, the LMS PolyMAX method was used on two 'historically' difficult data sets – a trimmed car body (high damping) and flutter data (high data noise). A summary of the analytical foundation of the LMS PolyMAX method is also included.

Using LMS PolyMAX on a Trimmed Car Body

A typical example of a challenging modal analysis application is the structural analysis of a trimmed car body. The trim material turns a nicely resonating car body into a highly damped system with large modal overlap. In this example, data from a Porsche 911 Targa Carrera 4 was used. The accelerations of the fully equipped car were measured at 154 Degrees Of Freedom (DOF), while four shakers were simultaneously exciting the structure. This gives a total of 616 FRFs (Frequency Response Functions) used in the modal analysis procedure.[1]

The data were analyzed in a frequency range from 3.5 to 30 Hz using three techniques:

- The Frequency-Domain Direct Parameter Identification (FDPI) technique which has traditionally been used to analyze data from highly damped structures.[2-5]
- The Least Squares Complex Exponential (LSCE) method which is considered an "industry-standard" time domain estimation method.[2,3,6]
- The new LMS PolyMAX method.

Figure 1 shows the stabilization diagrams for the 3 methods. FDPI yields a clearer diagram than LSCE, confirming the common assumption that FDPI is the preferred method in the case of high damping. However, both are clearly outperformed by the LMS PolyMAX method, especially at lower frequencies where it is much clearer than the FDPI diagram while also finding more stable poles.

For the subset of LMS PolyMAX poles that have an FDPI counterpart, Table 1 shows that resulting estimations for frequency and damping are very close. Also, the mode shapes are very similar which is evidenced by the MAC values represented in Figure 2.

The excellent identification results obtained with the LMS PolyMAX method are confirmed by comparing the measured FRFs with the FRFs that can be synthesized from modal parameters. Figure 3 shows both measured and synthesized FRFs at one response location (FRFs with respect to 2 out of 4 input points). Overall, an impressive 93% correlation was reached between measured and synthesized FRFs.

Figure 4 shows the FDPI and LMS PolyMAX stabilization diagrams of another trimmed body dataset where 2 inputs and 500 outputs were measured. In the previous example, an experienced user could have used FDPI to identify a subset of the structural modes. In this case, the FDPI stabilization diagram is hard to interpret at all. Again, LMS PolyMAX shows an easier to interpret stabilization diagram.

Using LMS PolyMAX on Flight Flutter Data

In some cases FRF data are highly contaminated by noise, such as flight flutter testing. In the example considered here, both wing tips of an aircraft are excited during the flight with so-called rotating vanes. These vanes generate a sine sweep through the frequency range of interest. The forces are measured by strain gauges. Next to these measurable forces, turbulences are also exciting the plane resulting in rather noisy FRFs. Figure 5 shows some typical multiple coherences and corresponding FRFs, which clearly show the noisy character of the data. During the flight, accelerations were measured at 9 locations, while both wing tips were excited (2 inputs). The data were analyzed using both the LSCE and LMS PolyMAX methods. Figure 6 shows both stabilization diagrams. Also in this example, the LMS PolyMAX method shows some clear advantages to the LSCE method – selecting poles is intuitive, clear and reliable. The synthesized FRFs (Figure 7) validate the LMS PolyMAX estimations, even in the presence of high amounts of data noise.

LMS PolyMAX – Historical background

The LMS PolyMAX method is a further evolution of the Least Squares Complex Frequency-Domain (LSCF) estimation method. That method was first introduced to find initial values for the iterative maximum likelihood method.[7] The LSCF

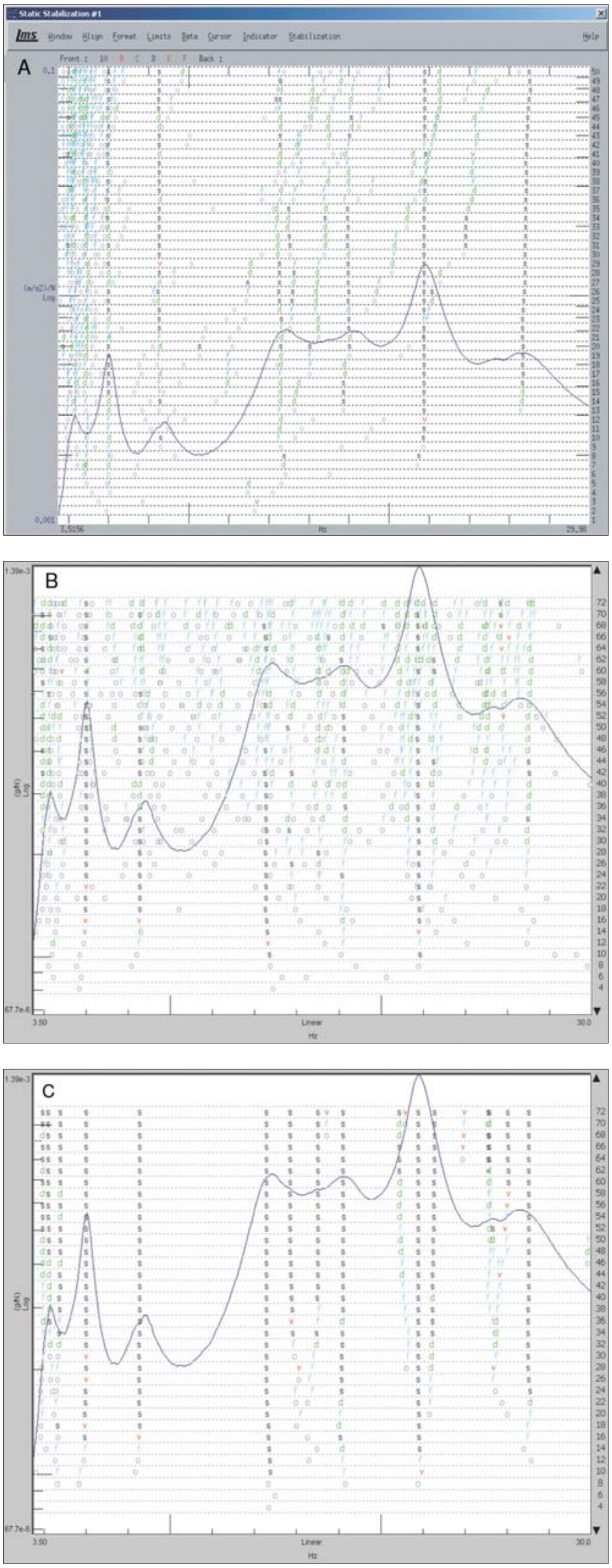

Figure 1. Stabilization diagrams obtained by applying different parameter estimation methods to the Porsche data: (A) FDPI; (B) LSCE; (C) the new LMS PolyMAX method.

estimates a so-called common-denominator transfer function model.[8] It was found that these "initial values" already yielded very accurate modal parameters with a very small computational effort.[7,9,10] The most important advantage of the LSCF estimator over available and widely applied parameter estimation techniques[2] is the fact that very clear stabilization diagrams are obtained. Further analysis and background information are available in the references.[10,11]

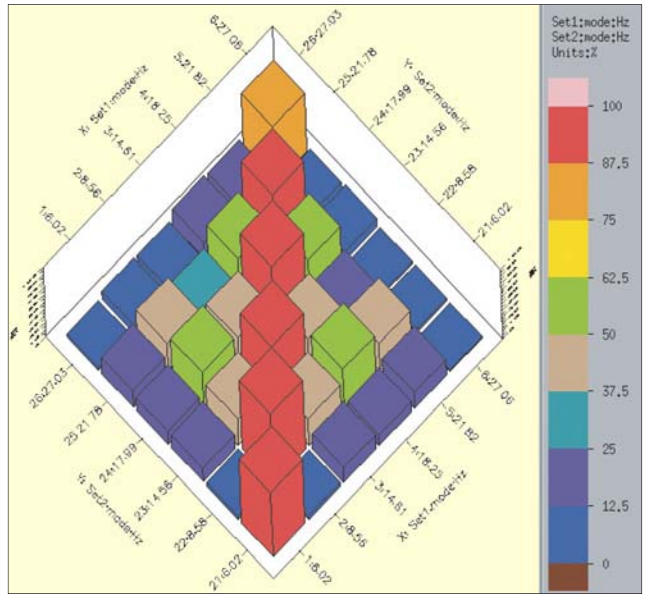

Figure 2. MAC values assessing the mode shape correlation between FDPI and corresponding LMS PolyMAX mode shapes.

It was found that the identified common-denominator model closely fit the measured frequency response function (FRF) data. However, when converting this model to a modal model by reducing the residues to a rank-one matrix using Singular Value Decomposition (SVD), the quality of the fit decreased.[9]

Another feature of the common-denominator implementation is that the stabilization diagram can only be constructed using pole information (eigenfrequencies and damping ratios). Neither participation factors nor mode shapes are available initially.[12] The theoretically associated drawback is that closely spaced poles will erroneously show up as a single pole.

These factors provided motivation for a polyreference version of the LSCF method, using a so-called right matrix-fraction model. In this approach, the participation factors are also available when constructing the stabilization diagram. The main benefits of the polyreference method are that the SVD step to decompose the residues can be avoided and closely spaced poles can be separated.[12-13] Here we briefly review the theory.

LMS PolyMAX – Theoretical Foundation

Data Model. Just like the FDPI (Frequency-Domain Direct Parameter Identification) method[4,5], the LMS PolyMAX method uses measured FRFs as primary data. Time-domain methods, such as the polyreference LSCE method[6], typically require impulse responses (obtained as the inverse Fourier transforms of the FRFs) as primary data. In the LMS PolyMAX method, the

Table 1. Eigenfrequencies and damping ratios obtained by applying the FDPI and LMS PolyMAX method.

FDPI method		LMS PolyMAX method	
f_i, Hz	ξ_i, %	f_i, Hz	ξ_i, %
–	–	3.96	5.4
–	–	4.24	9.6
–	–	4.81	11.6
6.02	4.1	6.02	4.2
8.58	6.4	8.57	6.5
14.56	6.1	14.59	5.8
–	–	15.74	6.3
–	–	17.05	5.8
17.99	5.5	18.25	4.9
–	–	20.91	2.7
21.78	2.9	21.81	2.7
–	–	22.58	3.4
–	–	23.98	0.9
–	–	25.12	2.4
–	–	25.23	3.1
–	–	26.05	2.1
27.03	5.6	27.06	5.2

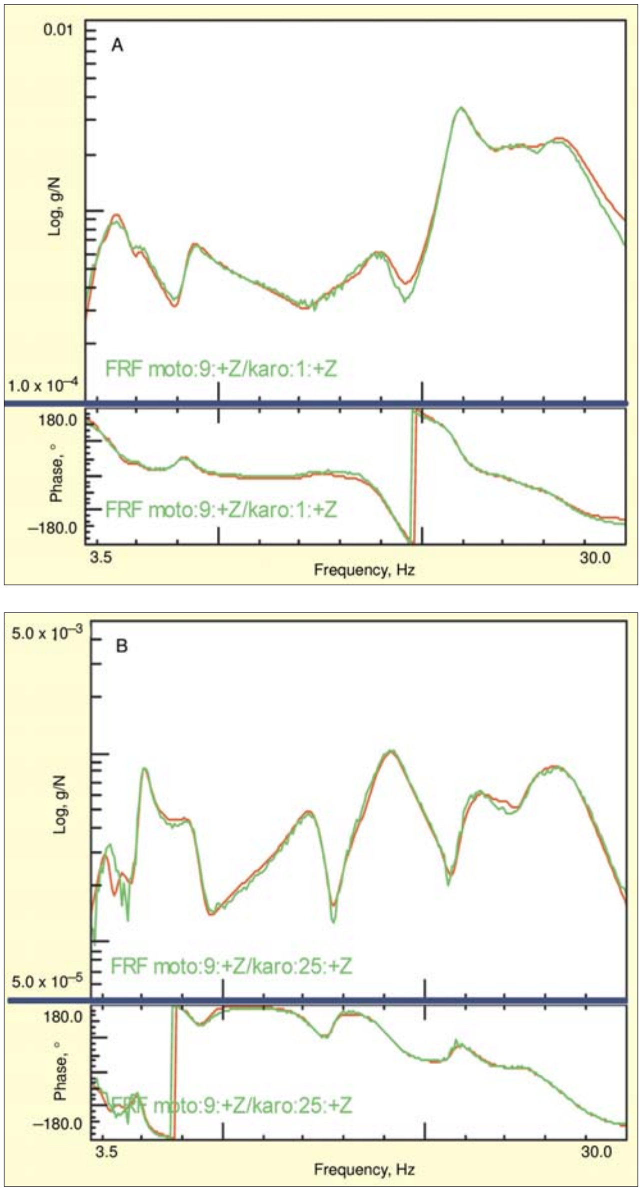

Figure 3. Comparison of measured FRFs (green) with FRFs synthesized from the identified modal model (red). The FRFs between 2 of the 4 inputs (A) and a typical output (B) are shown.

following so-called right matrix-fraction model is assumed to represent the measured FRFs:

$$\left[H(\omega)\right] = \sum_{r=0}^{p} z^r \left[\beta_r\right] . \left(\sum_{r=0}^{p} z^r \left[\alpha_r\right]\right)^{-1} \tag{1}$$

where $\left[H(\omega)\right] \in \mathbb{C}^{l \times m}$ is the matrix containing the FRFs between all m inputs and all l outputs; $\left[\beta_r\right] \in \mathbb{R}^{l \times m}$ are the numerator matrix polynomial coefficients; $[\alpha_r] \in \mathbb{R}^{l \times m}$ are the denominator matrix polynomial coefficients and p is the model order. Please note that a so-called z-domain model (i.e., a frequency-domain model that is derived from a discrete-time model) is used in Eq. 1, with:

$$z = e^{-j\omega\Delta t} \tag{2}$$

where Δt is the sampling time.

Eq. 1 can be written for all values of the frequency axis of the FRF data. Basically, the unknown polynomial coefficients $[\alpha_r]$, $[\beta_r]$ are then found as the least-squares solution of these equations (after linearization).[12,13]

Poles and Modal Participation Factors. Once the denominator coefficients $[\alpha_r]$ are determined, the poles and modal participation factors are retrieved as the eigenvalues and eigenvectors of their companion matrix:

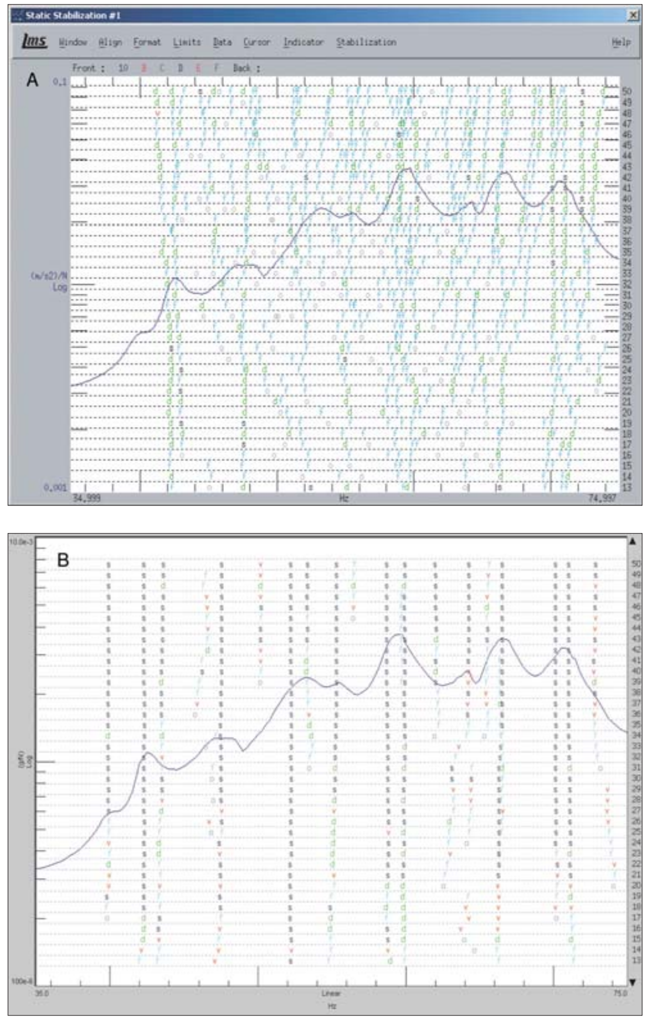

Figure 4. Stabilization diagrams obtained by applying different parameter estimation methods to data from a partially trimmed car: (A) FDPI; (B) the new LMS PolyMAX method.

$$\begin{pmatrix} 0 & I & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & I \\ -\left[\alpha_0^T\right] & -\left[\alpha_1^T\right] & \cdots & -\left[\alpha_{p-2}^T\right] & -\left[\alpha_{p-1}^T\right] \end{pmatrix} \bullet V = V\Lambda \tag{3}$$

The modal participation factors are the last m rows of $V \in \mathbb{C}^{mp \times mp}$; the matrix $\Lambda \in \mathbb{C}^{mp \times mp}$ contains the (discrete-time) poles $e^{-\lambda_i \Delta t}$ on its diagonal. They are related to the eigenfrequencies ω_i (rad/s) and damping ratios ξ_i (–) as follows (* denotes complex conjugate):

$$\lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j\sqrt{1-\xi_i^2}\,\omega_i \tag{4}$$

This procedure is similar to what happens in the time-domain LSCE method and allows a stabilization diagram to be constructed for increasing modal orders and using stability criteria for eigenfrequencies, damping ratios and modal participation factors.

Mode Shapes. Theoretically, the mode shapes could be derived from the coefficients $[\alpha_r][\beta_r]$, but we proceed in a different way. The mode shapes can be found by considering the so-called pole-residue model:

$$\left[H(\omega)\right] = \sum_{i=1}^{n} \frac{\{\upsilon_i\} < l_i^T >}{j\omega - \lambda_i} + \frac{\left\{\upsilon_i^*\right\} < l_i^H >}{j\omega - \lambda_i^*} - \frac{[LR]}{\omega^2} + [UR] \tag{5}$$

where n is the number of modes; ^{H} denotes complex conjugate transpose of a matrix; $\{\upsilon_i\} \in \mathbb{C}^l$ are the mode shapes;

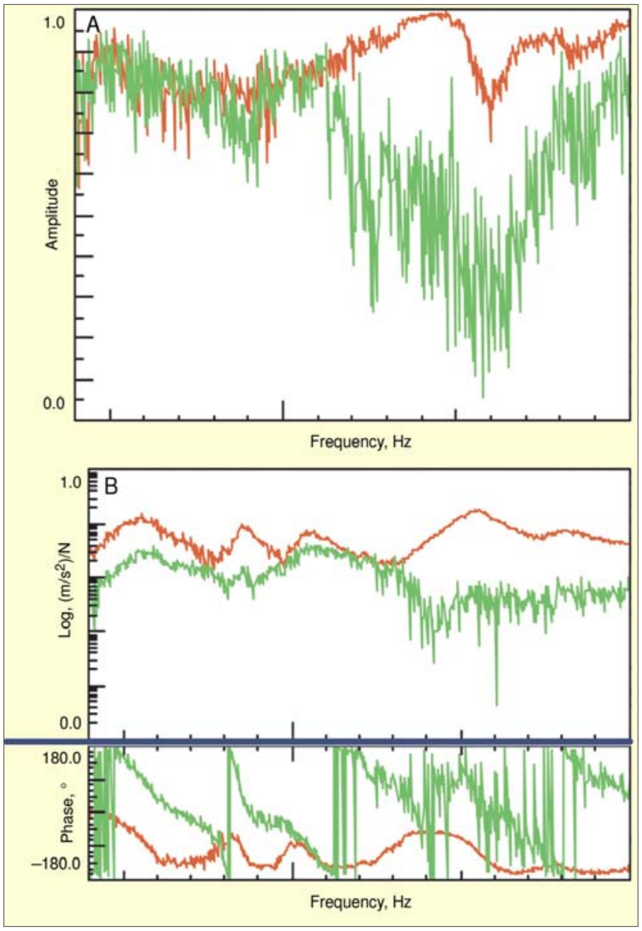

Figure 5. Flight flutter test data: (A) multiple coherences of a sensor at the wing tip close to the excitation (red) and a sensor at the back of the plane (green); (B) corresponding FRFs. The frequency axis is blind for confidentiality reasons.

$< l_i^T > \in \mathbb{C}^m$ are the modal participation factors and λ_i are the poles (Eq. 4). $[LR][UR] \in \mathbb{R}^{l \times m}$ are respectively the lower and upper residuals modeling the influence of the out-of-band modes in the considered frequency band. The interpretation of the stabilization diagram yields a set of poles λ_i and corresponding participation factors $<l_i^T>$. Since the mode shapes $\{v_i\}$ and the lower and upper residuals are the only unknowns, they are readily obtained by solving Eq. 5 in a linear least-squares sense. This second step is commonly called Least Squares Frequency Domain (LSFD) method.[2,3] The same mode-shape estimation method is normally also used in conjunction with the time-domain LSCE method.

Comparing LMS PolyMAX with Other Estimators

LMS PolyMAX versus LSCE. As may be clear from the previous section, the LMS PolyMAX method proceeds along similar lines as the polyreference LSCE time-domain method:

- Establishment of a set of linear equations for the maximum required modal order, from which the matrix polynomial coefficients $[\alpha_r]$ can be computed in a least-squares sense.
- Construction of a stabilization diagram by solving the eigenvalue problem (Eq. 3) for increasing model orders. The information regarding eigenfrequencies, damping ratios and modal participation factors is contained in this diagram.
- Based on the user-interpretation of the stabilization diagram, computation of the mode shapes and the lower and upper residuals by solving (Eq. 5) in a least-squares sense.

The difference between LSCE and LMS PolyMAX lies in the first step. LSCE uses impulse responses to find the polynomial coefficients, whereas LMS PolyMAX requires frequency response functions.

However, this seemingly small difference has major consequences for the modal parameter estimation process. It turns out that the new LMS PolyMAX estimator yields extremely clear stabilization diagrams making it very simple to select the 'physical' poles. In the LSCE method, the non-physical (and sometimes even the physical) poles tend to 'wander' in the stabilization diagram, especially at large modal orders. The LMS PolyMAX method has the interesting property that the nonphysical poles are estimated with a negative damping ratio so that they can be excluded before plotting them. Such a clear diagram does not mean that some of the poles are missing. On the contrary, more poles can be found with the LMS PolyMAX method, as evidenced by the examples in this article. Other validation studies also revealed that the LMS PolyMAX method has no problems in correctly estimating modes having a low damping ratio. It is sometimes stated that time-domain methods are preferred in case of low damping, and frequency-domain methods in case of high damping. The LMS PolyMAX method excels in both cases.

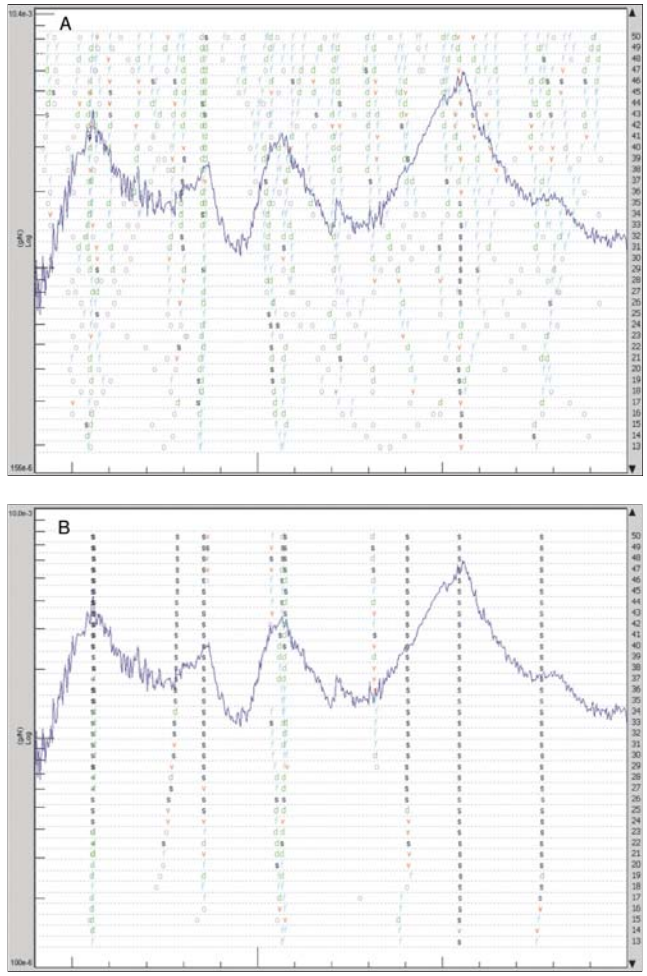

Figure 6. Stabilization diagrams obtained by applying different parameter estimation methods to the flight flutter test data: (A) LSCE; (B) the new LMS PolyMAX method.

LMS PolyMAX versus Other Frequency-Domain Methods. Many frequency-domain parameter estimation methods typically involve the inversion of a matrix containing powers of the frequency-axis of the data. Therefore, one quickly runs into numerical conditioning problems and severe constraints to both the frequency range and the modal order range of the analysis. In the past, it has been proposed to use an orthogonal polynomial basis for the frequency-domain model to solve numerical problems. However this significantly increases the computation time and memory requirements.

The LMS PolyMAX method does not suffer from numerical problems as it is formulated in the *z*-domain (i.e., a frequency-domain model that is derived from a discrete-time model), whereas the existing frequency-domain methods use a Laplace-domain formulation (i.e., a frequency-domain model that is

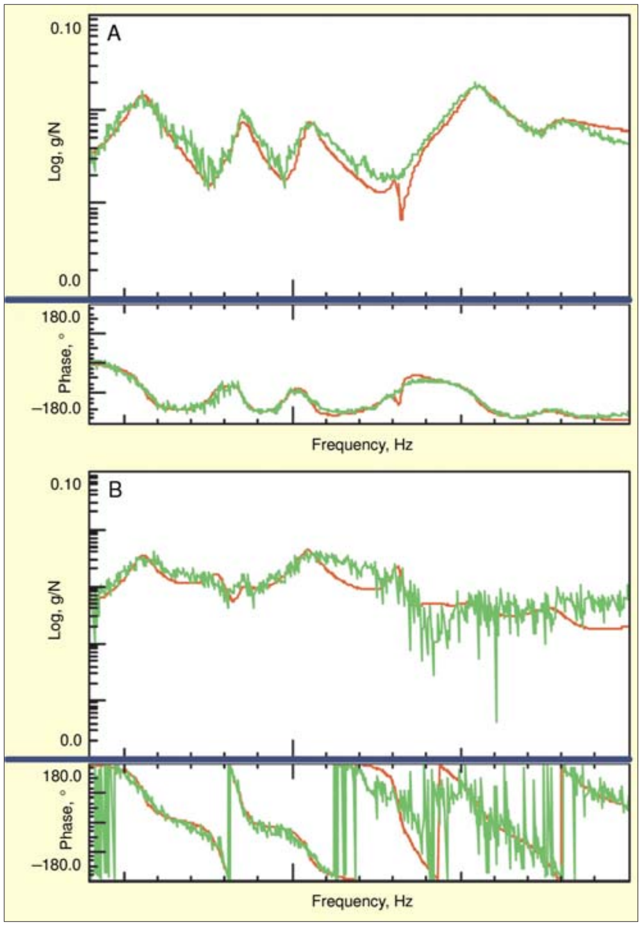

Figure 7. Comparison of the measured FRFs (green) with FRFs synthesized from the identified modal model (red). (A) Sensor at the wing tip; (B) Sensor at the back of the aircraft.

derived from a continuous-time model). In LMS PolyMAX, the frequency axis that extends between f_0 and f_{end} is shifted and mapped into a half unit circle in the complex plane (Eq. 2):

$$z = e^{-j\omega\Delta t},\ \omega = 2\pi\left(f - f_0\right),\ \Delta t = \frac{1}{2\left(f_{end} - f_0\right)} \tag{6}$$

Similar to other frequency-domain methods, the LMS PolyMAX method involves the inversion of a matrix containing powers of the frequency-axis of the data. The main advantage of LMS PolyMAX is that taking powers of the z-variable does not increase the range of the values, as it boils down to a rotation in the complex plane: $z^r = e^{-j\omega\Delta tr}$. As a result, the LMS PolyMAX method can deal with a large frequency range and very large model orders, speeding up the modal parameter estimation process considerably, as in many cases the complete frequency-band of interest can be processed at once.

There was some common belief that the numerical conditioning of frequency-domain methods is worse than time domain methods and that broadband analyses are preferably performed in the time-domain.[2] When using the LMS PolyMAX approach, these statements are no longer true.

Computational Efficiency. The advantages discussed here have no penalties in terms of computational time – LMS PolyMAX is as fast as LSCE. LSCE became the industry-standard because of its high speed even for a very large number of measured outputs. A lot of research was spent to achieve this computational efficiency. On current PC platforms, calculation and display of the stabilization diagram for a typical full car body model (like the trimmed car body example discussed here) is in the order of seconds.

Conclusions

With the new LMS PolyMAX method, a breakthrough in Experimental Modal Analysis has been achieved. Whereas the method equals or even outperforms the current standard LSCE technique on common test structures, it brings a solution for problems – like trimmed body and flutter data – where current EMA technology has shown its limits.

By substantially simplifying the analysis process, LMS PolyMAX will be enjoyed by many new users in the field. For advanced applications, its powerful clear stabilization and the quality of the modal parameter estimation are real breakthroughs, widely expanding the application range and drastically reducing the number of iterations needed.

LMS PolyMAX is not yet-another-parameter-estimation-technique, but a global solution for Experimental Modal Analysis. The new function is part of the LMS Test.Lab Structures solution for modal testing and analysis. LMS Test.Lab Structures is an integrated suite of applications covering the range of structural dynamic engineering completely. Dedicated applications serve impact hammer measurements, single shaker testing and advanced multiple-input, multiple-output (MIMO) analyses. These starting points measure the motion/force transfer or FRFs required for modal analysis. A dedicated modal analysis module automatically accesses these measurements to compute the modal parameters: mode shape vector, resonant frequency, damping factor and modal mass. All applications are tightly integrated so that data streams smoothly from acquisition, through analysis, to display and reporting.

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