A Practical Review of Rotating Machinery Critical Speeds and Modes

Erik Swanson, Xdot-Consulting, Chapel Hill, North Carolina
Chris D. Powell, Structural Technology Corporation, Zoar, Ohio
Sorin Weissman, Alfa Wasserman, Inc., West Caldwell, New Jersey

The goal of this article is to present a practical understanding of terminology and behavior based in visualizing how a shaft vibrates, and examining issues that affect vibration. It is hoped that this presentation will help the nonspecialist better understand what is going on in the machinery, and that the specialist may gain a different view and/or some new examples.

For the engineer unfamiliar with some of the unique characteristics of rotating machinery vibration, the terminology and behavior of a machine can appear to be overwhelming. Like most specialty areas, there are a number of excellent texts, but it can be difficult to quickly pull out the practical insight needed. At the other end of the spectrum, there is also a large number of troubleshooting resources that focus on identification of problems and characteristics, but only offer limited insight. Discussion of a recent combined experimental and analytical effort raised the possibility of an article that would attempt to provide a deeper insight into some of the basic characteristics of rotating machinery vibration from a less mathematical perspective.

Thus, we have set out to discuss several issues that are basic to an understanding of rotating machinery vibration:
- What are "critical speeds"?
- How do critical speeds relate to resonances and natural frequencies?
- How do natural frequencies change as the shaft rotational speed changes?
- How are shaft rotational natural frequencies different from more familiar natural frequencies and modes in structures?
- What effects do bearing characteristics have?

Vibration Intuition

As a part of exploring the world as children, everyone is familiar with the idea that banging on a structure will make it bounce back and vibrate. Some items vibrate more easily than others (a metal rod versus a wooden stick, for example). We also have intuition that it is easier to get things to vibrate or move back and forth at certain frequenies. For example, we tend to learn that a swing with long ropes moves back and forth more slowly than a swing with short ropes. 'Pumping' the swing at a rate that matches the rate at which it wants to naturally move back and forth will get you swinging much higher than rates that are faster or slower than the swing's natural frequency.

Many of us have also had some experience with stringed instruments. From this experience, we develop some idea that heavy objects (thick strings) tend to vibrate at a lower frequency than light objects (thin strings). We learn that increasing stiffness (tightening the string) raises the frequency of its vibration. Finally, we also learn that decreasing a major dimension (shorter string) results in higher frequency vibration.

A Brief Review of Structural Vibration

As engineers, we learn that vibration characteristics are determined by a structure's mass and stiffness values, with damping (ability to dissipate vibrational energy) playing an integral role by controlling amplitudes. This education generally starts with the simplest possible system – a rigid mass attached to a spring as shown in Figure 1.

With this simple system, we quantify our intuition about vibrational frequency (heavier objects result in lower frequency, stiffer springs yield higher frequency). After some work, we reach the conclusion that the free vibration frequency is controlled by the square root of the ratio of stiffness to mass.

$$\text{Natural Frequency} = \sqrt{\frac{\text{Stiffness}}{\text{Mass}}} \qquad (1)$$

Experimentally, we could (in principle) build a single degree of freedom system consisting of a rigid block sitting on a spring. Were we to push the block down and release it, we would find that the displacement versus time is a sinusoidal function at a single frequency, which is equal to the natural frequency as predicted by Equation 1 and shown in Figure 2.

We could then add a viscous damper parallel with the spring, and provide a sinusoidal force as shown in Figure 3. By carefully applying a constant amplitude sinusoidal force that slowly increases in frequency and recording the amplitude of the motion, we could then generate the classic normalized frequency responses of a spring-mass-damper system. By repeating the test with a variety of dampers, the classic frequency response shown in Figure 4 can be developed. Assuming we knew the mass, stiffness and damping of our system, this response is also predicted quite well by the standard frequency domain solution to the differential equation of motion for this system shown in Equation 2.

$$\text{Amplitude} = \frac{\dfrac{F_0}{k}}{\sqrt{\left(1-\dfrac{m\omega^2}{k}\right)^2+\left(\dfrac{c\omega}{k}\right)^2}} \qquad (2)$$

There are several noteworthy points about these frequency responses. The first is that at low excitation frequencies, the response amplitude is roughly constant and greater than zero. The amplitude is governed by the ratio of the applied force to the spring stiffness. The second is that the response increases to a peak, then rapidly decreases in the low and medium damping cases.

This peak frequency is approximately the damped natural frequency, (more technically correct, it is the peak response frequency, which moves down in frequency from the damped natural frequency as damping increases). The system is said to be "in resonance" when the excitation frequency matches the damped natural frequency. Very large amplitudes are possible when the excitation frequency is close to this frequency. The amplitude is controlled by the magnitude of the damping (more damping reduces the amplitudes). The high damping case has no real peak, and is said to be 'overdamped.' Finally, the amplitude continues to decrease for all higher frequencies. These characteristics will be contrasted with the response of a rotating system to unbalance excitation in a later section.

Moving from the simple single mass system to multimass systems, the basics do not change. Natural frequencies are still primarily related to mass and stiffness, with some changes due to damping. Excitation frequency equal to a damped natural frequency is a resonance. Excitation near a resonance can result in large amplitude responses. Response amplitudes are controlled by damping. With enough damping, the response peak can be completely eliminated. The biggest change is that there are now multiple natural frequencies and that each natu-

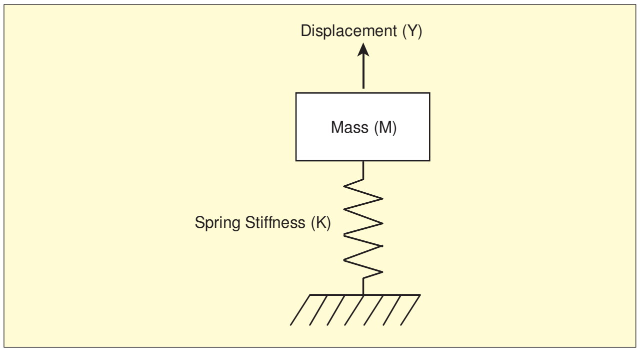

Figure 1. Simple spring-mass system.

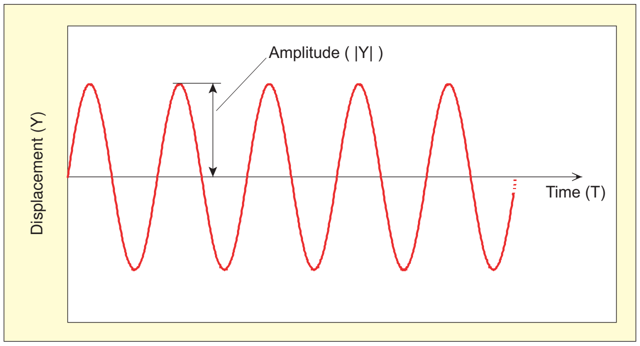

Figure 2. Free response of simple spring-mass system.

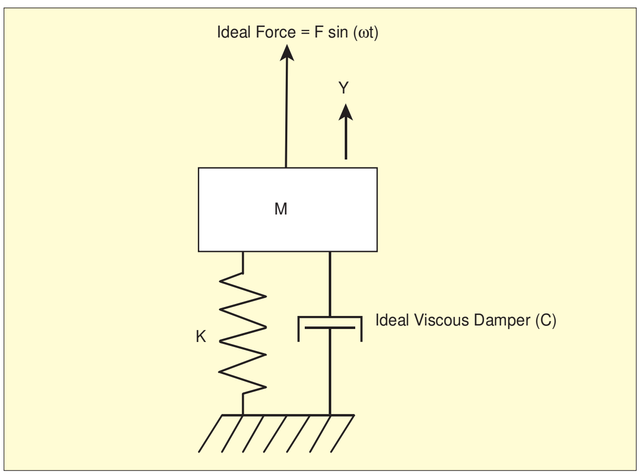

Figure 3. Simple spring-mass-damper system with forcing.

ral frequency has a corresponding unique "mode-shape" with different parts of the structure vibrating at different amplitudes and differing phases relative to one another.

Real structures can be viewed as a series of finer and finer lumped mass approximations that approach a continuous mass distribution. The continuous structure has an infinite number of natural frequencies, each with its own characteristic vibration shape (mode).

As an example, consider a simple beam structure supported by pin joints at each end. This structure is simple enough that a closed-form solution to the natural frequencies and mode shapes is possible. Equation 3 presents the resulting equation for natural frequencies, and the first three mode shapes are shown in Figure 5.

$$f_i = \frac{(i\pi)^2}{2\pi(\text{Length})^2}\sqrt{\frac{(\text{modulus of elasticity})(\text{area moment of inertia})}{\text{mass per unit length}}} \quad (3)$$

In essence, this equation is still just the square root of the ratio of stiffness to mass. The mode shapes shown in Figure 5 and throughout the article are the shape of the beam at the position of maximum displacement for a given (damped) natural frequency. The dashed lines show the positions of the beam during the vibration cycle. These intermediate positions are not

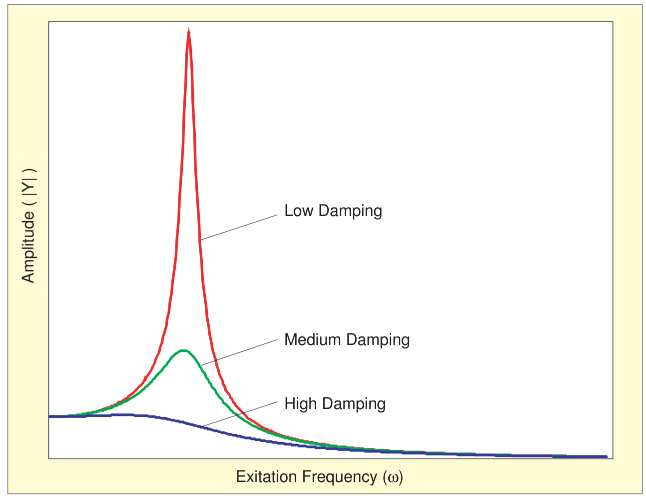

Figure 4. Frequency response of spring-mass-damper system to constant amplitude force.

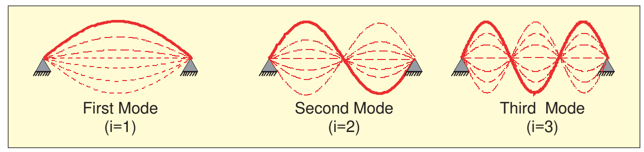

Figure 5. First three mode shapes of pinned-pinned beam.

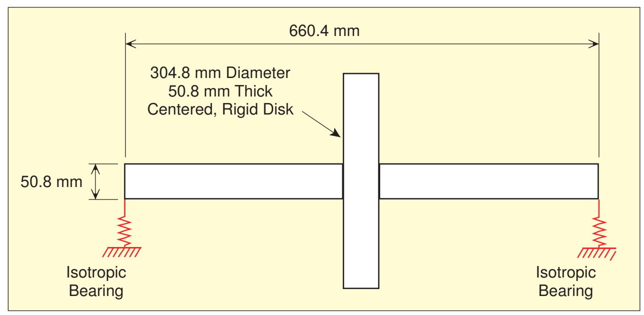

Figure 6. Basic machine model cross section.

shown in the remainder of the figures.

All of this background and intuition carries over into the rotating machinery world – with a few important differences, especially once the rotor starts to spin.

A Simple Rotating Machine

The rotating machinery equivalent to the single spring-mass-damper system is a lumped mass on a massless, elastic shaft. This model, historically referred to as a 'Jeffcott' or 'Laval' model, is a single degree of freedom system that is generally used to introduce rotor dynamic characteristics. For the purposes of this article, a slightly more complex multi-degree of freedom model corresponding to a physical rotor will be used. This model, shown in cross-section in Figure 6, consists of a rigid central disk, a shaft (with stiffness and mass) and two rigidly mounted bearings. To make the examples more concrete, dimensions shown were selected. Physically, this is somewhat similar to a center-hung fan, pump or turbine.

Nonrotating Dynamics

Suppose that our simple machine is not spinning, that the bearings have essentially no damping, and that the bearings have equal radial stiffness in the vertical and horizontal directions (all typical characteristics of ball bearings). Let us also suppose that there are three versions of this machine, one each with soft, intermediate and stiff bearings.

Through either analysis or a modal test, we would find a set of natural frequencies/modes. At each frequency, the motion is planar (just like the pinned-pinned beam). This behavior is what we would expect from a static structure. Figure 7 shows

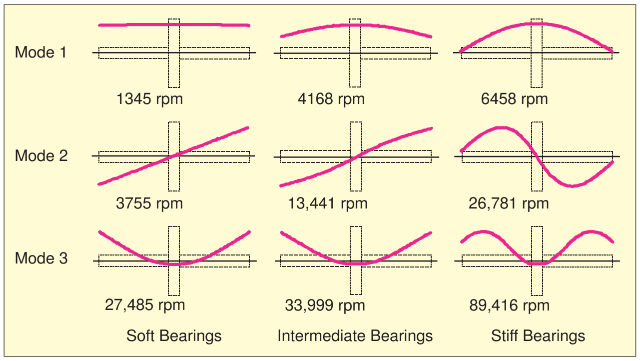

Figure 7. Mode shapes versus bearing stiffness, shaft not rotating.

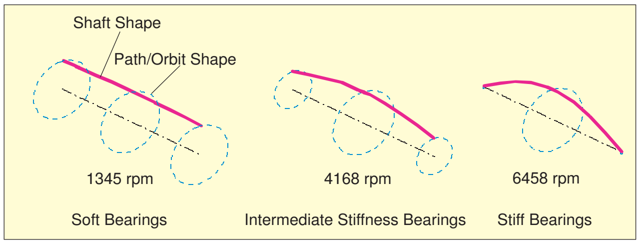

Figure 8. Shaft rotating at 10 rpm, 1st mode shapes and frequencies in rpm.

the first three mode shapes and frequencies for the three bearing stiffnesses. As with the beam, the thick line shows the shaft centerline shape at the maximum displacement. As it vibrates, it moves from this position to the same location on the opposite side of the undisplaced centerline, and back.

Note that the ratio of bearing stiffness to shaft stiffness has a significant impact on the mode-shapes. For the soft and intermediate bearings, the shaft does not bend very much in the lower two modes. Thus, these are generally referred to as "rigid rotor" modes. As the bearing stiffness increases (or as shaft stiffness decreases), the amount of shaft bending increases.

One interesting feature of the mode shapes is how the central disk moves. In the first mode, the disk translates without rocking. In the second mode, it rocks without translation. This general characteristic repeats as the frequency increases. If we moved the disk off-center, we would find that the motion is a mix of translation and rocking. This characteristic will give rise to some interesting behavior once the shaft starts rotating.

If we repeated the constant amplitude excitation frequency sweep experiment, we would get very similar behavior as with the spring-mass-damper system plot shown previously. There would be a spring-controlled deflection at low frequencies, a peak in amplitude, and a decay in amplitude with further increases in frequency.

Rotating Dynamics – Cylindrical Modes

Since rotating machinery has to rotate to do useful work, let's consider what happens to the first mode of our rotor once it is spinning. Again, we will have three different versions with increasing bearing stiffness, and we will assume our support bearings have equal stiffness in all radial directions. Let's repeat our analysis/modal test with the shaft spinning at 10 rpm, and look at the frequency and mode shape of the lowest natural frequency. Figure 8 below shows the frequencies and mode shapes for the lowest mode of the three machines.

Note that the shape of the motion has changed. The frequencies, though, are quite close to the nonrotating first mode. As in the nonrotating case, the bearing stiffness to shaft stiffness ratio has a strong impact on the mode-shape. Again, the case with almost no shaft bending is referred to as a rigid mode. These modes look very much like the nonrotation modes, but they now involve circular motion rather than planar motion. To visualize how the rotor is moving, first imagine swinging a jumprope around. The rope traces the outline of a bulging cylinder. Thus, this mode is sometimes referred to as a 'cylindrical' mode. Viewed from the front, the rope appears to be bouncing up and down. Thus, this mode is also sometimes called a 'bounce' or 'translatory' mode.

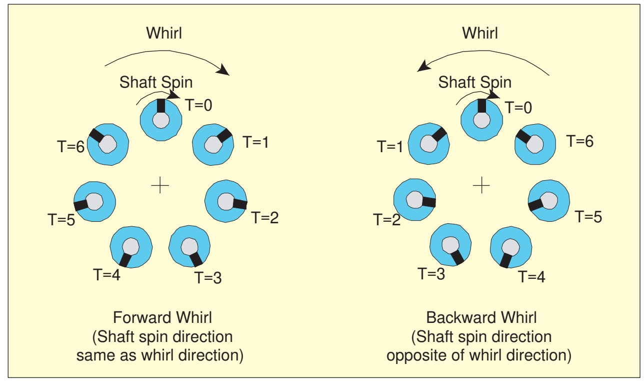

Figure 9. Whirl sense.

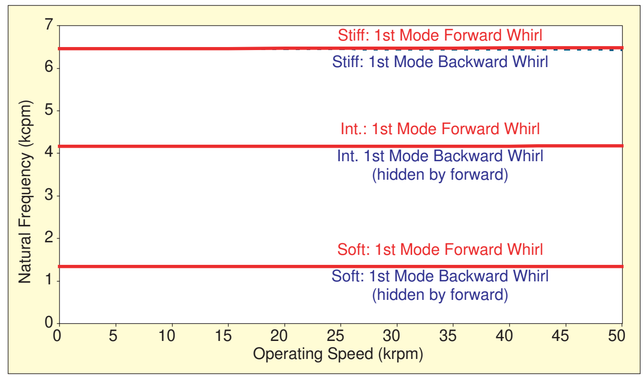

Figure 10. Effect of operating speed on 1st modes.

Unlike most jump-ropes, however, the rotor is also rotating. The whirling motion of the rotor (the 'jumprope' motion) can be in the same direction as the shaft's rotation or in the opposite direction. This gives rise to the labels "forward whirl" and "backward whirl." Figure 9 shows rotor cross sections over the course of time for both synchronous forward and synchronous backward whirl. Note that for forward whirl, a point on the surface of the rotor moves in the same direction as the whirl. Thus, for synchronous forward whirl (unbalance excitation, for example), a point at the outside of the rotor remains to the outside of the whirl orbit. With backward whirl, on the other hand, a point at the surface of the rotor moves in the opposite direction as the whirl to the inside of the whirl orbit during the whirl.

To see how a wider range of shaft speeds changes the situation, we could perform the analysis/modal test with a range of shaft speeds from nonspinning to high speed. We could then follow the forward and backward frequencies associated with the first mode. Figure 10 plots the forward (red line) and backward (black dashed line) natural frequencies over a wide shaft speed range. This plot is often referred to as a "Campbell Diagram." From this figure, we can see that the frequencies of this cylindrical mode do not change very much over the speed range. The backward whirl mode drops slightly, and the forward whirl mode increases slightly (most noticeably in the high stiffness case). The reason for this change will be explored in the next section.

Rotating Dynamics – Conical Mode

Now that we have explored the cylindrical mode, let's look at the second set of modes. Figure 11 shows the next frequencies and mode-shapes for the three machines. The frequencies are close to the nonrotating modes where the disk was rocking without translating. The modes look a lot like the nonrotating modes, but again involve circular motion rather than planar motion.

To visualize how the rotor is moving, imagine holding a rod

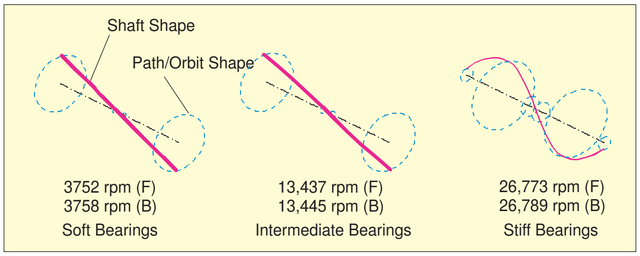

Figure 11. Shaft rotating at 10 rpm, 2nd mode shapes and frequencies in rpm.

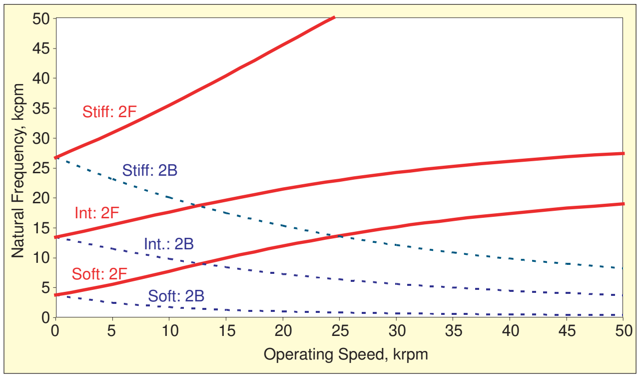

Figure 12. Effect of operating speed on 2nd natural frequencies.

stationary in the center, and moving it so that the ends trace out two circles. The rod traces the outline of two bulging cones pointed at the center of the rod. Thus, this mode is sometimes referred to as a 'conical' mode. Viewed from the side, the rod appears to be rocking up and down around the center, with the left side being out-of-phase from that on the right. Thus, this mode is also sometimes called a 'rock' mode or a 'pitch' mode. As with the first mode and the nonrotating modes, the low bearing stiffness mode is generally referred to as a rigid mode, and a high bearing stiffness pulls in the rotor ends. As with the cylindrical mode, the whirl can be in the same direction as the rotor's spin ("forward whirl"), or the opposite direction ("backward whirl").

To see the effects of changing shaft speeds, we could again perform the analysis/modal test from nonspinning to a high spin speed and follow the two frequencies associated with the conical mode. Figure 12 plots the forward (red line) and backward (black dashed line) natural frequencies over a wide speed range. From this figure, we can see that the frequencies of the conical modes do change over the speed range. The backward mode drops in frequency, while the forward mode increases.

The explanation for this surprising behavior is a gyroscopic effect that occurs whenever the mode shape has an angular (conical/rocking) component. First consider forward whirl. As shaft speed increases, the gyroscopic effects essentially act like an increasingly stiff spring on the central disk for the rocking motion. Increasing stiffness acts to increase the natural frequency. For backward whirl, the effect is reversed. Increasing rotor spin speed acts to reduce the effective stiffness, thus reducing the natural frequency (as a side note, the gyroscopic terms are generally written as a skew-symmetric matrix added to the damping matrix – the net result, though, is a stiffening/softening effect).

In the case of the cylindrical modes, very little effect of the gyroscopic terms was noted, since the center disk was whirling without any conical motion. Without the conical motion, the gyroscopic effects do not appear. Thus, for the soft bearing case, which has a very cylindrical motion, no effect was observed, while for the stiff bearing case, which has a bulging cylinder (and thus conical type motion near the bearings), a slight effect was noted.

Exploring Gyroscopic and Mass Effects

Now that we have seen how gyroscopic effects act to change the rotating natural frequency whenever there is motion with some conical component, let's look at two sets of single disk rotors. In each case, there will be a nominal rotor, a heavy disk rotor, and a smaller diameter, longer disk rotor. The heavy disk differs from the nominal in that a fictitious mass equal to the disk mass is attached (i.e., mass increases, but mass moment of inertia is unchanged) The smaller, longer disk is the same weight, but smaller in diameter and greater in length. This smaller disk has reduced the mass moment of inertia about the spin axis ('polar' moment Ip) by a factor of 0.53, and reduces the mass moment of inertia about the disk diameter (Id) by a factor of 0.65.

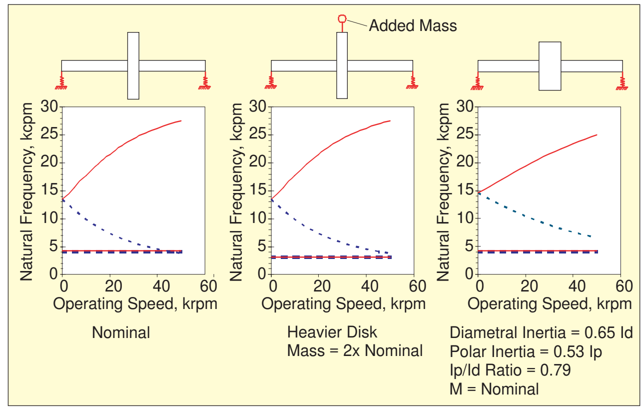

Figure 13. Comparison of different disk properties, center disk configuration.

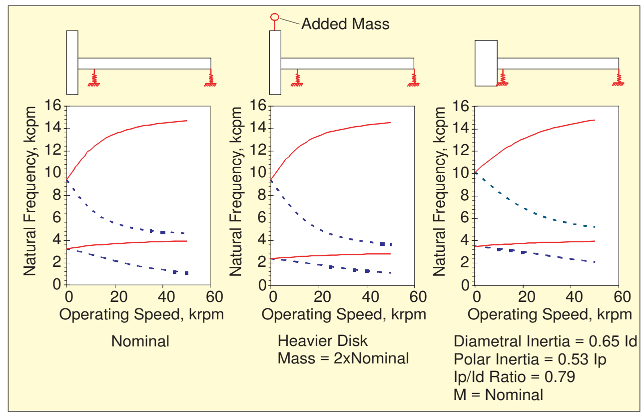

Figure 14. Comparison of different disk properties, overhung configuration.

For the first case, let's use a symmetric, center disk rotor again. Figure 13 shows the three models, and the three sets of natural frequencies versus speed. Comparing the nominal model to the two modified versions, note that:

- The increased mass lowers the first mode frequencies (mass is at a point of large whirling motion).
- The increased mass leaves the second mode unchanged (increased mass is at a point of little whirling motion).
- The reduced mass moment of inertia version does not change the first mode (disk center of gravity has very little conical motion).
- The reduced mass moment of inertia increases the frequency of the second mode, and decreases the strength of the gyroscopic effect (disk center of gravity has substantial conical motion).

For the second case, let's move the disk to the end, and move the bearing inboard to result in an overhung rotor with the same mass and overall length. Figure 14 shows the three models and the three sets of natural frequencies versus speed. Comparing the nominal model to the two modified versions, the important things to note are:

- The increased mass lowers the first mode frequencies and very slightly lowers the second mode frequencies.
- The reduced mass moment of inertia version increases the

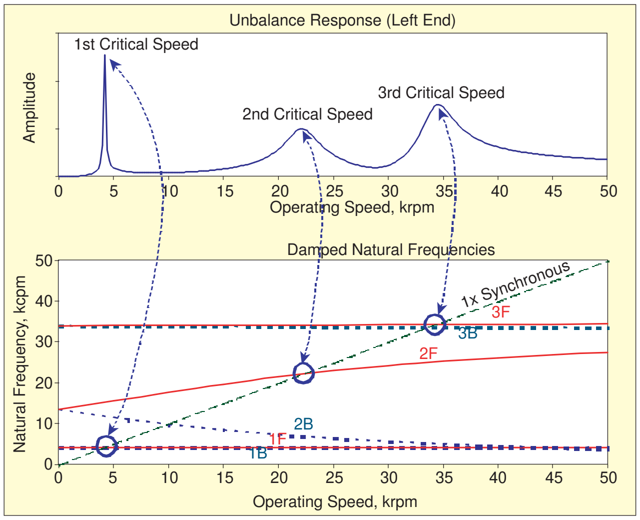

Figure 15. Comparison between natural frequencies and critical speeds.

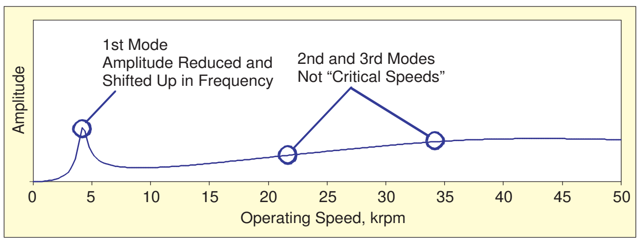

Figure 16. Critical speeds with additional bearing damping.

frequency of both the first and second modes, and decreases the strength of the gyroscopic effect.

If we looked at the mode shapes and these plots, we would again see that the reasons are the same as for the center disk rotor. Changes in the mass at a point of large whirl orbit strongly affect the natural frequency of that mode, but have little effect if it is at a node. Changes to mass moment of inertia at a location of large whirl orbit, on the other hand, have little effect. Changes to mass moment of inertia at a node with large conical motions have a strong effect on the corresponding mode.

Although not entirely obvious from the plots presented, changes in the ratio of polar mass moment of inertia to diametral mass moment of inertia change the strength of the gyroscopic effect. Indeed, for a very thin disk (a large ratio), the forward conical mode increases in speed so rapidly that the frequency will always be greater than the running speed. Indeed, there will be no conical critical speed as defined below.

Gyroscopic and Mass Effects – Summary So Far

Before moving on to critical speeds and unbalance response, let's summarize the last few sections related to natural frequencies in rotating systems.

- Machines with a nonrotating shaft behave much like familiar structures. However, once the rotor is spinning, the modes are no longer planar. With radially symmetric bearings, the rotor center traces out a circle.
- The rotor whirls either in the same direction as rotation, or against rotation, resulting in both forward and backward whirl modes.
- The frequencies are affected by both the mass and diametral mass moment of inertia.
- The mass has the greatest effect at points of large circular motion (anti-nodes), while the mass moment of inertia has the greatest effect at points of large rocking motion (nodes).
- Changes in mass precisely at a node do not change the corresponding natural frequency, and changes in mass moment of inertia at points of no conical motion (center of an axially symmetric rotor, for example), do not change the corresponding natural frequency.
- The modes affected by the mass moment of inertia (the conical mode, for example), are strongly affected by changes in speed. Assuming the bearing characteristics do not change, the backward whirl mode will decrease in frequency with increasing shaft speed, while the forward mode frequency will increase. The extent to which this occurs is related to both the mode shape and the ratio of the polar mass moment of inertia to the diametral mass moment of inertia.

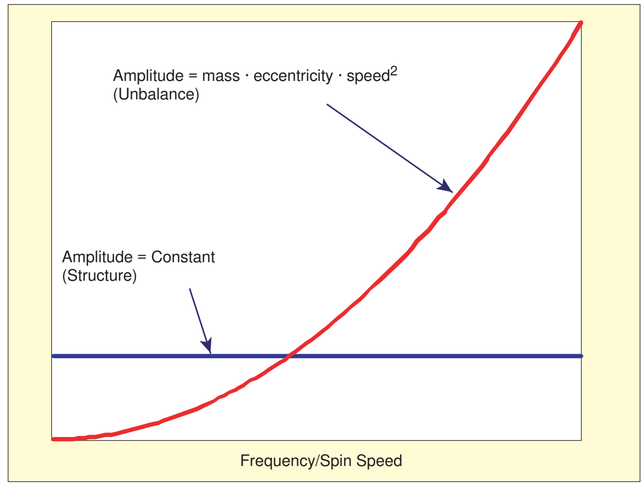

Figure 17. Characteristics of unbalance excitation.

Thus, a machine with a big disk/fan blade will probably show strong speed dependent effects in at least some modes. A fairly symmetric machine will probably have some modes that are relatively constant with shaft speed.

Critical Speeds

With some insight into rotating machinery modes, we can move on to "critical speeds." The American Petroleum Institute (API), in API publication 684 (First Edition, 1996), defines critical speeds and resonances as follows:

Critical Speed – A shaft rotational speed that corresponds to the peak of a noncritically damped (amplification factor > 2.5) rotor system resonance frequency. The frequency location of the critical speed is defined as the frequency of the peak vibration response as defined by a Bodé plot (for unbalance excitation).

Resonance – The manner in which a rotor vibrates when the frequency of a harmonic (periodic) forcing function coincides with a natural frequency of the rotor system.

Thus, whenever the rotor speed passes through a speed where a rotor with the appropriate unbalance distribution excites a corresponding damped natural frequency, and the output of a properly placed sensor displays a distinct peak in response versus speed, the machine has passed through a critical speed. Critical speeds could also be referred to as "peak response" speeds. As with the structural case, one can also consider a speed (i.e., unbalance excitation frequency) that coincides with a damped natural frequency (i.e., a resonance), generally termed "damped critical speeds." Numerically, these are distinct from critical speeds as defined by the API specification. For very light damping, they are fairly close. For increasing levels of damping, they become noticeably different.

As a critical speed example, we will use the medium stiffness, center disk model, and add an unbalance distribution that excites the first three modes. We will also add a small amount of damping at the bearings. Figure 15 shows the resulting vertical displacement response due to the unbalance forces at the left bearing as a function of speed. The damped natural frequency versus speed plot (Campbell Diagram) is drawn below for reference. Note that a line corresponding to 1× synchronous speed has been added to the damped natural frequency plot for reference.

As in the definition, critical speeds occur at the peak response speed when a system natural frequency is excited by the shaft unbalance. As with any resonance, very large ampli-

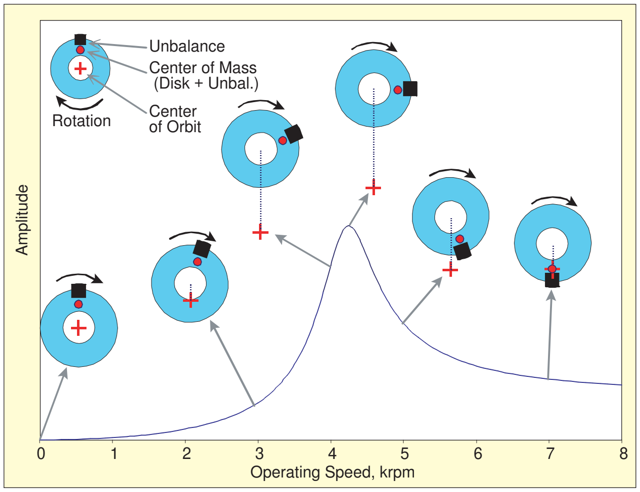

Figure 18. Phase relationship of center of orbit versus center of mass through critical speed.

tudes are possible, and are controlled only by the system damping. For this rotor-bearing system, the bearing damping is not as effective at controlling the amplitude through the first mode as it is through the second and third modes. Since unbalance does not excite the backward modes in this rotor-bearing system, there are no critical speeds corresponding to the intersections shown in the Campbell diagram between the synchronous (1×) line and backward modes.

If we increase the bearing damping somewhat, we would get the response shown in Figure 16. In this case, the additional bearing damping completely eliminates the response peaks at the second and third natural frequencies. Thus, these would no longer be considered "critical speeds" by the API definition, even though there is an intersection between operating speed and the corresponding damped natural frequencies.

All of the critical speeds in this case are forward modes, which is generally the case. Some more complex machines can have mixed modes, with some portions of the rotor whirling with rotation (forward), and some portions whirling against rotation (backward). With these machines, it is quite possible that a critical speed will be a mixed mode. It is also possible to have the special case of a lightly damped (i.e., ball bearing) machine that has a large difference between the vertical and horizontal bearing mount stiffness. For this unusual case, it is possible – though not very common – to have unbalance excitation of a backward mode.

Generally, machinery is designed not to run close to a critical speed due to the high vibration amplitudes associated with the resonance. As such, most machinery specifications require a minimum separation between the normal operating speed range and any critical speed.

Characteristics of Unbalance Excitation

A careful comparison of the previous set of unbalance response plots in Figures 15 and 16 for the center disk machine, and the frequency response plot for the spring-mass-damper structure in Figure 4 reveals two significant differences. At low frequencies, the structural plot shows a response equal to the static response, whereas the unbalance response plot starts out with no response. Likewise, at higher frequencies, the structural response decays, while the unbalance response tends to a constant value at higher speeds.

These two differences are the result of the frequency dependency of the constant amplitude sinusoidal force versus unbalance excitation. The structural excitation was assumed to be a constant force at all frequencies, while unbalance excitation has a speed-squared characteristic as shown in Figure 17. At zero rpm, there is no force from unbalance excitation, which explains the first difference noted.

The second difference – that the unbalance response amplitude goes to a constant value above the critical speed – has a

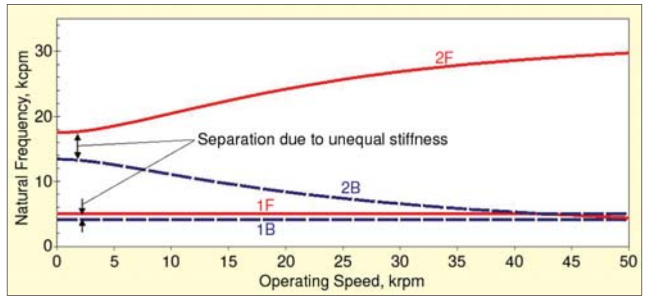

Figure 19. Natural frequencies vs. speed, nominal model, 2× vertical stiffness.

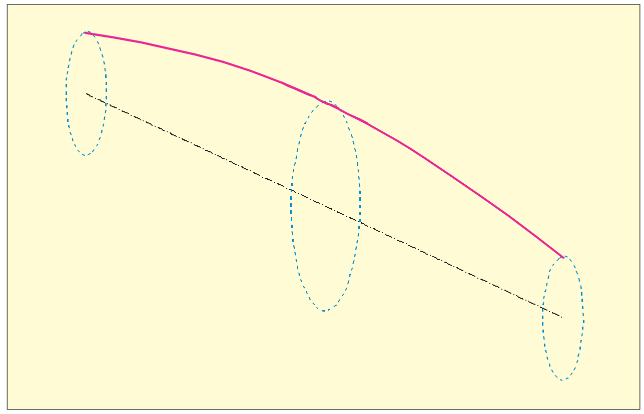

Figure 20. Non-circular (elliptical) modes with asymmetric bearing stiffness.

more interesting explanation. We can see what occurs by referring to Figure 18. This figure plots the relative angular relationship between the unbalance location and rotor response as rotor speed passes through a critical speed. Below the critical speed, the unbalance acts to pull the disk out into an orbit that grows increasingly large with speed. At the critical speed, the rotor response lags the unbalance by approximately 90°. However after passing through the critical speed, the phase between the unbalance force and the response direction has changed by 180°. As a result, the disk now rotates around the mass center of the disk/unbalance. Once the disk achieves this state, further increases in speed do not change the amplitude until the effects of the next mode are observed.

Some Complications

So far, our discussion has been relatively straight-forward. Structural modes that are planar when the shaft is not rotating become circular when the shaft starts rotating, and split into forward and backward modes. The forward modes increase in frequency with increasing speed, while the backward modes decrease in frequency with increasing speed. How much the mode changes depends on the distribution of mass and diametral mass moment of inertia and the shape of the corresponding mode shape. Unless the bearings have high damping, there is a critical speed every time the rotor speed passes through with a forward whirling damped natural frequency. In some cases, the modes have mixed forwards and backwards motion. Finally, there is a pathological case where backwards whirl can be excited by unbalance forces. However, we work in the real world, and in the real world there are always complications.

The first has already been foreshadowed. Many real machines do not have equal bearing/mount stiffness in all directions. For rolling element bearings, the support structure is frequently asymmetric. For fluid-film bearings, both the support and the bearing can be asymmetric. So what happens?

There are basically three effects. In the first, there is additional frequency separation between the pairs of modes as shown in Figure 19. In the second, the orbits traced out by points on the rotor will no longer be circular and become elliptical (see Figure 20). Otherwise, the picture remains much

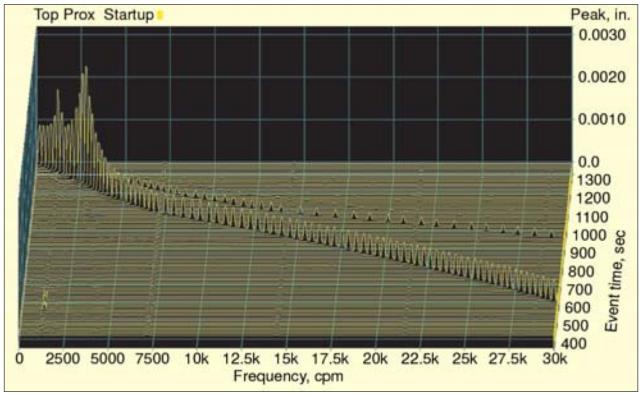

Figure 21. Event time waterfall of startup.

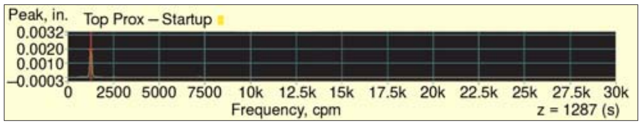

Figure 22. Spectral line at 1274 rpm first critical.

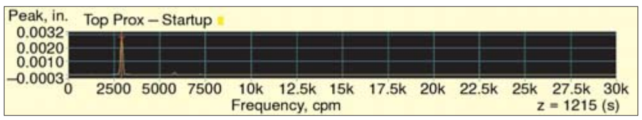

Figure 23. Spectral line at 2900 rpm second critical.

the same.

The second complication applies primarily to fluid-film bearings. The stiffness and damping characteristics for these bearings are a strong function of shaft speed. Fluid-film bearings also have cross-coupling between the vertical and horizontal axes. A force in the vertical axis will generate a displacement in the horizontal axis, and vice-versa. Thus, it is possible for these bearings to change the neat generalization that forward whirl increases with speed and backward whirl decreases with speed. Depending on the rotor and bearing characteristics and how the latter change with speed, it is possible that a forward mode might actually appear to decrease with speed, and/or a backward mode increase with speed.

A third noteworthy complication is that rotating machinery is rarely proportionally damped and the modes are generally complex rather than real. The upshot of this technical detail is that the phase angle between various points on the shaft is frequently not 0° or 180°, as is generally seen in lightly damped structural modes. The details of this circumstance are beyond the scope of this article, but it is an important point to be aware of for the more advanced reader.

Other complications might include nonlinear effects, shaft bow, thermal bow, etc. For more details, the reader is encouraged to consult the various texts in the field, or take one of the excellent short courses available from several organizations.

Case History – Introduction

While all of this critical speed and rotating natural frequency 'stuff' is nice, how might it apply to the real world? As an answer to this question, we will present a case study based on some of the data and results for the project that inspired this article. Alfa Wassermann, Inc., a manufacturer of medical centrifuges, desired to establish a baseline of rotor dynamic characteristics for a long standing product line. The goal was to develop an experimentally validated rotor dynamic model for this machine. This project is of special interest to this current article because over the speed range considered, this machine behaves very much like the soft bearing (rigid rotor), center disk machine we have been considering.

The technical approach for establishing the product's baseline was to develop an analytical rotor dynamics model using Eigen Technologies' DyRoBeS, qualify the model through experimental modal analysis using Vibrant Technology's

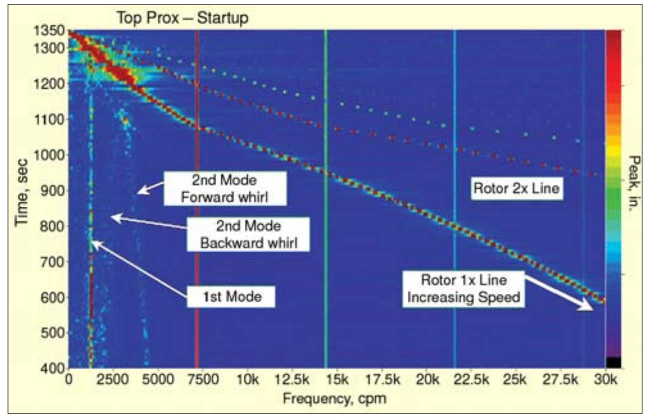

Figure 24. Color map of startup showing rotor criticals as faint background lines.

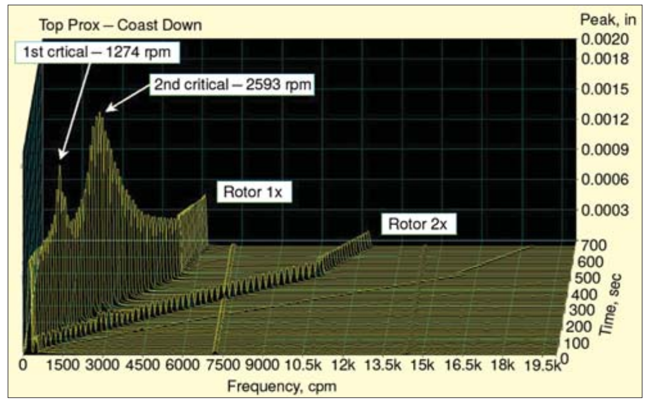

Figure 25. Event time waterfall of coastdown data.

ME'scope, acquire operational vibration data in the form of waterfall plots using Dactron's RTPro Dynamic Signal Analysis software, use the operational data to fine tune and validate the rotor dynamics model, and evaluate various aspects of the machine's predicted performance.

Case History – Operating Data

Vibration was measured by existing proximity probes installed at each end of the rotor. The rotor was, however, completely sealed from access and a once-per-revolution pulse signal was not available. Without a pulse signal, waterfall data could not be plotted with a traditional 'rpm' z-axis. Instead, an "event time" waterfall was used to position spectral lines versus time along the z-axis.

The first acquired spectral line is positioned at the rear of the waterfall and has an event time label equal to the number of seconds before the present. The most recent spectral line is at the front of the plot and has an event time label of 0.0 seconds. For example, the startup waterfall presented in Figure 21 has the zero speed spectrum located at the rear-most position with an event time label of 1300 seconds. New spectral lines were added every nine seconds. The line of 1× peaks moves to the right and forward with increased rotor speed. Fortunately, startup speed versus time was almost linear, making the 1× line appear to be almost linear and therefore easy to visually interpret.

This startup data shows that the rotor passes through its first critical at 1274 rpm followed by a second critical at 2900 rpm. Spectral lines associated with each of these criticals have been extracted from the waterfall and are presented for information in Figures 22 and 23. Modal analysis determined that the first rotor natural frequency results in a 'cylindrical' critical speed while the second natural frequency results in a 'conical' mode as would be expected.

Replotting the waterfall of Figure 21 as a color map results in Figure 24. Of primary interest in this figure is not the series of peaks along the 1× line, but evidence of the rotor's damped

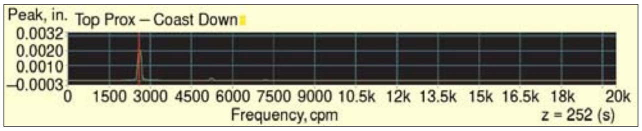

Figure 26. Spectral line at 2593 rpm second critical.

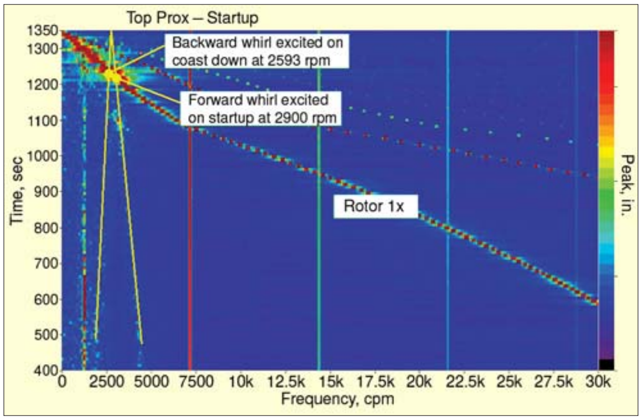

Figure 27. Tracking of conical modes.

natural frequencies being excited by 'background' vibration. These are seen in the color map as faint lines in the background vibration, unassociated with operational harmonics.

The first faint line seen in Figure 24 is at a constant frequency of 1274 cpm. Rotor dynamic analysis and nonrotating modal testing determined that the first mode would be 'cylindrical,' which as previously discussed will remain at a fairly constant frequency regardless of running speed because it is not influenced by gyroscopic effects. A cylindrical shape was confirmed by end-to-end proximity data shown being in-phase.

Rotor analysis calculated the second mode to be a 'conical' shape, where motion at one end of the rotor is out-of-phase with that at the other end. As previously described, this mode will be influenced by gyroscopic effects that split the nonrotating rocking mode into two modes, one whirling in the forward direction and the other whirling in the reverse direction. As previously described, the forward mode frequency increases with increased shaft speed while the backward mode decreases with increased shaft speed. This is shown by the faint lines in Figure 24 that form an upside down '∨' where the apex occurs at zero speed at the top of the plot. The apex occurs at the frequency where experimental modal analysis finds the rocking mode when the shaft is not spinning. The modes can be seen to gain greater frequency separation as shaft speed increases. The left leg of the '∧' is the backward component that decreases in frequency with increased speed, while the right leg is the forward component that stiffens and increases in frequency.

Figure 25 presents coast down data, again showing two criticals. On coast down, higher speed data are at the back of the waterfall with the greater time label and the spectral line for zero speed is at the front with a label of 0.0 seconds. While the first critical occurs at the same speed as during startup, 1274 rpm, the second critical interestingly is now observed at 2593 rpm rather than at 2900 rpm. Figure 26 shows the spectral line with this frequency. If the practitioner had only viewed peak frequencies from startup and coast down waterfalls, he should be teaming with curiosity as to why the second critical has different frequencies.

If we again plot Figure 24 and apply lines to help track the forward and backward whirl conical modes as shown in Figure 27, we get a clue. It appears that forward whirl is excited during startup at 2900 rpm, and the backward whirl is excited at 2593 rpm during coast down. Figure 28 shows a plot of the corresponding prediction of damped natural frequencies versus operating speed from the analysis. The predicted frequencies for the second mode are slightly higher than measured during the coast down, but show a separation of about 250 rpm, which is within the frequency sample spacing of the experimental data. The first mode prediction is in good agreement.

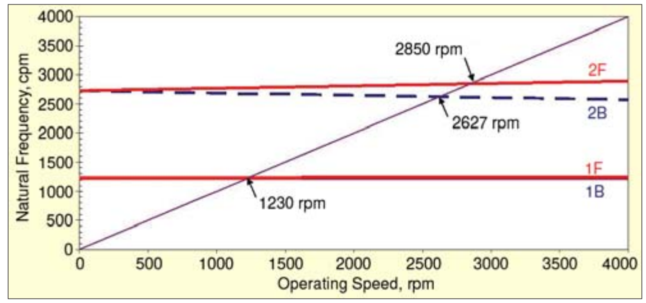

Figure 28. Damped natural frequency vs. operating speed predictions.

We can also postulate that the reason for the shift in peak response speed between startup and coast down could be the expected excitation of the forward critical speed during run-up, and an unexpected excitation of the backward mode during coast down. Given that the machine under consideration should have fairly symmetric bearings, it does not fit the typical profile of a machine that would exhibit backward whirl due to unbalance excitation; thus, the explanation would lie with seals and several other potential nonlinearities. A second possible explanation would be a nonlinear stiffness effect related to larger amplitudes on run-up than during coast down.

Conclusion

It was shown that cylindrical rotor modes are not influenced by gyroscopic effects and remain at a fairly constant frequency versus rotor speed. Conversely, conical rotor modes are indeed influenced and caused to split into forward and backward whirl components that respectively increase and decrease in frequency with increased rotor speed. S|V

The authors can be contacted at cpowell@structuraltechnology.com.