Predicting Footfall-Induced Vibrations of Floors

Eric E. Ungar, **Jeffrey A. Zapfe**, and **Jonathan D. Kemp**, Acentech, Incorporated, Cambridge, Massachusetts

The forms of vibration criteria for sensitive equipment are discussed and generic facility criteria are reviewed. The parameters of idealized footfall force pulses associated with walking at different speeds are summarized. Expressions are presented for a floor's displacement, velocity and acceleration resulting from idealized footfall pulses, with the floor modeled in terms of its fundamental mode. Graphs are presented for the estimation of the maximum velocities and accelerations and for the root mean square velocities and accelerations resulting from a footfall. Approaches to estimation of the vibrations due to continuous walking and due to multiple walkers are discussed and the practical applicability of the various prediction means is delineated.

The floors of buildings that are to support vibration-sensitive equipment must provide acceptable vibration environments. Vibrations induced by footfalls – that is, by walking personnel – generally are a major concern and need to be addressed by structural and layout design considerations. Therefore, footfall-induced floor vibrations should be considered early in the design process, and means are needed for predicting these vibrations relatively simply and conservatively. This article presents such means.

Vibration Criteria for Sensitive Equipment

Equipment Criteria. Limits on the vibrations of floors that support precision equipment may be stated in terms of several different measures. The suppliers of such equipment typically have indicated vibration criteria in terms of: (a) maximum velocity or acceleration; (b) root mean square velocity or acceleration; or (c) maximum velocities or accelerations observed at given frequencies. Some equipment items may be affected by short-duration transient disturbances, whereas others are affected only by relatively persistent vibrations. Although many suppliers' specifications are incomplete (for example in that they fail to delineate the applicable frequency bandwidths, averaging and measurement durations), the vibration predictions one needs to perform should be done in relation to the applicable criteria as far as possible.

Generic Facility Criteria. In many situations specific equipment items are not known and/or corresponding criteria are not available. To assist the designer in such situations, generic criteria summarized in Figure 1 and Table 1 (adapted from Reference 1) have been developed and have found wide acceptance. Diagrams similar to Figure 1 have appeared in several publications.[1-4] In order to avoid confusion that may result because different publications have used different letter designations for the various curves, it is recommended that these curves be identified by the velocity values that correspond to their horizontal sections, and not by letters.

Note that the Figure 1 criteria are stated in terms of root mean square (rms) velocity in 1/3 octave frequency bands and that the given curves extend from 4 to 80 Hz. Outside of this frequency range, greater vibrations are assumed to be acceptable. For equipment that includes internal vibration isolation with associated natural frequencies in the 1-2 Hz range, it has been suggested that the horizontal portions of the curves be extended down to 1 Hz.

There are good reasons for the criteria being stated in terms of 1/3 octave bands. Several equipment criteria have been developed by subjecting a specific item to increasing vibrations at a given frequency, observing when adverse effects begin to appear, and repeating the process at several frequencies. Criteria developed in this way provide reasonable guidance in cases where an item's natural frequencies are relatively widely separated and where the excitation also consists of rather widely separated frequency components. However, if the natural frequencies are spaced relatively closely and/or the excitation is broadband or consists of a number of closely spaced components, then several natural frequencies may be excited in concert. How many depends on the inherent structural damping.

With 10% of critical viscous damping (or, equivalently, with a loss factor of 0.2) the bandwidth of a modal response is equal to 20% of the band's center frequency – that is, one may essentially expect all modes with natural frequencies between 0.9*f* and 1.1*f* to respond to excitation at a frequency *f*. The aforementioned 20% bandwidth is approximately equal to the 23% bandwidth of 1/3 octave bands. Because 10% of critical damping is believed to be a reasonable and conservative assumption, and because available instruments can measure data conveniently in 1/3 octave bands, they have been recommended for

Table 1. Generic facility criteria.

Facility Equipment or Use	Velocity Limit* µm/s	mils/s
Ordinary Workshops	800	32
Offices	400	16
Residences, Computer Systems***	200	8
Operating Rooms, Surgery, Bench Microscopes up to 100×, Laboratory Robots	100	4
Bench Microscopes up to 400×, Precision Balances, Metrology, Class A Equipment**	50	2
Micro and Neuro-Surgery, Bench Microscopes at Greater than 400×, Optical Equipment on Isolation Tables, Class B Equipment**	25	1
Electron Microscopes at up to 30,000×, Microtomes, Magnetic Resonance Imagers, Mass Spectrometers, Class C Equipment**	12	0.5
Electron Microscopes at Greater than 30,000×, Cell Implant Equipment, Class D Equipment**	6	0.25
Unisolated Optical Systems, Class E Equipment**	3	0.13

* Corresponds to Figure 1 curves from 8-80 Hz. (1 mil = 1000 µin.).
** Equipment typically used in microelectronics and photolithography:
Class A – Inspection, probe test, and other manufacturing support equipment
Class B – Aligners, steppers, photolithography tools for line widths of 3 µm or more
Class C – As above, for line widths of 1 µm
Class D – As above and electron beam systems, for line widths of 0.5 µm
Class E – As above, for line widths of 0.25 µm or more.
*** Curve of Figure 1 corresponds to standard mean whole-body threshold of perception.

Table 2. Footfall force pulse parameters.

Walking Speed Designation	Steps/min	Rise Time, τ/2 (s)	Plateau Time, D (s)	Force Ratio $R = F_m/W$
Slow*	75	0.23	0.48	1.13
Moderate**	100	0.20	0.39	1.20
Fast***	125	0.16	0.32	1.29

* Moving about in typical congested laboratory spaces.
** Ordinary walking along corridors.
*** Purposefully rapid walking along corridors.

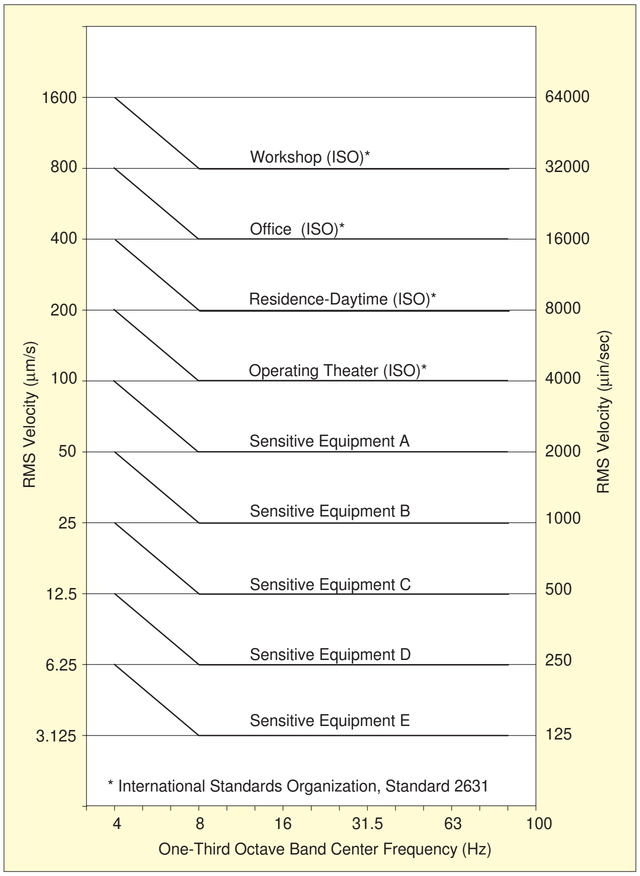


Figure 1. Generic facility criteria.

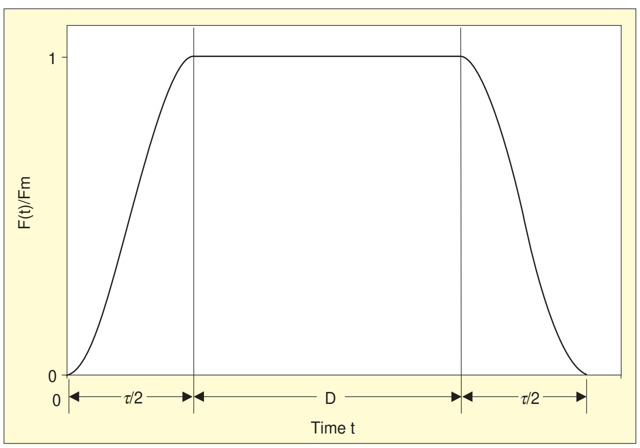


Figure 2. Idealized footfall force pulse.

stating generic vibration criteria.[5]

Footfall Force Pulses

Individual Step. Measurements[6-8] have shown that a force pulse acting on a floor due to a person walking on it has a shape that may be idealized as shown in Figure 2. The rise time $\tau/2$ of the versed-sine portion (and the drop time, which is equal to the rise time), the plateau or 'dwell' duration D, and the plateau force F_m, all depend on the walking speed, typically expressed in terms of steps (footfall impacts) per minute. Furthermore, the plateau force also is proportional to the weight of the walking person.

Figure 3 shows data points extracted from three sources[5-7] indicating how the dwell duration varies with walking speed, together with a polynomial curve fit to the data. Figure 4 similarly shows information for the rise time, and Figure 5 shows data for the ratio of the plateau force F_m to the weight W of a walking person. The values of these parameters depend also to some extent on the type of footwear and on the flexibility of

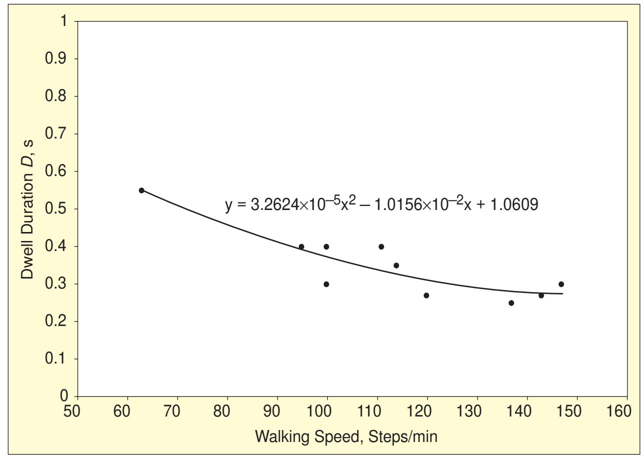


Figure 3. Dependence of dwell duration on walking speed.

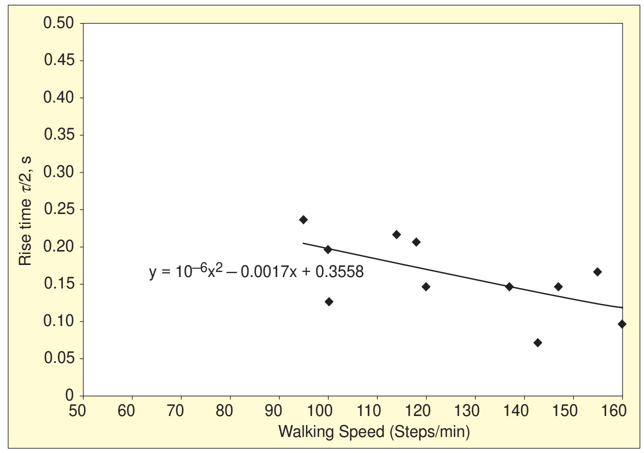


Figure 4. Dependence of pulse rise time on walking speed.

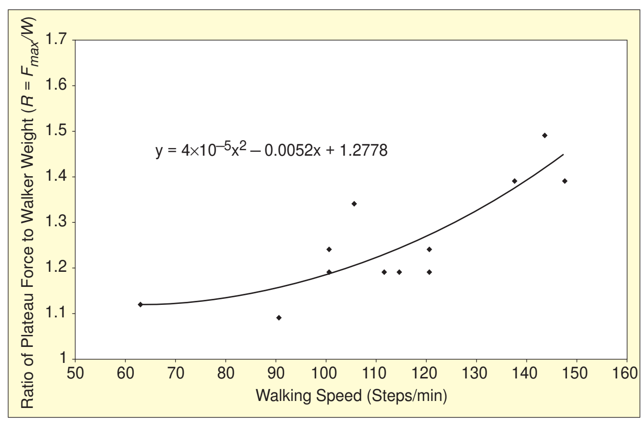


Figure 5. Dependence of plateau force on walking speed.

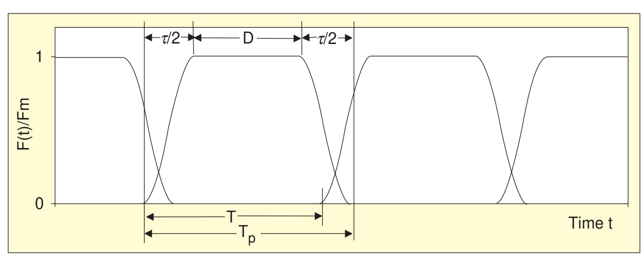


Figure 6. Train of footfall forces pulses in continous walking.

the floor surface, but the present discussion is focused on personnel wearing conventional shoes and walking on ordinary floors, as relevant for industrial or research facilities.

Table 2 presents typical values of the footfall pulse parameters for three walking speeds of general interest. Descriptions of these speeds, adapted from References 7 and 9, appear in

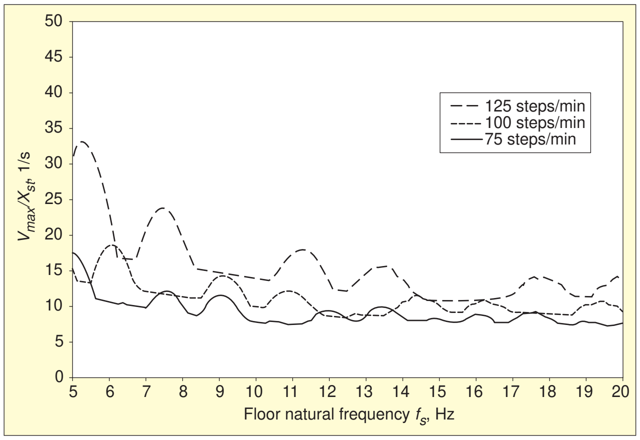


Figure 7. Maximum velecities induced by idealized footfall force pulses.

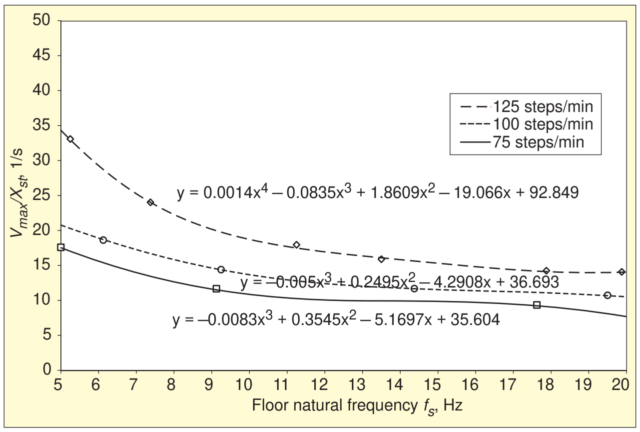


Figure 7B. Upper bounds to maximum velocities induced by idealized footfall pulses.

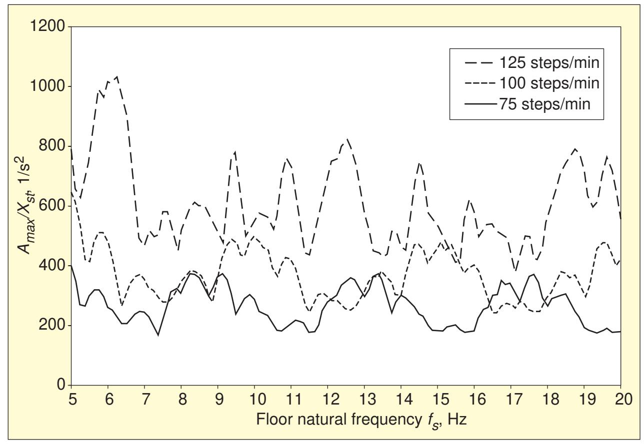


Figure 8. Maximum accelerations induced by idealized footfall force pulses.

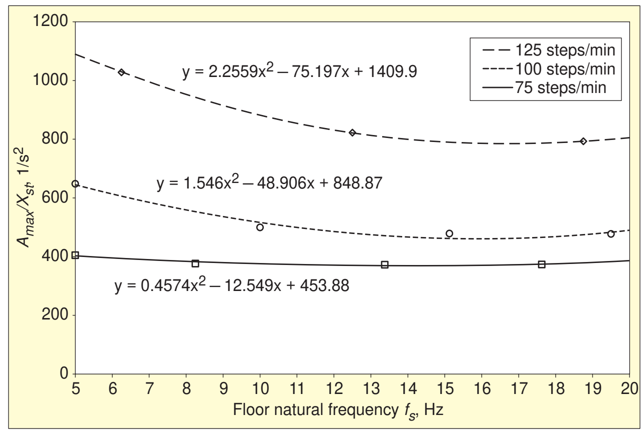


Figure 8B. Upper bounds to maximum accelerations induced by idealized footfall force pulses.

the table's footnotes.

In typical continuous walking, one step follows another and the footfall force pulse from one step begins before the pulse from the previous step ends, as shown in Figure 6. The duration of a pulse, $T_p = D + \tau$, is greater than the step repetition period T (which is the reciprocal of the walking speed). Values of these parameters and of the step frequency $f_1 = 1/T$ are indicated in Table 3.

Floor Vibrations due to Walking

Dominant Floor Motion. The dominant footfall-induced motion of a floor typically is associated with the floor's fundamental mode. The motion of this mode can be analyzed in terms of an equivalent spring-mass system and depends on the effective stiffness, natural frequency f_s, and damping of the system, as well as on the parameters of the footfall force pulse. (The natural frequency f_s encompasses the contribution of the effective mass m, since $2\pi f_s = \sqrt{k/m}$, where k denotes the effective stiffness.)

The motion of a spring-mass system due to an individual force pulse is similar to that of a plucked guitar string. While the system is being 'plucked' – that is, while the pulse acts on the system – it deflects as forced by the pulse. After the end of the pulse, the system vibrates at its natural frequency with an initial magnitude that is established by the action of the pulse, and this vibration subsequently decays at a rate that depends on the system's damping.

Table 3. Parameters for repeated footfalls.

Walking Speed Designation	Steps/min	Duration of Pulse, T_P (s)	Step Repetition Period, T (s)	Step Freq. f_1, (Hz)
Slow	75	0.94	0.80	1.250
Moderate	100	0.79	0.60	1.667
Fast	125	0.64	0.48	2.083

The motion of a spring-mass system due to a continuing train of force pulses is like that due to steady excitation at a number of frequencies, where the frequencies and magnitudes of the excitation components are those of the Fourier components of the pulse train. The greatest vibrations of the system may be expected due to a component whose frequency matches the system's natural frequency. At such a resonance condition, the magnitude of the system's motion is inversely proportional to the system's stiffness and damping.

Maximum Floor Velocity and Acceleration due to Individual Footfall. Appendix A presents analytical expressions that describe the idealized force pulse of Figure 2. It details the calculation of the displacement that such a pulse induces in a spring/mass system that represents the fundamental mode of a floor. Equations are provided for the system's time-dependent displacement, velocity and acceleration.

Figure 7, obtained from numerical calculations based on Equations A5a-d of Appendix A, indicates how the ratio of the greatest velocity V_{max} to X_{st} varies with the natural frequency f_s of the floor for footfalls at the three previously cited typical walking speeds. The symbol X_{st} denotes the static displacement of the floor due to a constant force F_m; that is, $X_{st} = F_m/k = RX_W$, where $R = F_m/W$ (as given in Table 2) and where $X_W = W/k$ denotes the static deflection due to the weight W.

One may observe that the calculated maximum velocities vary markedly with the floor natural frequency. Because the natural frequency of a floor generally cannot be predicted precisely and because actual footfall force pulses are likely to differ from the idealizations used in the calculations that led to Figure 7, it may not be conservative to evaluate a floor strictly on the basis of this figure. In order to develop an upper bound that may be used for conservative estimation, simple curves were fit to the points at the greatest peaks of the curves of Figure 7. These upper bound curves are shown in Figure 7B, to-

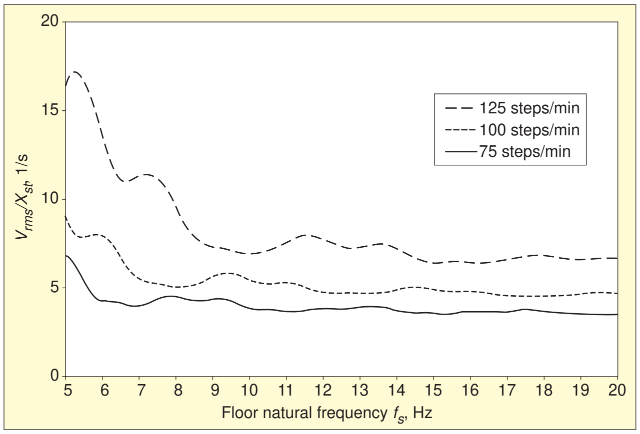


Figure 9. Root mean square (rms) velocity during idealized footfall.

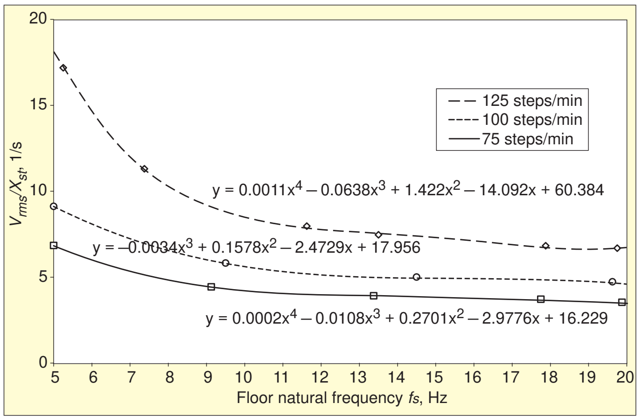


Figure 9B. Upper bounds of rms velocity during idealized footfall.

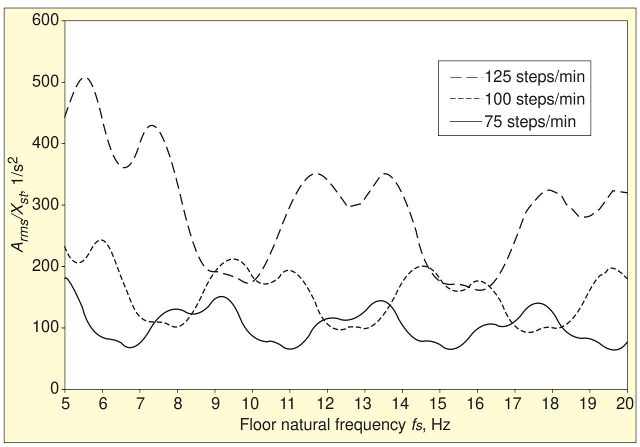


Figure 10. Rms acceleartion during idealized footfall.

gether with equations that describe them. Note that in Figure 7B, as well as in all figures that present upper bounds, y in the curve-fit equations corresponds to the ratio shown on the ordinate and x corresponds to the floor's fundamental natural frequency f_s.

From calculations based on the acceleration expressions of Equations A6a-d, one may obtain the curves of Figure 8, which show how the maximum acceleration of a floor due to a force pulse varies with the floor's natural frequency. In analogy to Figure 7B, Figure 8B presents curves fit to the greatest acceleration peaks.

Root Mean Square Velocity and Acceleration During Single Footfall. The root mean square velocity resulting during a footfall force pulse is given by

$$V_{rms} = \frac{1}{T_P}\sqrt{\int_0^{T_P} V^2(t)dt}$$

Figure 9 show the corresponding results obtained by numerical integration of the velocity equations of Appendix A. Again, Figure 9B presents an upper bound to these results, obtained by fitting a curve to the highest peaks of Figure 9. Figures 10 and 10B show analogous information pertaining to the rms acceleration.

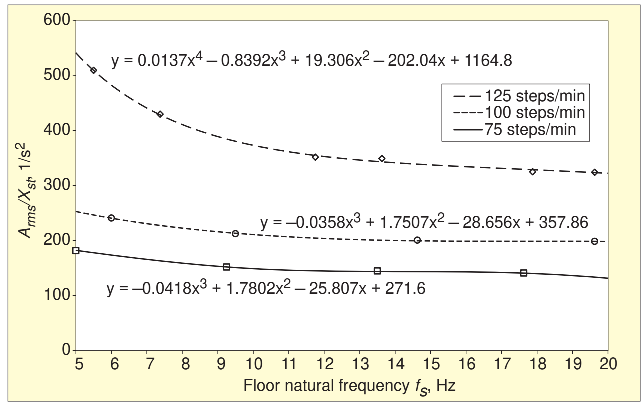


Figure 10B. Upper bound to rms acceleration during idealized footfall.

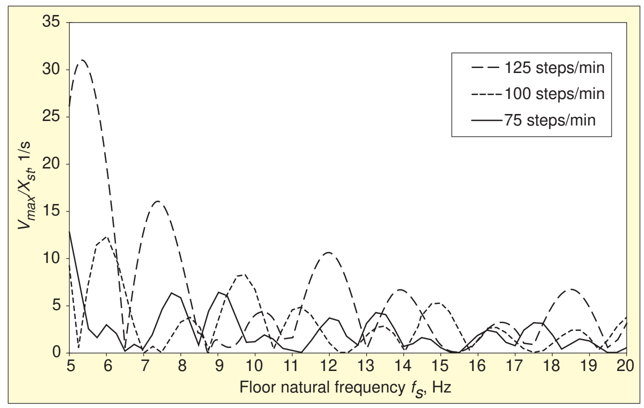


Figure 11. Velocity after pulse passage.

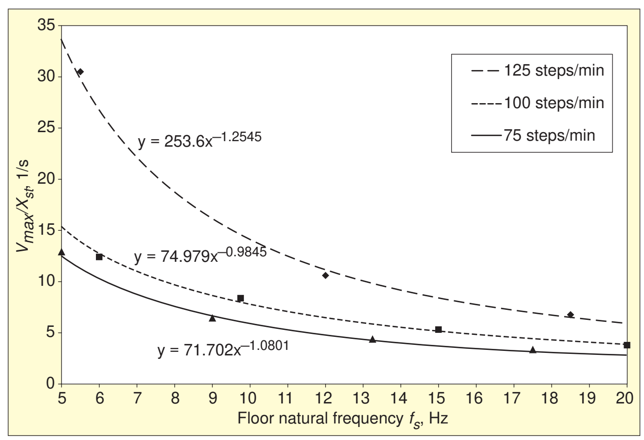


Figure 11B. Upper bound to velocity after pulse passage.

Velocity and Acceleration After Footfalls. In some instances only the persistent, relatively steady vibrations that are present after passage of a footfall pulse are of interest, rather than the transient motions that occur during a footfall. The amplitudes of the 'residual' velocities after passage of a single footfall, calculated from Equations A5d and A4c, are shown in Figure 11. Corresponding upper bounds are presented in Figure 11B.

It may be shown[11] that the rms residual velocity V_{rms} that results from a long series of footfalls, where the individual footfalls are separated by an interval T, is related to the residual velocity amplitude V that results from a single footfall by $V_{rms} = V[4\zeta\pi f_s T]^{-1/2}$. ζ represents the structure's damping ratio. The residual vibrations occur at the floor's natural frequency f_s, and therefore the acceleration amplitude A corresponding to velocity amplitude V is given by $A = 2\pi f_s V$.

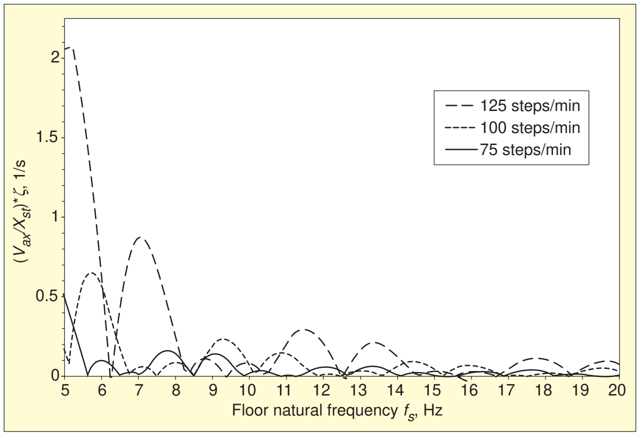


Figure 12. Velocities at floor resonances with steady walking.

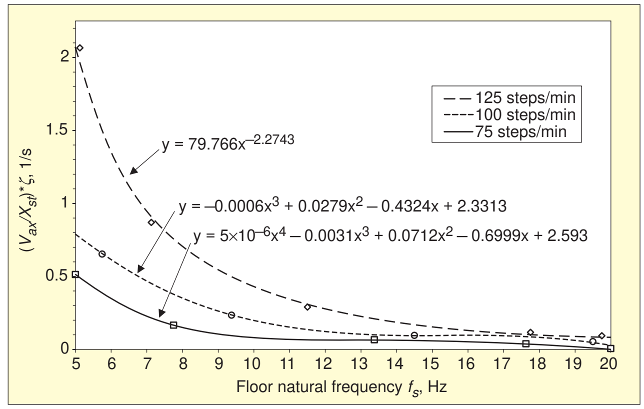


Figure 12B. Upper bound to velocities at floor resonances with steady walking.

Velocity and Acceleration due to Continuous Walking. Figure 12, based on Equation B6 of Appendix B, shows how the steady-state velocity of a floor subjected to continuous walking varies with the floor's fundamental natural frequency for the "worst case" where a Fourier component of the pulse train matches the floor's natural frequency. Figure 12B presents upper bounds to the curves of Figure 12 and may be used for conservative estimation purposes. In view of the most severe vibration occurring at the floor's natural frequency f_s, one may obtain the acceleration magnitude A that corresponds to a velocity magnitude V at a given natural frequency from $A = 2\pi f_s V$.

Application Notes

The floor of a structural bay (delineated by column lines) typically is most easily deflected at locations near the middle of the bay and is least flexible near the column lines and particularly near the columns themselves. Consider a walker's path that traverses a relatively flexible part of the bay (say, near midbay) only briefly, with most of the path extending across less flexible parts. In this case one may reasonably consider that the most significant floor motions are associated with the single footfall that occurs at the most flexible point along the path. On the other hand, for paths that go over extended flexible portions of a bay, the floor vibrations tend to result from a series of footfalls.

Generally only a small number of floor vibration cycles occur during the action of a footfall pulse. Many more cycles occur during the decaying vibrations after passage of the pulse. Thus, in situations where a vibration criterion places a limit on the velocity or acceleration, regardless of how many cycles of an oscillation occur – that is, where the limit applies to transient motions – these criterion limits should be compared to the predicted maximum velocity and acceleration values. On the other hand, in cases where the criterion only places a limit on the steady oscillations, the criterion limits should be compared to the predicted residual vibrations.

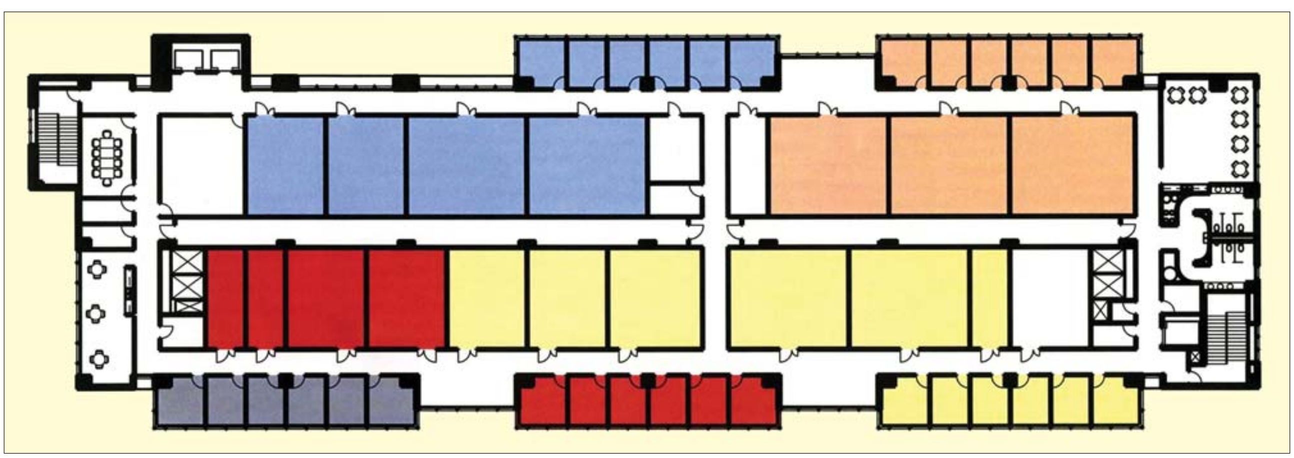
The approach presented in this article estimates the vibrations induced at the point where walking occurs on the floor. The static deflection X_{st} refers to this point. One may obtain an estimate of the corresponding vibrations that result at another point on the floor by taking the ratio of the vibration magnitudes at the two points (i.e., the point of interest and the point at which walking occurs) to be equal to the ratio of the displacements at the two points in the floor's fundamental mode shape.

The foregoing analyses account only for one person walking at a time. If N similar persons walk simultaneously and *in step* at or near a given location, the resulting vibrations may be expected to be at most N times as great as those resulting from a single walker. If the same number of persons walk *out of step* at or near a given location, then one may estimate the resulting vibrations to amount to about $\sqrt{N}$ times that due to a single walker. The magnitude of the velocity resulting at a given point due to persons walking simultaneously at different locations may be estimated as

$$\sqrt{\sum V_j^2}$$

where V_j represents the velocity resulting at the given point due to walker number j and where the summation extends over

Typical Research Building Floor Plan



The figure above illustrates a typical research building floor plan, with laboratory spaces in the interior and offices along the exterior. The different colors indicate different research groups. A typical large laboratory here (like the bigger blue, yellow and orange rooms) measures approximately 25 ft × 30 ft. The solid black rectangles and squares represent columns. A structural bay is a rectangular area demarked by columns at its corners.

all walkers. The magnitudes of the expected maximum accelerations may be estimated similarly.

It should be noted that realistic assessment of the most severe vibration environment at a given location may require consideration of several walking scenarios – i.e., of personnel walking at different speeds and along different paths.

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The authors can be contacted at eungar@acentech.com, jzapfe@acentech.com and jkemp@acentech.com.

Appendix A – Motions due to Footfall Force Pulses

Analytical Representation of Idealized Force Pulse. The pulse shown in Figure 1 may be described by:

$$\frac{F(t)}{F_m} = \frac{1-\cos\omega t}{2} \quad \text{for } 0 \le t \le \frac{\tau}{2} \text{ (rise)} \tag{A1a}$$

$$\frac{F(t)}{F_m} = 1 \text{ for } \frac{\tau}{2} \le t \le \frac{\tau}{2} + D \text{ (dwell)} \tag{A1b}$$

$$\frac{F(t)}{F_m} = \frac{1-\cos\omega(t-D)}{2} \quad \text{for } \frac{\tau}{2} + D \le t \le \tau + D \text{ (drop)} \tag{A1c}$$

where ω is the radian frequency associated with the pulse rise and is defined by $\omega = 2\pi/\tau$.

In the foregoing the initial interval has been labeled as 'rise,' the second as 'dwell' and the third as 'drop.' These designations will be used in the following discussion for the sake of expedience, and the time after pulse passage, $t > \tau + D$, will be designated by 'residual.'

Calculation of Displacement Due to Force Pulse. The displacement $x(t)$ of a mass m that is supported on a spring of stiffness k to a force pulse $F(t)$ may be found by means of Duhamel's integral,[10] which for the present purposes may be written as

$$\frac{x(t)}{X_{st}} = \left[\frac{v_0/p}{X_{st}} + \frac{p}{F_m}\int_0^t F(s)\cos ps \cdot ds\right]\sin pt + \left[\frac{x_0}{X_{st}} - \frac{p}{F_m}\int_0^t F(s)\sin ps \cdot ds\right]\cos pt \tag{A2}$$

Here $X_{st} = F_m/k$ denotes the static displacement of the mass due to a constant force F_m and $p = \sqrt{k/m} = 2\pi f_s$ represents the radian natural frequency of the spring-mass system, with f_s denoting its cyclic natural frequency.

Displacements Due to Idealized Footfall Pulse. By substitution of the foregoing footfall pulse relations into Duhamel's integral one may obtain the expressions given below, generally after considerable manipulation. These expressions apply for $p \neq \omega$; corresponding expressions for $p = \omega$ and their consequences can be readily derived, but are omitted here for the sake of brevity.

$$\frac{2x(t)}{X_{st}} = 1 - \cos pt + \frac{p^2}{p^2-\omega^2}\left[\cos pt - \cos\omega t\right] \text{ (rise)} \tag{A3a}$$

$$\frac{2x(t)}{X_{st}} = 2 + \frac{\omega^2}{p^2-\omega^2}\left[\cos pt + \cos p\left(t-\frac{\tau}{2}\right)\right] \text{ (dwell)} \tag{A3b}$$

$$\frac{2x(t)}{X_{st}} = 1 + \frac{\omega^2}{p^2-\omega^2}\left[\cos pt + \cos p\left(t-\frac{\tau}{2}\right) - \cos p\left(t-D-\frac{\tau}{2}\right) + \frac{p^2}{\omega^2}\cos\omega\left(t-D-\frac{\tau}{2}\right)\right] \text{(drop)} \tag{A3c}$$

$$\frac{2x(t)}{X_{st}} = \frac{\omega^2}{p^2-\omega^2}\left[A\sin p(t-D-\tau) + B\cos p(t-D-\tau)\right] = \frac{\omega^2}{p^2-\omega^2}C\sin p(t-D-\tau+\phi) \text{ (residual)} \tag{A3d}$$

$$A = \sin\left(p\frac{\tau}{2}\right) - \sin p(D+\tau) - \sin p\left(D+\frac{\tau}{2}\right) \tag{A4a}$$

$$B = -1 - \cos\left(p\frac{\tau}{2}\right) + \cos p(D+\tau) + \cos p\left(D+\frac{\tau}{2}\right) \tag{A4b}$$

$$C^2 = A^2 + B^2 = 4\left[1+\cos\left(p\frac{\tau}{2}\right)\right]\left[1-\cos p\left(D+\frac{\tau}{2}\right)\right]; \quad \tan p\phi = \frac{B}{A} \tag{A4c}$$

Velocities Due to Idealized Footfall Pulse. Expressions for the velocity $v(t)$ may be obtained readily by differentiation of the displacement expressions. One finds the following relations:

$$\frac{2v(t)/p}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = \frac{p}{\omega}\sin\omega t - \sin pt \text{ (rise)} \tag{A5a}$$

$$\frac{2v(t)/p}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = -\sin pt - \sin p\left(t-\frac{\tau}{2}\right) \text{ (dwell)} \tag{A5b}$$

$$\frac{2v(t)/p}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = -\sin pt - \sin p\left(t-\frac{\tau}{2}\right) + \sin p\left(t-D-\frac{\tau}{2}\right) - \frac{p}{\omega}\sin\omega\left(t-D-\frac{\tau}{2}\right) \text{ (drop)} \tag{A5c}$$

$$\frac{2v(t)/p}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = A\cos p(t-D-\tau) - B\sin p(t-D-\tau) = C\cos p(t-D-\tau+\phi) \text{ (residual)} \tag{A5d}$$

Accelerations Due to Idealized Footfall Pulse. By differentiation of the velocity expressions one may obtain the following relations for the accelerations $a(t)$:

$$\frac{2a(t)/p^2}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = \cos\omega t - \cos pt \tag{A6a}$$

$$\frac{2a(t)/p^2}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right) = -\cos pt - \cos p\left(t-\frac{\tau}{2}\right) \text{ (dwell)} \tag{A6b}$$

$$\frac{2a(t)/p^2}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right)=-\cos pt-\cos p\left(t-\frac{\tau}{2}\right)$$
$$+\cos p\left(t-D-\frac{\tau}{2}\right)-\cos\omega\left(t-D-\frac{\tau}{2}\right)\ \text{(drop)} \tag{A6c}$$

$$\frac{2a(t)/p^2}{X_{st}}\left(\frac{p^2}{\omega^2}-1\right)=-A\sin p(t-D-\tau)$$
$$-B\cos p(t-D-\tau)=-C\sin p(t-D-\tau+\phi)\ \text{(residual)} \tag{A6d}$$

Appendix B – Motions Due to Continuous Walking

A diagram indicating overlapping force pulses corresponding to continuous walking appears in Figure 6. If one takes the pulse train as symmetric about $t = 0$, one can develop a Fourier cosine series to describe the continuing pulse train.

The force pulse present at time $t = 0$ and symmetric about $t = 0$ obeys the following expressions in the half period from $t = 0$ to $t = T/2$:

$$F_1(t)=F_m \text{ for } 0<t<D/2 \tag{B1a}$$

$$F_1(t)=\frac{F_m}{2}\left[1-\cos\frac{2\pi}{\tau}\left(\frac{T_P}{2}-t\right)\right] \text{ for } \frac{D}{2}<t<\frac{T}{2} \tag{B1b}$$

The contribution of the subsequent pulse during the aforementioned half period is given by

$$F_2(t)=F_m \text{ for } 0<t<T-T_P/2 \tag{B1c}$$

$$F_2(t)=\frac{F_m}{2}\left[1-\cos\frac{2\pi}{\tau}\left(T-\frac{T_P}{2}-t\right)\right] \text{ for } T-\frac{T_P}{2}<t<\frac{T}{2} \tag{B1d}$$

Thus, the net force pulse for the entire half period may described as $F(t) = F_1(t) + F_2(t)$.

For a Fourier cosine series of the form

$$F(t)=a_0+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi nt}{T}\right) \tag{B2}$$

the coefficients of the various components for $n = 1,2,3,\ldots$are given by

$$a_n=\frac{4}{T}\int_0^{T/2}F(t)\cos\left(\frac{2\pi nt}{T}\right)dt \tag{B3}$$

The a_0 term corresponds to a time-independent component and is of no interest here. Corresponding to the above expression for $F(t)$ one finds after some manipulation that

$$a_n=\frac{(-1)^n F_m}{n\pi\left[\left(\frac{n\tau}{T}\right)^2-1\right]}\left[\sin\left(\frac{n\pi D}{T}\right)+\sin\left(\frac{n\pi T_P}{T}\right)\right] \tag{B4}$$

The step frequency $f_1 = 1/T$ usually is considerably less than the fundamental natural frequency f_s of floor structure. Thus, there exists the possibility of a floor being excited at resonance by one of the higher order components of the footfall pulse train. At such a resonance, $f_s = nf_1$.

Since the steady-state velocity amplitude V_{res} at resonance of a spring-mass-damper system (with natural frequency f_s) driven by constant force F_j obeys

$$V_{res}=\frac{F_j\pi f_n}{\zeta k} \tag{B5}$$

where ζ represents the (viscous) damping ratio or the fraction of critical damping, one may determine that the velocity of a floor structure at resonance due to a footfall force pulse train is given by

$$\frac{|V_{res}|}{X_{st}}\varsigma=f_1\left|\frac{\sin(\pi Df_s)+\sin(\pi T_P f_s)}{(f_s\tau)^2-1}\right| \tag{B6}$$