Combining Test-Based and Finite Element-Based Models in NASTRAN

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Components often exist within critical load paths of complex systems that are not readily modeled with conventional finite element techniques. Furthermore, the owner of the system model often has no design authority or interest in some components other than their effects on the remaining system. Frequency dependent stiffness and damping for these components can also significantly add to the modeling difficulty. This article describes a process for combining test-based and finite element-based models in NASTRAN that circumvents many of these limitations. Advantages of including test-based models within a complex system model include: (1) It is generally faster and more accurate to develop a test-based model than a finite element-based model; (2) Computer run times and computer resources are reduced; and (3) Minimizing errors, which are frequently present in complex system models, is possible. In this combined analysis approach, test-based models are determined using admittance modeling. A test-based model for the component's dynamics is derived at the locations where it interfaces with the system finite element model. This model is then converted into a NASTRAN readable format and analytically coupled to the finite element model, enabling prediction of the combined system response. Several examples are presented to demonstrate the efficacy of this approach.

A brief introduction to admittance modeling[1] is presented as background for the work referenced in this article. Admittance modeling is a mathematical technique where subsystems can be combined or separated in a building-block approach by using accelerance (acceleration over force) frequency response functions (FRFs). An example is shown in Figure 1, where we have two structures (A and B) joined at a single point. The desired subsystem FRFs are H_B, the base structure. The FRFs for the add-on structure are H_A and represent all of the required test fixtures. The combined structure, H_C, is the total system.

It is straightforward to derive the relationships between driving-point FRFs where each FRF is the ratio of the acceleration response to the input force at the connection point of each structure. At a given frequency, H_A, H_B and H_C are 6×6 matrices of complex numbers that are the ratio of the six acceleration responses (three translational and three rotational) at the connection point due to each of the six input forces (three forces and three moments) applied at the connection point. For an applied force (f_C) at the connection point, force equilibrium requires that $f_C = f_A + f_B$, where f_A and f_B are the forces on structures A and B at the connection point. Since compatibility at the connection point requires that $a_A = a_B = a_C$, we can divide the force equation by acceleration to rewrite it in terms of FRFs ($H = a/f$) as

$$H_C^{-1} = H_A^{-1} + H_B^{-1} \tag{1}$$

If H_A and H_B are measured, then H_C can be found by

$$H_C = \left[H_A^{-1} + H_B^{-1}\right]^{-1} \tag{2}$$

Similarly, if H_A and H_C are measured, then H_B can be found by

$$H_B = \left[H_C^{-1} - H_A^{-1}\right]^{-1} \tag{3}$$

which can be rewritten as

$$H_B = \left[I - H_C H_A^{-1}\right]^{-1} H_C \tag{4}$$

These equations are for acceleration responses and applied forces at the connection point. Similar relations can be derived for applied forces and acceleration responses at arbitrary locations on the structures.

Nomenclature

- C = damping matrix
- CG = center of gravity
- F = force vector
- FRF = frequency response function
- H = acceleration-over-force FRF
- I = identity matrix
- K = stiffness matrix
- M = mass matrix
- X = displacement vector
- κ = complex stiffness
- ω = frequency

Inclusion of H_B^{-1} in NASTRAN

The type of dynamic solution required in NASTRAN here is a direct frequency response analysis. In other words, there is no modal reduction performed via the normal modes of the structure. NASTRAN provides multiple ways to read in externally-generated matrices, including Direct Matrix Input at a Grid (DMIG) and INPUTT4.[2] In the discussion that follows, the DMIG option will be utilized.

Note that H_B is a matrix with units of acceleration divided by force, consequently H_B^{-1} can be viewed as a complex mass matrix. This virtual complex mass can be multiplied by $-\omega^2$ to obtain complex stiffness. The complex stiffness $\kappa(\omega)$ relates the response to the force by the equation

$$\kappa(\omega) X(\omega) = F(\omega) \tag{5}$$

where

$$\kappa(\omega) = \left[-\omega^2 M + i\omega C + K\right] \tag{6}$$

NASTRAN refers to $\kappa(\omega)$ as K2PP stiffness, and it is added to the matrices generated by conventional finite elements. Another option would have been to use H_B^{-1} directly and input the matrices as M2PP mass. Both methods were tried, and the end results were identical.

NASTRAN has two limitations on K2PP (or M2PP). First, it requires that the matrices be symmetric. This is really no limitation, as one expects reciprocity for these FRF matrices, which results in their symmetry. In reality, due to numerical and test uncertainty, the matrices are not perfectly symmetric, but the data was post-processed to enforce symmetry. The second limitation is that NASTRAN allows for the input of only one unique set of K2PP and M2PP matrices. Because H_B^{-1} is a function of frequency, it is necessary to read in a different H_B^{-1} for each frequency line. To circumvent this limitation, a NASTRAN DMAP (Direct Matrix Abstract Program) alter (modification of DMAP) was developed that uses a different subcase to compute

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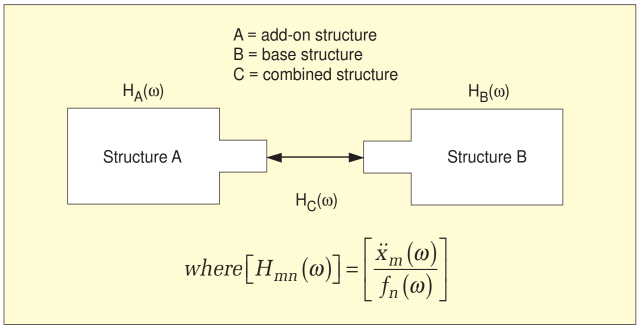

Figure 1. Admittance model example.

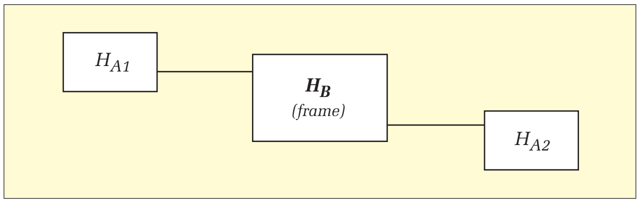

Figure 2. Base structure and add-on structure.

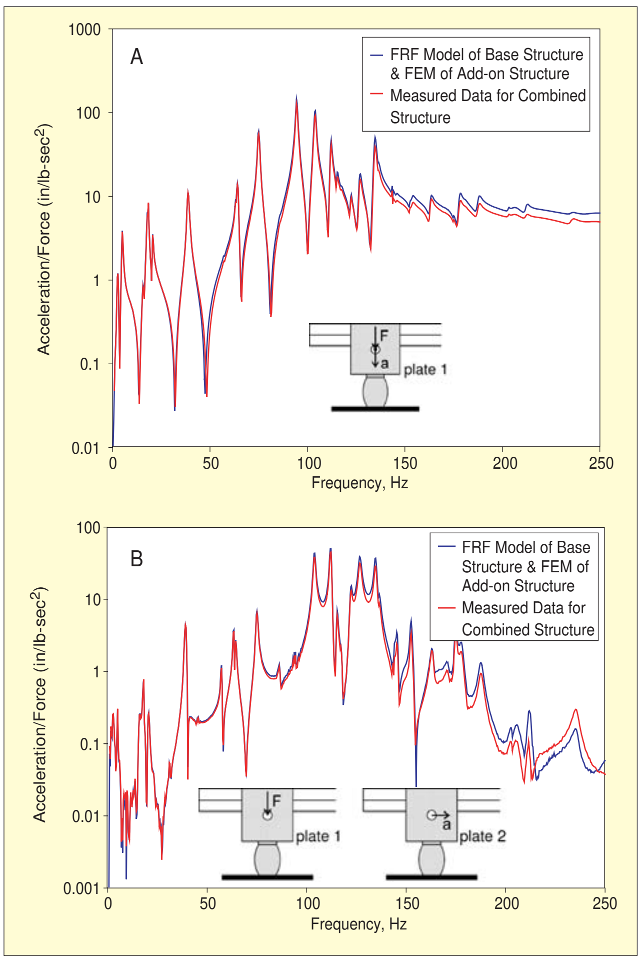

Figure 3. Initial verification. (A) Vertical driving point response. (B) Horizontal response diagonally across base structure.

the response at each frequency line; therefore a different H_B^{-1} is read in for each subcase. The original DMAP alter was developed for NASTRAN version 70.7 and a subsequent version was developed for NASTRAN version 2001.

The typical analysis conducted so far has 800 frequency lines between 0 and 250 Hz. A software code running in MATLAB processes all the FRF data measured on the combined structure to create H_C. Then, using the known H_A for the add-on structure (which can be obtained either via further experiments or from an analytical solution), the MATLAB code performs the admittance modeling calculations given previously to calculate H_B^{-1} for the base structure alone and outputs the complex stiffness in NASTRAN DMIG bulk data cards.

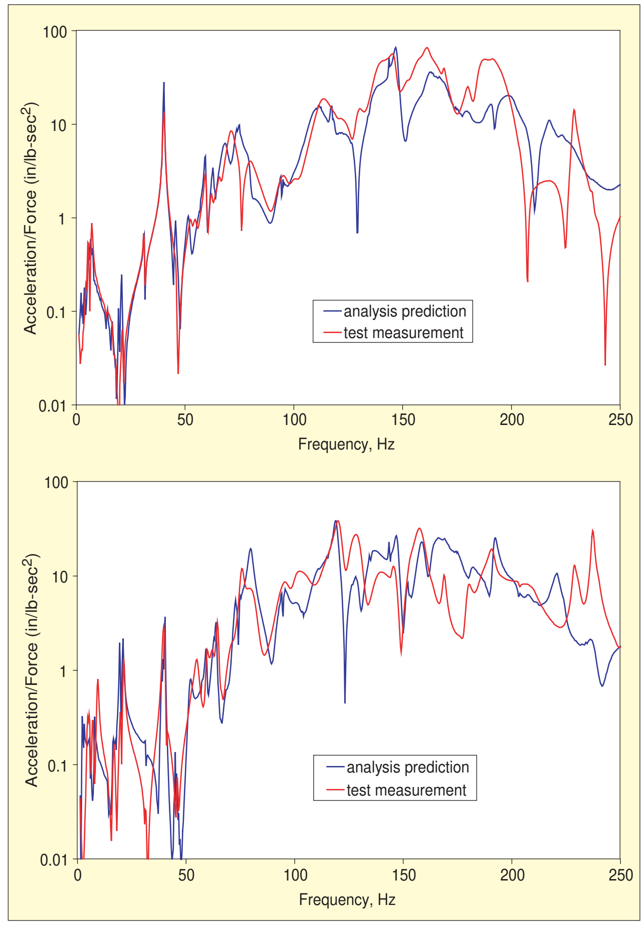

Figure 4. Horizontal input on attached structure, test versus analysis comparison. (A) Horizontal response due to horizontal input. (B) Vertical response due to horizontal input.

Initial Validation of Analysis Approach

To validate the admittance technique and its usage with NASTRAN, initial experiments were performed on a simple frame structure with multiple connection points to which test fixtures could be attached. H_C was measured for the system, which included the simple frame structure (H_B) plus the add-on structure (H_A) consisting of H_{A1} and H_{A2} in Figure 2. The admittance process was then used to calculate H_B by removing H_A from H_C.

Initial checkout of the approach compared admittance code predictions against NASTRAN predictions for the base structure with the add-on structure. Structure B was the frame structure and the test fixture mass, with structure A consisting only of the test fixture stiffness and damping. Admittance processing of the measured H_C gave H_B, the test-based FRF model of structure B. H_B^{-1} was then converted into NASTRAN DMIG bulk data cards. In NASTRAN, a finite element model of structure A was created and attached to the FRF model of structure B. Very good agreement was observed between the NASTRAN predictions and the measured data. For a vertical input at one attachment point, Figure 3 shows an FRF of the vertical driving-point response and an FRF of the horizontal response at the other attachment point.

Additional validation data were then created by applying different forces (i.e., forces at different locations than those used to measure H_C and calculate H_B) to the add-on structures and measuring the response to compare with NASTRAN predictions. Excellent agreement (not shown here) was found between predictions based on H_B and the newly measured validation data.

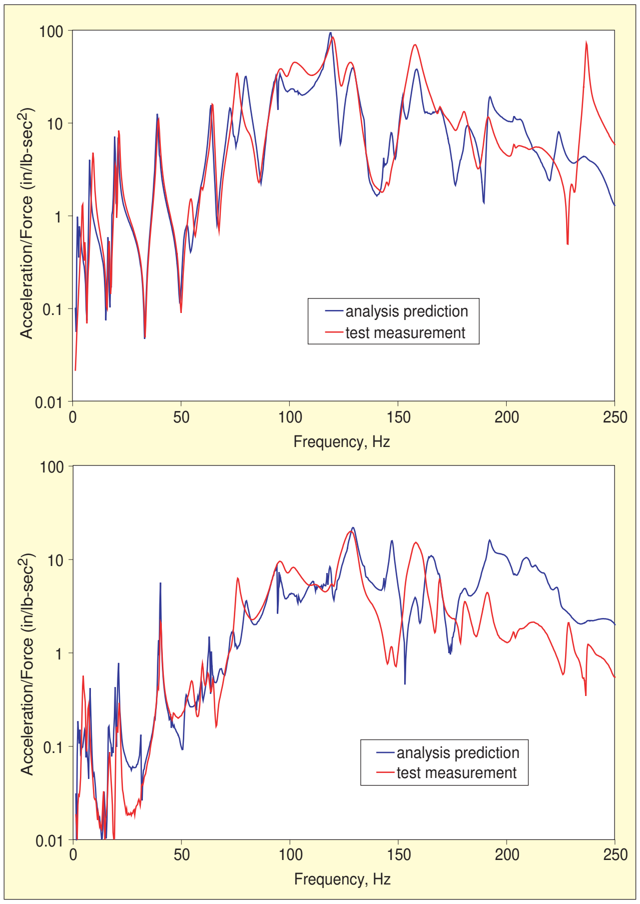

Figure 5. Vertical input on attached structure, test versus analysis comparison. (A) Vertical response due to vertical input. (B) Horizontal response due to vertical input.

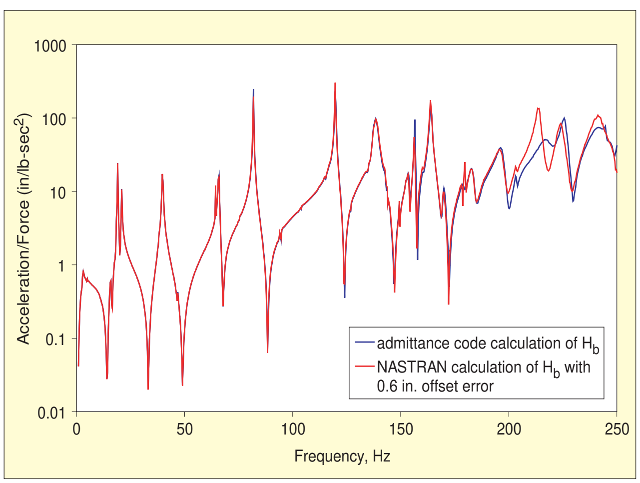

Figure 6. Effect of mass offset.

Verification with Real Structures

Given the agreement shown in Figure 3, the next logical step was to use H_B^{-1} of the frame structure alone (with the mass, stiffness and damping of the add-on structures removed) attached to a finite element model of a more complex add-on structure. For comparison with the analysis results, validation data were acquired for the frame structure attached to the more complex add-on structure with well-characterized boundary conditions. Figures 4 and 5 show axial and vertical attachment responses for axial and vertical excitation forces, respectively.

Other Response Locations

While it is very valuable to know the structural response at attachment locations, it is generally more valuable to know the responses at interior locations of a structure. These quantities are not recoverable during the analysis just described, as there is no driving-point information available to build the necessary H_B matrix. However, these quantities are recoverable in a post processing operation as long as the transfer functions between the attachment locations and the locations of interest are measured. A motion/motion transfer function is used as shown below,

$$x^r = \left[H^r{}_C H_C^{-1}\right] x^s \qquad (7)$$

where x^r are the desired interior responses, H_C is the same as used previously, $H^r{}_C$ is the acceleration/force transfer function between the desired interior responses and the attachment point forces, and x^s is the attachment point response. After the product of $H^r{}_C$ and H_C^{-1} is computed, it can either be converted to NASTRAN DMIG format in the same manner as H_B and a NASTRAN DMAP written to perform the requisite matrix multiplication, or x^s can be output from NASTRAN and the matrix multiplication can be performed in MATLAB or similar software.

Other Lessons Learned

During the process of validating this analysis method, H_B was calculated with and without portions of the add-on structure, in particular the test fixtures used to apply the input loads. Subsequently this mass was subtracted off in NASTRAN with negative mass (and inertia) elements. The add-on structures are in the form of 2-in. thick aluminum plates, creating a 1-in. offset of its CG away from the attachment location of the test structure. This offset was included in the analytical model for H_A, which was subtracted from H_C to produce H_B. The initial comparison between the analysis prediction and the measured value for H_B is shown in Figure 6 for a driving-point vertical response at an attachment location. The comparison looks good, but it is not line-for-line identical. Evaluation of the hardware and the MATLAB admittance code revealed that there is also a vertical offset needed to compensate for the add-on structure mass, which shifts the CG downward. When this additional offset (of only 0.6 in.) was included, the comparisons were almost line-for-line accurate. This exercise was very valuable as it provided more confidence in the analysis process and showed how important it is to physically locate items accurately, as small offsets have significant effects, especially at higher frequencies.

Conclusions

An analysis procedure for combining test-based and finite element-based models in NASTRAN has been developed and validated. This procedure allows for the accurate representation of complex systems in which some component is not readily modeled with conventional finite element techniques. This combined modeling approach potentially minimizes computer run times and necessary computer resources, and produces a more accurate system model.

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