Linking FEA with Test

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This article introduces the motivation and expected benefits of linking FEA and testing. Typical applications are found in the field of modal analysis where frameworks and efficient procedures are now available for calibrating analytical models to better correspond with the results of experimental models. An overview of the tools is provided together with a scenario for including uncertainty management as well as a future outlook of the technology.

For many years there has been a wide gap between finite element analysts and test engineers. Although engineering managers at all levels have long ago recognized the benefits of processes that combine the analytical and experimental approaches to product design and analysis, their implementation in many cases was prohibited because of practical and cultural reasons. Analysts and experimentalists use their own vocabulary, work at different locations and often use different types of hardware and incompatible software and file formats. A number of technological and organizational changes in recent years have made it possible to remove these obstacles:

- Powerful and affordable workstations now offer sufficient horsepower to run industrial finite element analysis as well as all kinds of test systems.
- Computers are connected by networks (LAN, WAN or the Internet) so that data files can be easily moved from one system to another.
- Analysts and test engineers have access to more courses, literature and conferences that address the topic of linking FEA and test.
- New technical centers bring people with different backgrounds together under one roof with easy access to test labs and simulation tools.

In addition, there now exist several dedicated integration frameworks that further help to remove any remaining barriers. They are designed around a database that can contain analytical and test data imported using data translators with commercial or in-house developed FEA and test systems. Depending on the needs and goals, add-on analysis tools can exploit these hybrid data resources in decision- and knowledge-based synergistic processes from which the entire engineering team can benefit (Figure 1). Some examples:

- FEA results can be used to optimize the experimental setup (pretest analysis, virtual testing).
- Test results are used as reference data to validate, calibrate or refine a finite element model using error localization, correlation analysis and model updating tools.
- Unknown or badly known physical properties (e.g., damping) can be identified and uncertainties in finite element models better assessed.
- Hybrid models that partially contain FE and test models can be developed to build more complete models to include all essential components that contribute to the overall response, while maintaining a good balance between model size and performance.

It is of utmost importance that the integration and analysis software does not become an obstacle in itself. Unlike CAD or FEA, these tools may not be used on a daily basis and therefore should be easy to learn, with a user interface that conforms to standards, is simple and logical. It is essential that the user interface, graphics, analysis and reporting tools can be customized and extended so that they can be integrated in any existing CAE environment.

By better exploiting the combined test and analysis data, engineering staff and management will gain more confidence in the simulations they increasingly need to rely on for design optimization, acoustics, fatigue analysis and so on. They will also gain expertise for future modeling, build up reusable knowledge and more rapidly acquire essential skills like engineering judgment.

More Testing Needed

In our competitive world, engineers face the challenge to design increasingly complex products that need to satisfy more acceptance criteria. They must be stronger, lighter, quieter, safer and less expensive to build and maintain. To keep development time and cost competitive, industry relies on computerized simulation tools. Finite element analysis (FEA) is a powerful technique to simulate and improve the behavior of a product under various types of loadings. The FEA method has matured over the past three decades to a point where design, meshing, analysis and postprocessing are becoming highly integrated and automated. This predictive approach relies on the quality of the simulation model, the software to analyze it and the engineering judgment of the analyst interpreting the results.

In order to keep up with quality requirements, simulation models and procedures must be validated. Among the different ways of doing so, testing is intuitively still the preferred method. A trial-and-error design and analysis approach involving a series of prototypes is too time-consuming and expensive. It is therefore necessary to reduce the number of iterations on prototypes. This can be achieved by deriving more information on prototype testing and by doing more upfront simulation.

Ultimately it is unlikely that we will be able to one day *design-right first-time* and altogether eliminate testing, given that analysts become increasingly ambitious and will want to simulate the entire product lifecycle. It will be extremely difficult to remain on top of the increasing complexity encountered when modeling entire assembled products (versus components), using new materials (composites versus metals) and include the effects of manufacturing and environmental loads.

To successfully make the move to digital prototyping and thereby reduce the number of physical prototypes, predictions of performance should be provided with a measure of confidence and validated against experimental data. This requires quantifying the physical and numerical uncertainty. To reach this goal, fundamental change is required that will lead to some other form of simulation models that are capable of representing the underlying and relevant physics in a more realistic way. While current FEA is based on nominal values for input parameters, these new models will be probabilistic by nature. More testing is needed, not less, to provide the enormous amounts of data that can be statistically postprocessed and be converted into knowledge and insight.[1]

Uncertainty is Everywhere

Uncertainty in numerical simulation results manifests itself in two main classes: physical uncertainty and numerical uncertainty. There exist four main levels at which physical uncertainty, or scatter, becomes visible, namely:

- Boundary and initial conditions – impact velocity, impact angle, mass of vehicle, characteristics of barriers, etc.
- Material properties – yield stress, strain-rate parameters, density, local imperfections, etc.
- Geometry – shape, thickness, manufacturing and assembly tolerances, etc.
- Loads – earthquakes, wind gusts, sea waves, blasts, shocks, impacts, etc.

Uncertainty is further increased because many of these properties may vary substantially with temperature, frequency or load level. Information on these forms of scatter can be obtained by measurement. A sufficiently large number of samples needs to be evaluated to distinguish the natural and intrinsic scatter

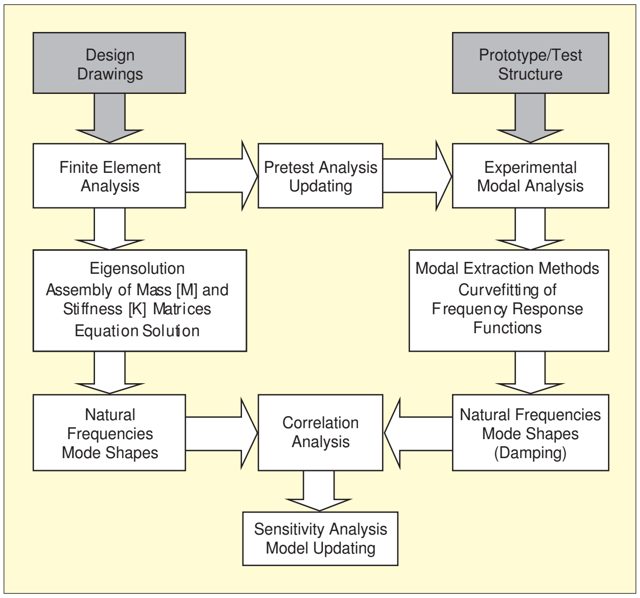

Figure 1. General procedure for linking test and analysis in modal analysis.

from the (often high) scatter that may be attributed to a small number of statistical samples.

Probability distribution functions and their associated properties can be obtained from statistical analysis of the test data. For example, the elastic modulus of isotropic material can be described using a normal (Gaussian) distribution that is characterized by a mean value and standard deviation.

The following numerical uncertainties can be identified:

- Conceptual modeling uncertainty – lack of data on the physical process involved, lack of system knowledge.
- Mathematical modeling uncertainty – accuracy of the mathematical model validity.
- Discretization error uncertainties – the choice of element types, mesh density, level of geometrical detail.
- Numerical solution uncertainty – rounding-off, convergence tolerances, integration step.
- Human mistakes – programming errors in the code or wrong utilization of the software, mistakes in data or units.

These types of scatter may or may not exist regardless of the physics involved. An example of the exhibit of numerical uncertainty is the different results that may be obtained by two finite element codes, using the same finite element model. Indeed changing solver, computing platform or element formulation can be possible causes of significant differences.

It is clear that uncertainty also exists in testing. Possible causes of physical uncertainty are related to:

- Test definition – fixture, mounting procedure, excitation method, transducer location, sensor weight, dynamic loading.
- Instrumentation – calibration, distortions, cabling noise.
- Data acquisition – digital signal processing, measurement and filtering error.

Techniques like experimental modal analysis are also subject to numerical uncertainty in the mathematical models that are used for modal parameter estimation.

Recognizing the existence of uncertainty and scatter is good reason to spend more time on validating simulation models. However, validation of a single model using a single test leads to only a snapshot result. While this may be valuable for roughly calibrating the input parameters, it should be understood that information on the broader picture is not taken into account unless scatter of the input parameters and reference test data are taken into account. Depending on the purpose of the simulation and the amount and type of reference test data that are available, different situations can be encountered:

- Selection of parameters based on a hierarchy defined by uncertainty and sensitivity level (joints, materials, etc.). This can require different types of activities like equivalent geometrical parameter updating, material identification, mesh refinement or load identification.
- The mesh itself determines the updating parameters. Some examples are: stick models for wing flutter analysis (Figure 2), component models that need to be reduced and included in an assembly (synthesis) or just for computational efficiency, refined models to be used for both dynamics and static analysis (stress, optimization), models to be used for acoustic analysis, etc.
- Scatter of output responses, obtained from repeated testing of the same structure or testing many samples, can be used to identify or adjust estimates of random properties of input parameters leading to a validated and updated stochastic simulation model.

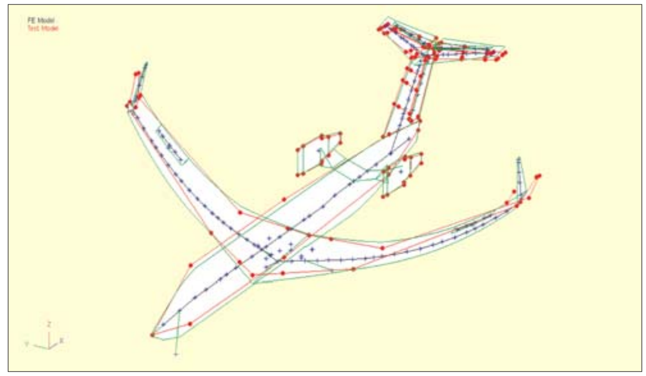

Figure 2. Stick model of a regional aircraft used to correlate wing bending modes with vibration test data. Beam elements are used to model the fuselage and wings (blue). Sensor locations are marked in red. The green lines are non-structural plot elements that are used to connect the sensor data to the finite element nodes.

From the previous paragraphs it should be clear that developing the proper model validation and updating strategy is often a complex issue and should be looked at as a process that must be adapted to specific needs. This requires access to a library of tools in a flexible, programmable environment.

Deterministic Model Updating

Most correlation and updating techniques for structural dynamics applications are based on the functional relationship between the measured responses and the structural parameters that can be expressed in terms of a Taylor series expansion limited to the linear term.[2] This relationship can be written as:

$$\{R_e\} = \{R_a\} + [\mathbf{S}]\left(\{P_u\} - \{P_o\}\right) \tag{1}$$

or

$$\{\Delta R\} = [\mathbf{S}]\{\Delta P\} \tag{2}$$

Where

- $\{R_e\}$ = Vector containing the reference system responses (experimental data)
- $\{R_a\}$ = Vector containing the predicted system responses for a given state $\{P_o\}$ of the parameter values
- $\{P_u\}$ = Vector containing the updated parameter values
- $[S]$ = Sensitivity matrix

The discrepancy in the initial model predictions and the test data is resolved by minimizing a weighted error E, given by:

$$Min\left(E = \{\Delta R\}^t [\mathbf{C_R}]\{\Delta R\} + \{\Delta P\}^t [\mathbf{C_P}]\{\Delta P\}\right) \tag{3}$$

and subject to constraints

$$g_i(P) \le 0;\ P_{\min} \le P \le P_{\max} \tag{4}$$

The matrices $[\mathbf{C_R}]$ and $[\mathbf{C_p}]$ respectively express the confidence of the user in the reference system responses and initial parameter estimates. In case the confidence matrices are derived from statistical postprocessing of multiple tests, they can be obtained from the covariance matrices.

Deriving Equation 3 and minimizing E with respect to the parameter values leads to an updated value for the parameter

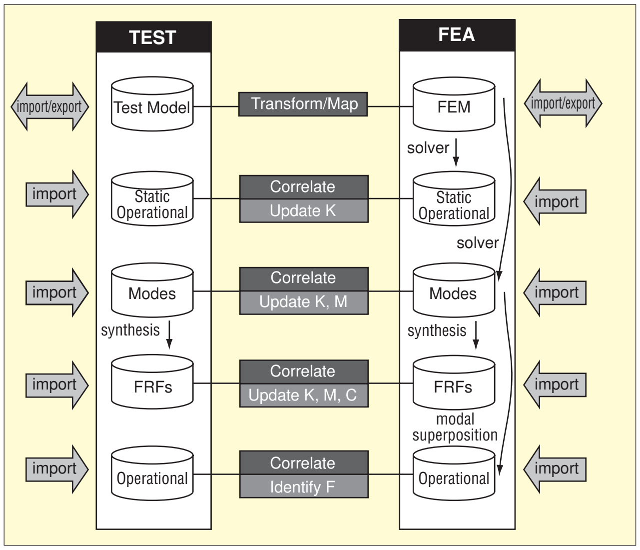

Figure 3. Test-analysis correlation and FE model updating for structural analysis can involve static or dynamic response data to update mass (M), stiffness (K), damping (C) or loads (F).

values that reduce the distance between the simulation and test results, but at the same time keeping the distance between the original and updated model minimal (in terms of parameter changes).

The similarity of model updating defined by Equations 3 and 4 with a general design optimization problem is striking. However, whereas the "objective function" in design optimization usually expresses the quality of the design in terms of cost, weight, reliability, etc., model updating is concerned about improving the finite element model to better predict the observed behavior while at the same time limiting the changes to the model. This translates to different choices of targets, variables and constraints. In design optimization, an optimum is acceptable if the parameter constraints are satisfied. In model updating, the changes to the model should remain within the range of expected variance of the input parameters. This is not only guaranteed by satisfying the proper parameter constraints but also by updating the most suitable parameters.

Another major issue is that the system of equations (Equation 1) is usually undetermined, i.e., the number of updating parameters largely exceeds the number of responses. Possible solutions are to reduce the number of updating parameters or increase the number of responses. Some ways to reduce the number of updating parameters are:

- Using techniques like local correlation analysis, sensitivity analysis and uncertainty analysis to reduce the model size by number of critical combinations of input parameters.
- Grouping elements and selecting global parameters for the group instead of updating at local element level.
- Defining many parameter relations (Equation 4) and thus reducing the number of independent parameters. It is to be noted that the $[\mathbf{C_p}]$ matrix in Equation 3 also offers the potential to express relations between parameters.
- Using a bottom-up modeling and test methodology. In this approach the different components that constitute an assembly are first modeled, tested and updated separately. This is followed by the repeated tests at different phases of the assembly that allows focusing on modeling of joints.

Alternatively, the number of responses can be increased by:

- Adding correlation targets. Computed correlation targets like MAC, or mode shape orthogonality (Figures 3 and 4) can be included in addition to responses that are directly measurable like mass, displacements, frequency response functions or resonance frequencies.
- Simultaneously updating the parameters that are common in variants of the FE model.[3] For example, solar panels for satellites can be tested during different stages of deployment and for each stage there is a FE model. This provides a richer set of test data to serve as references for updating element properties that are common in all configurations. Such properties can be, for example, the joint stiffness or material properties. Other examples are a launcher tested with different levels of fuel, or differently shaped test specimens made of a composite material that needs to be identified.
- Using full field measurement data (optical).

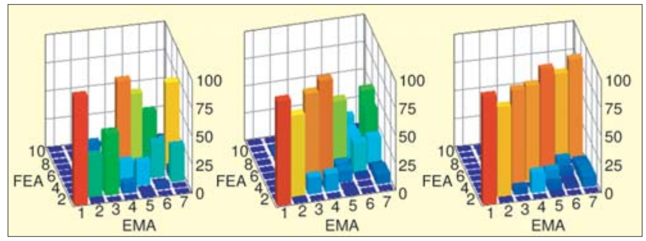

Figure 4. Modal assurance criterion (MAC) before updating the FE model (left), using test resonance frequencies as targets (middle) and using test resonance frequencies + MAC = 100 as targets (right).

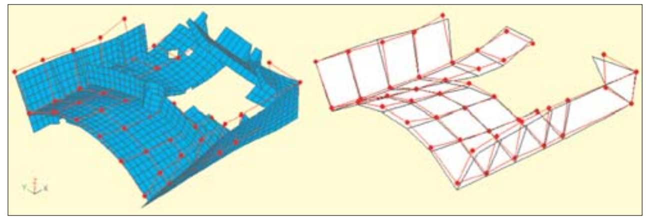

Figure 5. FE and test mode shape overlay of a hard disk plate using full model view (left) and the model reduced to the test sensor locations for enhanced interpretation (right).

A major benefit of dedicated commercial model updating software like FEMtools[4-5] is that it offers a wide range of response and parameter types to explore the behavior of the simulation model and quickly try different strategies to improve correlation with test. The following supporting and related tools need to be readily at hand to increase efficiency:

- Data translators – Two-directional translators are available with the most popular FEA and test database formats (NASTRAN, ANSYS, I-DEAS, ABAQUS, Universal File, etc.).
- Solver integration – For reanalysis of a modified FE model, virtually any solver can be used. Process integration commands, translators and drivers are available to automate the iterative updating cycle.
- Database management – Imported models and results are converted into a relational database of tables that can be edited, visualized or processed in any imaginable way. Typical operations are coordinate system transformations, mode shape normalization and creation of a set of elements based on topology, material or geometry.
- Parameter and response selection – All physical element properties (material, geometry), lumped properties (e.g., mass) and damping (modal, viscous, structural) can be selected as local or global updating parameters. The software manages the property cards and removes the hassle of manually creating new property cards as new element sets are created. All structural responses that can be measured can be selected as updating targets or correlation targets can be specified for criteria like MAC.
- Pretest analysis – When a baseline finite element model of a structure is available, it can be used to simulate tests. Using different observability or mode shape orthogonality criteria, test engineers can locate optimal locations and directions to measure and excite the structure. The FE model can be reduced to these locations and converted into a test model.[6-7]
- Correlation analysis – This task includes mapping the test model on the FE model, visual correlation, FRF correlation, local and global shape correlation, and computation of the quantitative level of correlation that can be used to monitor convergence during iterative model updating.[8] See Figures 5 and 6 for some examples of mode shape correlation.
- Sensitivity analysis – Sensitivity coefficients quantify the

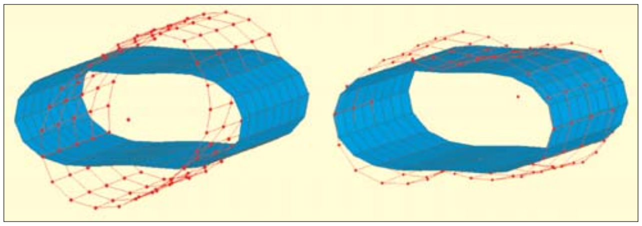

Figure 6. Mode shape correlation of axisymmetrical structures requires automated scaling and local rotation of double modes. The left figure shows the original overlay of FE mode (blue) and test mode (ref). The right figure shows the overlay after transforming the modes.

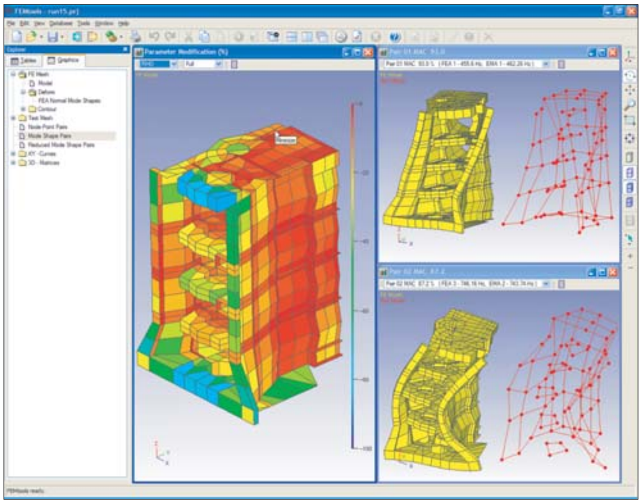

Figure 7. Modal correlation and sensitivity analysis of an engine block using multi-window toolbox.

variation of a response value (e.g., resonance frequency or mass) as a result of modifying a parameter value.

- Parameter estimation – The model updating problem that is defined by Equations 3 and 4 is in essence a constrained multi-objective optimization problem that potentially involves many parameters (up to the number of elements in the FE model). Although different methods can be considered, the Bayesian estimator, which is a weighted least squares method, has proven to be very suitable in terms of speed and the capacity to obtain a balanced and smooth convergence.
- Structural dynamics modification – This tool is used to rapidly apply and analyze the effect of structural changes to the dynamic response of structures without the need for re-meshing and reanalyses in a full FEA solver.[9] In case no acceptable parameter updates can be found, it may be required to reinvestigate the geometrical level of detail that was used in the modeling. For example, the effect of adding or removing lumped mass or a stiffener beam can be quickly evaluated.

These tools are provided in a framework environment that also provides the following facilities:

- Graphics – It is imperative that data at all stages of the process can be visualized using 2D and 3D graphics. Multiple graphics windows can be opened for side-by-side comparison. Animations and special graphics like FEA modes reduced to test locations are helpful to gain understanding of the structure (Figures 7 and 8).
- User interface – All tools are accessible via customizable menus and toolbars, or a command language for automated procedures.
- Scripting language – A built-in scripting language and API function library provides access to all FEA and test data, analysis tools and graphics for unlimited integration, automation and customization.

Probabilistic Model Validation and Updating

In the presence of scatter, the single deterministic response represents only one point of a cloud of values and therefore carries little information on likeliness and trends. Point clouds

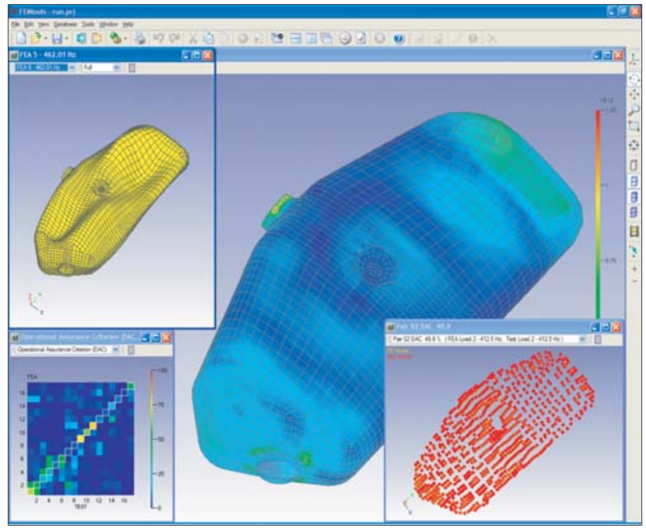

Figure 8. Identification of distributed pressure forces in a muffler cavity from measurements of operational deflection shapes (ODS) at the outside surfaces, elastic modes of the shell, and acoustic modes of the cavity. Measurements done with a laser scanning device (red markers).

on the other hand can be interpreted in terms of probability of a response value lying below or above a prescribed level. At a minimum, all responses are now defined as an interval with information on the confidence an analyst can have that the true response value will be within this interval. Additional statistical information can be derived if necessary.

Obtaining point clouds requires repeated testing of the same product (to reveal physical scatter) as well as testing a series of similar products to identify product variability. From the simulation side, the updating parameters can be randomized (i.e., apply a statistical probability distribution) and, using a probabilistic analysis tool like Monte Carlo Simulation, obtain point clouds for every parameter-response combination. There is a point for each state (also called sample) of the input variable. On statistical grounds, the collection of all point clouds, one for each input-output variable combination, constitutes a new concept of model, often referred to as Meta-Model in the literature. From these point clouds, statistical postprocessing results like histograms, the mean and standard deviation of output responses are obtained.

The correlation between simulated and experimentally obtained point clouds should now be analyzed using statistical measures. An example is the Mahalonobis distance:

$$d_M = (\mu_1 - \mu_2)^t \, COV_p^{-1} \, (\mu_1 - \mu_2) \tag{5}$$

where the vectors μ_1 and μ_2 represent the centers of gravity of each meta model and COV_p the pooled covariance matrix. Note the similarity with Equation 3. Whereas a deterministic measure of correlation, like the average relative error on resonance frequencies, provides only a snapshot measure that could be good or bad depending on coincidence, the Mahalonobis distance is clearly a much safer measure because it is based on position and shape of point clouds. Coincidence, good or bad luck with parameter estimations or variable measurement conditions can hardly influence this result.

The concept of meta models, both for numerical simulation and testing, together with the Mahalonobis metric, enables comparing responses in a statistically sound and rigorous manner. Position, shape and size of point clouds should be compared with the test meta-model being the reference. For example, consider the scatter plots shown in Figure 9. Differences in the principal axes of the two ellipses suggest either a major shortcoming in the discretization of structure geometry, a physical discrepancy between the two models or simply modelling errors. It should be clear that relative translation and overall size of point clouds are easier to correct than relative rotations. The former merely indicate systematic or global errors whereas the latter usually indicate (local) physical errors.

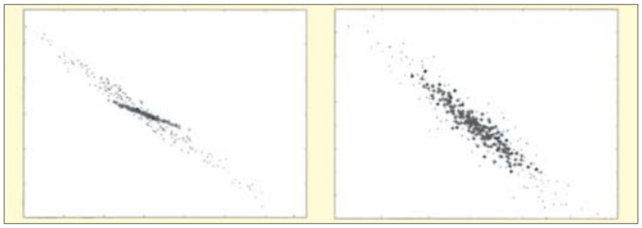

Figure 9. Example of physically distant (left) and close (right) models represented by means of point clouds.

Secondly, the level of scatter in the two models is clearly different. Although this may be desirable in some cases, it is in general preferable to obtain a simulation model that exhibits a level of scatter that is in balance with the scatter on the test data.

A fundamental contribution of meta-model analysis towards model updating is the possibility of pinpointing the dominating parameters of a system and to quantify the correlations between the input and output variables. This is the equivalent of sensitivity analysis in deterministic analysis. However, the concept of sensitivities, or gradients, no longer exists in the presence of scatter. So unless scatter is very low and can be neglected, other procedures to identify the dominant parameters need to be applied. In a similar way, not all available responses may be of equal relevance. Indeed, statistical postprocessing may reveal hidden relations and identify dependent and independent responses. As a result, the analyst can reduce the order of the system to include only the most dominant parameters and independent responses. Using regression analysis, relations between the dominant parameters and independent responses are established. This holds promise to solve some of the remaining difficulties in deterministic model updating like selection of updating parameters, definition of targets (i.e., when is correlation satisfactory) and interpretation of results. Without the additional insight that can be gained from probabilistic analysis, these decisions have to be made mainly based on engineering judgment.

The objective of probabilistic model updating is then to solve the system of equations for unknown parameter properties that change the center of gravity, the principal directions and the density of point clouds resulting from probabilistic analysis to match the corresponding test point clouds. In fact this comes down to 'updating' the Probability Density Function (PDF) of input parameters such that the PDF of the outputs correspond with the PDF of the experimental reference responses. In its simplest form, assuming a normal probability distribution, this means that in addition to the nominal value (like in deterministic model updating), the standard deviation of model parameters should also be adjusted.

It should be noted that the ranking of input parameters based on how much they influence the performance of the system offers additional benefits in the subsequent design improvement phase. Indeed, a designer or engineer does not need to spend time with input parameters that have only minor influence. Instead, the functional performance of the design can be modified most efficiently by working with the most dominant parameters only. Reducing the scatter on these parameters (for example by specifying more severe manufacturing tolerances) is the most rewarding in terms of robustness of the design. On the other hand, the engineer should relax tolerances on the parameters that do not significantly influence the performance, and in the process save money on manufacturing costs.

Summary and Future Outlook

Finite element analysis has become an essential tool to support virtual product development. To successfully make the move to digital prototyping and thereby reduce the number of physical prototypes, predictions of performance should be provided with a measure of confidence and validated against experimental data. Linking FEA and test is a complex process that touches all aspects of the engineering design and analysis cycle. The complexity and nature of this task requires dedicated software tools. Implementing this process is not an option, it is a must. Owing the added focus of model validation in standard codes of practice and quality assurance programs, industry is accelerating the pace at which this technology is adopted.

Some trends and recent developments may determine the future outlook of FEA and test integration technology:

- Frameworks dedicated to bridge the gap between FEA and test will continue to mature. They will provide seamless interfacing with more FEA and test systems and provide a growing range of diagnostic tools, updating algorithms and supporting wizards.
- Expect to see tools like design of experiments, response surfaces methods, and higher-order sensitivity analysis becoming part of model updating toolboxes to provide real-time parameter variational analysis for faster exploration of solution space.
- Statistical correlation and probabilistic model updating are naturally related with robust design and reliability analysis methods and an alignment of these technologies would be to their mutual benefit.
- Model updating will play an important role in applications like structural health monitoring[10] and material identification.[11] These applications will benefit from advancements in testing equipment (full field 2D and 3D measurement, wireless sensors) and data processing (output-only modal analysis algorithms, photogrammetry, digital image correlation). Easier measurement of displacement, velocity, acceleration, strain or temperature will enable model validation and updating in every field of simulation, e.g., thermal, acoustics, nonlinear static and dynamic analysis.

These extended capabilities, enabled by massive computing capacity, will contribute to creating more reliable and more usable simulation models, which is a prerequisite to realizing the full potential of CAE in the next decades.

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