An Introduction to Shaker Thermodynamics

"Y'all want them eggs over easy, or just shook real good?"

George Fox Lang, Associate Editor

Large electrodynamic shaker systems produce significant heat. This article provides some guidance in analyzing the heat generated by your testing system. By knowing *how much heat* is generated and *where*, you can plan for appropriate laboratory cooling before you install a new shaker.

Heat is generated by electrical power dissipation (I^2R loss) in the voice-coil and in the exciter field-coil of an electrodynamic shaker. A supporting blower directs airflow through the shaker to extract this heat and convey it out of the laboratory space. However, the blower never extracts *all* of the heat, and thus the shaker remains a heat load to the lab cooling system. Additionally, the blower is electrically driven and is never perfectly efficient; it is also a potential heat load. Finally, the power amplifier/field exciter electronics throw heat into the lab as these electrical power conversions have some associated inefficiency.

Hence it is necessary to analyze the thermal performance of three elements – the shaker, the blower and the amplifier. We will start with the shaker, and the simple heat-balance model illustrated in Figure 1.

Modeling the Shaker

The shaker is modeled as a lump of iron heated by an electrical source and cooled by an airflow. Cool air enters the shaker at a temperature of T_{inlet}, which is assumed constant. The air is heated to the body temperature of the shaker, T_{shaker}, and remains at this temperature until it is expelled from the cooling system. That is, the ducting is considered thermally lossless.

Prior to operation, the shaker is presumed to be at the ambient T_{inlet} temperature. Once it begins a shaking operation, the shaker body temperature and exhaust air temperature increase by T_{rise}. Hence, T_{rise}, T_{out}, Q_{out} and Q_{lab} are functions of *time* described by the energy balance:

$$Q_{in} - Q_{out} - Q_{lab} = M_T C_{fe} T_{rise} \tag{1}$$

where:

Q_{in} = electrically-generated heat input to shaker (J)
Q_{out} = exhaust heat extracted by cooling air (J)
Q_{lab} = heat lost be shaker directly into laboratory (J)
M_T = thermal mass (kg)
C_{fe} = specific heat of iron (J/kg °C)
T_{rise} = temperature rise = $T_{shaker} - T_{inlet}$ (°C)

Differentiating Eq. 1 with respect to time yields the corresponding power balance.

$$P_{in} - P_{out} - P_{lab} = M_T C_{fe} \frac{dT_{rise}}{dt} \tag{2}$$

where:

P_{in} = heat power input to shaker (W)
P_{out} = exhaust power extracted by cooling air (W)
P_{lab} = power lost be shaker directly into laboratory (W)

Input power, P_{in}, is known to be constant and equal to the sum of the $i_{rms}{}^2 R$ (Watts) dissipated by the voice coil and the $e_{field}\, i_{field}$ (DC) power dissipated by the exciter coil.

Adding the Blower

Output power, P_{out}, may be expressed in terms of the airflow parameters and the instantaneous temperature rise. Specifically:

$$P_{out} = G C_A \rho_A T_{rise} \tag{3}$$

where:

G = volumetric air flow (m/s)
C_A = specific heat, air (J/kg °C)
ρ_A = density, air (kg/m³)

The power, P_{lab}, lost directly from the shaker to the laboratory is less clearly understood, depending strongly upon geometric specifics of the shaker. We will model this simply as a time-variant conduction power loss proportional to the input power and to the rise in shaker temperature. That is:

$$P_{lab} = \alpha P_{in} T_{rise} \tag{4}$$

where:

α = loss power coefficient (°C^{-1})

The value of α will be subsequently determined from more intuitive parameters.

Combining Shaker and Blower

Substituting Eqs. 3 and 4 into Eq. 2 yields the following system definition.

$$\frac{dT_{rise}}{dt} + \left(\frac{\alpha P_{in} + G C_A \rho_A}{M_T C_{fe}} \right) T_{rise} = \frac{P_{in}}{M_T C_{fe}} \tag{5}$$

The solution to this ordinary, linear, first-order, non-homogeneous differential equation with constant coefficients is given by:

$$T_{rise} = T_{rss} \left(1 - e^{-\frac{t}{\tau}} \right) \tag{6}$$

where the *time constant* τ is:

$$\tau = \frac{M_T C_{fe}}{\alpha P_{in} + G C_A \rho_A} \text{ (s)}$$

and the *steady-state temperature rise* T_{rss} is:

$$T_{rss} = \frac{P_{in}}{\alpha P_{in} + G C_A \rho_A} \text{ (°C)}$$

The time t_T required to reach any shaker temperature, $T < T_{rss}$ such as a maximum specified operating temperature, may thus be evaluated as:

$$t_T = -\tau \ln \left(\frac{T_{inlet} + T_{rss} - T}{T_{rss}} \right) \tag{7}$$

As asserted by Eqs. 3 and 4, the exhaust heat power P_{out} and the heat power lost directly to the laboratory P_{lab} are proportional to T_{rise} and thus start at zero value and increase exponentially toward the "steady state" values of Eqs. 8 and 9 as the shaker warms. That is:

$$P_{outSS} = GC_A \rho_A T_{rss} \tag{8}$$

$$P_{labSS} = \alpha P_{in} T_{rss} \tag{9}$$

Further, when the shaker temperature rise stabilizes at T_{rss}, a simple power balance between P_{in} and the thermal cooling is established. Specifically:

$$P_{in} = P_{outSS} + P_{labSS} \tag{10}$$

When the shaker has reached T_{rss}, we can define the "cooling efficiency" ε as the ratio of the Watts of heat power extracted by the blower to the total electrical Watts applied to the shaker, or:

$$\varepsilon = \frac{P_{outSS}}{P_{in}} \tag{11}$$

A little algebra applied to Eqs. 8 through 11 allows us to evaluate the somewhat cryptic α coefficient used to define P_{lab} in Eq. 4 in terms of ε, whose concept is intuitive and clear. Specifically:

$$\alpha = \frac{1-\varepsilon}{T_{rss}} = \frac{(1-\varepsilon)}{\varepsilon} \cdot \frac{GC_A \rho_A}{P_{in}} \tag{12}$$

The cooling blower accepts a specified electrical power input P_{inb} (Watts) and returns a volumetric airflow G (m^3/s) against a specified resistance pressure or 'head' H (n/m^2). Thus the useful blower output power P_{outb} (Watts) may be stated:

$$P_{outb} = GH \tag{13}$$

The electromechanical efficiency ε_b of the blower is simply:

$$\varepsilon_b = \frac{P_{bout}}{P_{bin}} = \frac{GH}{P_{in}} \tag{14}$$

The difference between P_{inb} and P_{outb} is converted to heat P_{heatb} thrown off from the blower to its surroundings. P_{heatb} is defined by:

$$P_{heatb} = P_{inb} - P_{outb} = P_{inb} - GH = (1-\varepsilon_b) P_{in} \tag{15}$$

The blower normally sucks in cool air at temperature T_{lab} from the laboratory. When this is done, the air conditioning system must make up the flow with an attendant cooling burden to chill the air from an outside temperature of $T_{outside}$. Chilling this "make-up air" adds an important thermal power load of P_{makeup}, where:

$$P_{makeup} = GC_A \rho_A (T_{outside} - T_{lab}) \tag{16}$$

About the Amplifier

The system's power amplifier is responsible to deliver P_{in} to the shaker. It does so with a rated power efficiency ε_{pa}. Thus the required power input to this component P_{inpa} may be stated:

$$P_{inpa} = \frac{P_{in}}{\varepsilon_{pa}} \tag{17}$$

The heat generated by the power amplifier P_{paheat} is thrown off into the laboratory and is defined by:

$$P_{paheat} = P_{inpa} - P_{in} = \frac{1-\varepsilon_{pa}}{\varepsilon_{pa}} P_{in} \tag{18}$$

Modern digital amplifiers provide very high efficiency (90% typical), while analog amplifiers can provide no more than 50% efficiency, by definition. This is clearly an important difference. A 90% efficient amplifier only draws 56% of the line power *and only generates 11% of the heat* associated with a 50% efficient amplifier serving the same shaker load.

In Summary

The maximum cooling system thermal load will depend on the geometric arrangements of components. In general, the air-conditioning must bear the heat load of:

1. P_{labSS} – steady-state shaker heat lost directly to the laboratory (Eq. 9).
2. P_{makeup} – the cooling of air passed through the shaker blower (Eq. 16).
3. P_{paheat} – the amplifier heat due to inefficiency (Eq. 18).

If the blower is housed within the laboratory space, add:

4. P_{heatb} – the heat thrown off by the blower (Eq. 15).

If the blower is vented within the air-conditioned laboratory space we must add:

5. P_{outSS} – the steady-state blower heat exhaust (Eq. 8), but we may delete P_{makeup}.

At least six "lab-plumbing" arrangements are possible. Each presents the cooling system with a different thermal load.

1. In the most common arrangement, the blower is outside of the laboratory space housing the shaker and power amplifier and sucks cool air from the lab through the shaker ($T_{inlet} = T_{lab}$), discharging it outside. This minimizes blower noise in the lab and presents no cooling-exhaust or blower heat loads to the workspace.
2. The blower can be better protected and maintained by installing it within the workspace with its discharge ported to the outside. This increases the air-conditioning load by the blower heat P_{heatb} and raises the noise level within the laboratory (unless a sound enclosure is employed).
3. In (unusual) circumstances with extremely high outside temperature, it may prove cost-effective to house the blower in the lab and allow it to discharge in the laboratory space. This adds the blower exhaust P_{out} load to the cooling burden, but eliminates the P_{makeup} load, as cooling air is re-circulated within the laboratory. However, the exhaust noise becomes an issue.

In cooler climates, there may be benefit in using *outside* air instead of laboratory air to cool the shaker. Using outside air eliminates the P_{makeup} burden from all configurations. When using outside air ($T_{inlet} = T_{outside}$), the blower and discharge may be arranged:

4. . . . with the blower outside of the laboratory space.
5. . . . with the blower in the lab, discharging to the outside.
6. . . . with the blower in the lab, discharging within the workspace (winter heating).

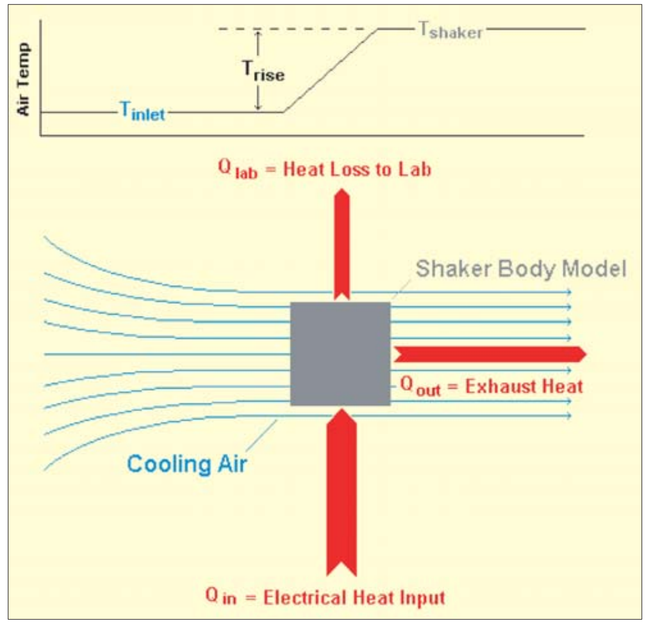
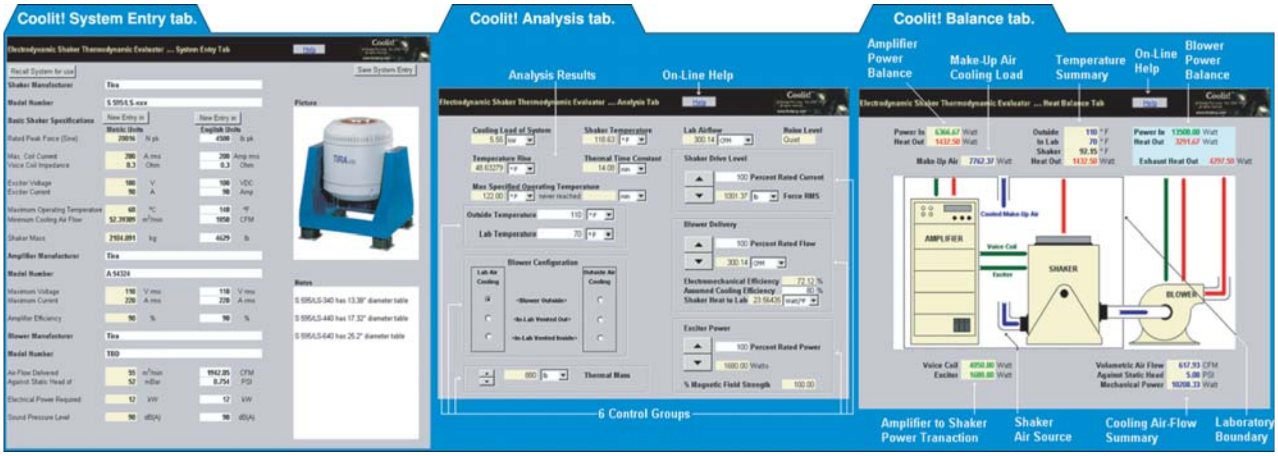


Figure 1. The basic thermal model of an air-cooled electrodynamic shaker.

Important Thoughts About the Thermal Input Power

Reference 2 provided derivation of the following set of four coupled differential equations modeling the electromechanical behavior of a shaker:

The Coolit! Spreadsheet



Coolit!™ is an elegant Excel® spreadsheet for thermal analysis of electrodynamic shakers. It uses the mathematical model described in this article and provides operating conveniences including push-button engineering unit selection and library storage/recall of shaker systems. Coolit! is simply organized using three dedicated sheets or tabs.

The 15 required shaker system parameters are entered on a pre-formatted "System Entry" tab in either ISO metric or common English units. Once entered, this data tab may be saved to the library of systems for subsequent recall and use. A photograph, notes and user calculations may be stored along with the basic entries.

The 'Analysis' tab provides six groups of controls describing the configuration of the equipment and its operating condition. Summary analysis results immediately reflect any control change and each may be viewed in any desired combination of engineering units. In addition to outside/laboratory temperature specification, shaker drive level adjustment and blower configuration selection, additional controls allow simulation of a blower duct obstruction or change in the field-exciter settings.

The 'Balance' tab provides the power flow for every system component, superimposed on a sketch of the chosen (1 of 6) configuration of the equipment. The Balance sheet tallies the detailed heat power, electrical power and airflow parameters that lead to the Analysis summary.

Please visit www.foxlang.com ⇒ Products ⇒ Coolit! for more details.

$$\begin{bmatrix} M_C & 0 & 0 & 0 \\ 0 & M_T+M_D & 0 & 0 \\ 0 & 0 & M_B & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{X}_C \\ \ddot{X}_T \\ \ddot{X}_B \\ 0 \end{Bmatrix} + \begin{bmatrix} C_C & -C_C & 0 & 0 \\ -C_C & C_C+C_S & -C_S & 0 \\ 0 & -C_S & C_B+C_S & 0 \\ k_2 & 0 & -k_2 & L \end{bmatrix}$$

$$\begin{Bmatrix} \dot{X}_C \\ \dot{X}_T \\ \dot{X}_B \\ di/dt \end{Bmatrix} + \begin{bmatrix} K_C & -K_C & 0 & -k_1 \\ -K_C & K_S+K_C & -K_S & 0 \\ 0 & -K_S & K_B+K_S & k_1 \\ 0 & 0 & 0 & R \end{bmatrix} \begin{Bmatrix} X_C \\ X_T \\ X_B \\ i \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ e \end{Bmatrix} \quad (19)$$

One solution to this set of coupled equations is provided by four transfer functions, all with current as the reference or denominator term. Three of these have absolute motions (table, coil and body) as the numerator. The fourth (voice-coil impedance) has the drive voltage as its numerator.

The Figure 2 shows typical results, in this instance for a 1200 lb force shaker with trunnion base isolation. The transfer functions presented reflect a payload of 150 kg (the maximum specified payload) rigidly mounted to the table. Note that the analysis frequency span shown exceeds the manufacturer's 3000 Hz maximum operating limit, which is imposed to avoid overstressing the armature near its resonance.

Observe that all four traces reflect three resonances (to varying degrees) when the shaker is heavily laden. The isolation resonance occurs at less than 1 Hz. The suspension resonance may be clearly seen just above 3 Hz and the armature resonance peaks at 3250 Hz.

Most importantly, note that the electrical impedance (green trace) is *nearly constant* at 0.5 Ω from 10 Hz to 1000 Hz. This value corresponds to the coil resistance R. The voice-coil impedance rises steadily at frequencies above $2\pi R/L$ Hz, due to the presence of inductance L. It peaks locally near the mechanical resonances as the back-EMF generated in the voice-coil reflects increased relative velocity between the shaker body and voice-coil.

These four solution transfer functions may be used in concert with the various operating limits of the system to understand how these limits restrict operation. Figure 3 presents the maximum voltage and current that may be applied to our example shaker in a *maximum drive* test. In this test, the shaker is driven with the largest input that may be applied without exceeding:

1. Maximum Shaker Current
2. Maximum Shaker Force Rating
3. Maximum Shaker Stroke Rating
4. Maximum Amplifier Voltage Available
5. Maximum Amplifier Current Available

In Figure 3, the left-hand figure (A) is a test run with a bare table; the right figure (B) presents the same test with the maximum payload (150 kg) that is supported by the shaker's load-leveling pneumatic sub-system.

The multi-colored bar running across the top of each figure denotes the input-limiting parameter as a function of frequency. Blue denotes shaker stroke, red denotes amplifier voltage, black denotes voice-coil current and purple denotes shaker force (armature stress).

Stroke is invariably the limit-variable at low frequency. When the shaker is lightly laden, amplifier voltage proves inadequate near the suspension resonance due to high back-EMF. (Manufacturers reflect this as a *shaker velocity limit* specification; most systems can actually exceed their specified velocity limit.) In the mid-range, voice-coil current is the limiting variable. Exceeding this limit will overheat the coil, resulting in serious damage. At frequencies approaching the upper operating limit (near the armature resonance), the limit variable can rapidly change between voltage, current and shaker force in a pattern that is model-specific. In a properly sized system, the amplifier current capacity should never be overtaxed in a sine or random test.

Note from these figures that the voice-coil current is the limiting variable over most of the operating range. Note further that

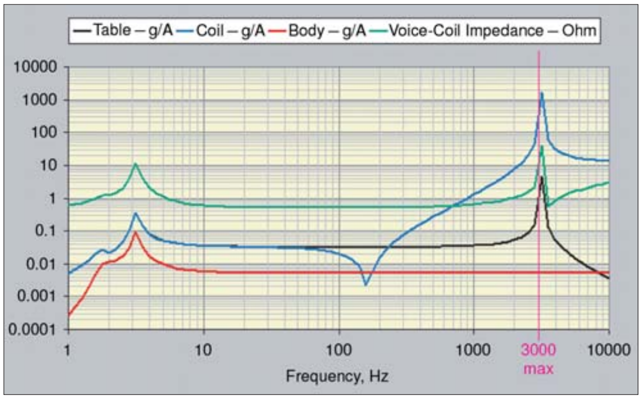


Figure 2. Solution transfer functions with a 150 kg payload.

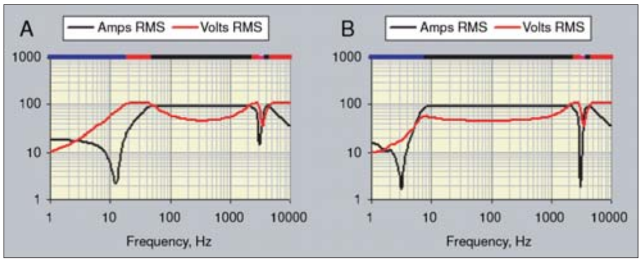


Figure 3. Maximum drive parameters for bare-table (A) and fully laden (B, 150 kg payload) tests.

the bandwidth over which current is the limiting factor increases as the payload gets larger. In this age of "do the most with the least," this implies that voice-coil current will be the dominant limiting factor of any test.

In Figure 4, six plots illustrate the *electrical* performance of our example system from several perspectives. The left-hand figures present voice-coil impedance (magnitude) and power factor (cosine of impedance phase angle). The right-hand figures plot the total electrical power (Watts) dissipated by the shaker, the $I_{real}{}^2 Z_{real}$ (I^2R) power dissipated by the voice-coil and the difference between these real powers during a *maximum drive* test. The top row presents bare-table results, the middle row illustrates a light payload of 25 kg and the bottom row shows results with a 150 kg payload.

From the left-hand figures we see that the impedance is exactly equal to R wherever the power factor is unity (impedance phase angle equal to 0°). Over a broader frequency range surrounding this condition, we find the power factor diminishes symmetrically against increases in the (complex) impedance. Hence, the product of power factor and impedance magnitude remains essentially equal to R over a much broader frequency range. This verifies that I^2R is a valid estimate of power dissipated by the voice-coil over a frequency range that extends (almost) from the suspension resonance to the armature resonance. This is a particularly important finding for thermal analysis!

From the right-hand figure we note that the power dissipated in the voice-coil essentially 'overlays' the total power dissipated by the shaker. The (blue) difference between these wattages is that power which drives the mechanical system. It is clear from these figures that an electrodynamic shaker is a very inefficient energy converter! Virtually all of the electrical drive power applied to the machine is dissipated as heat by the voice-coil.

Hence, for practical purposes, the conservative estimate of the "worst case" heating power input P_{in} is provided by:

$$Max(P_{in}) = I^2_{maxrms} R + V_{ExciterDC} I_{ExciterDC} \qquad (20)$$

I_{maxRMS}, $V_{Exciter\,DC}$ and $I_{ExciterDC}$ are standard shaker spec-sheet items. Resistance R is well estimated by the manufacturer's *nominal voice-coil impedance* specification.

This conclusion also implies that P_{in} is proportional to the square of the rms force delivered by the shaker against its payload,[3] plus a constant term reflecting the cost of generating the magnetic field surrounding the voice-coil.

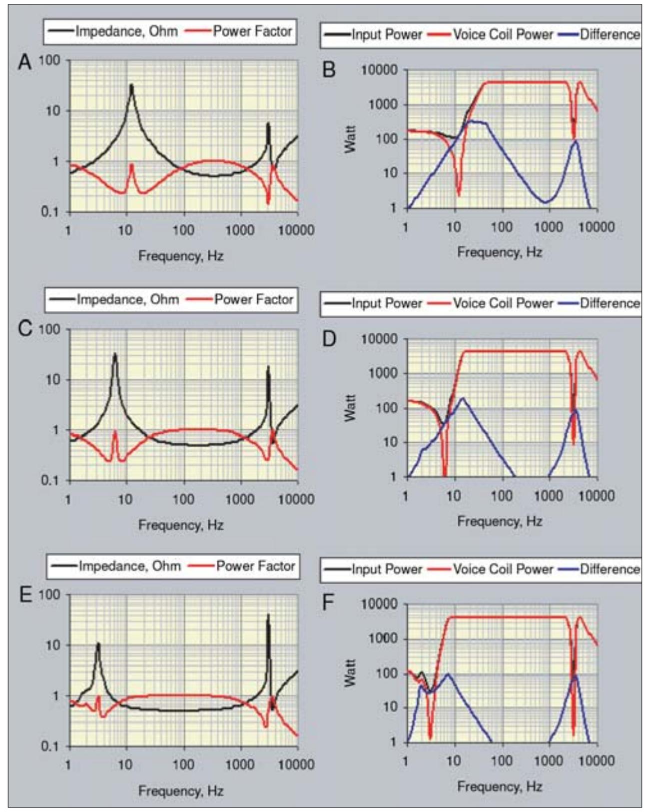


Figure 4. Shaker electrical performance with various payloads. A – voice coil properties with bare table, B – power dissipation with bare table, C – voice coil properties with 25 kg payload, D – power dissipation with 25 kg payload, E – voice coil properties with 150 kg payload, F – power dissipation with 150 kg payload.

Thermal Dynamics and Thermal Statics

The time-dependent thermal behavior of the system is described by Eq. 6. However, the system time constant, τ, is difficult to predict accurately because the value of the *thermal mass*, M_T, is not a typically specified item. We enter the examination of a new shaker knowing only that M_T is less than the total mass of the shaker (which is specified).

Fortunately, none of the steady-state power or temperature values asserted by the remaining equations are influenced by an error in the evaluation of M_T and therefore τ. Hence, our heat-load estimates are *unaffected by this frailty*.

We can improve our estimate of M_T and τ for an *existing* shaker installation through simple temperature/time observation. τ is that time required for the shaker temperature rise, T_{rise}, to reach 63% of its steady state value, T_{riseSS}. In two time constants, the temperature rise achieves 86% of its final value; in three time constants, 95% of the terminal value is reached.

Conclusions

A little thermal planning can head off an unpleasant surprise. If your new shaker is going to overtax the existing facility cooling, it is far better to know this prior to the installation. The simple model presented here is far from a detailed simulation, but it provides you with enough insight to make a sound judgment call regarding the need to upgrade the facility cooling in support of a new system.

References

1. Lang, G. F., "Coolit! Manual", www.foxlang.com, October 2003.
2. Lang, G. F. and Snyder, D., "Understanding the Physics of Electrodynamic Shaker Performance," *Sound and Vibration*, October 2001.
3. Lang, G. F. "Fundamentals of Electrodynamic Shakers," *Sound and Vibration*, April 1997.

The author can be contacted at: docfox@comcast.net.