New Technologies for Test-Analysis Correlation

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This article presents a discussion of new technology developments in test-analysis correlation software. The Attune software program is introduced and described, and an example of its use is presented by correlating an aircraft model to representative test data.

Recent developments in finite element analysis (FEA) have made significant strides in performing "real world" simulations. With the ever increasing computational power at the disposal of analysts and new technology and algorithm developments, commercial software codes are allowing analysis of increasingly complex models of full vehicles with more realistic environment loading conditions. Routine analysis increasingly involves advanced features such as nonlinear materials, failure, and contact. Additionally, more and more components of the vehicle system are included in the FE model in an increasing amount of detail. Whereas several years ago, a joint between two components might have been modeled with a linear spring stiffness, that same joint today may be represented with a detailed FEM of the joint with contact and friction defining the joint behavior.

With fewer assumptions made, the simulations are more realistic. With more realistic simulations of the product, there is an increasing pressure to eliminate the testing phase of product development. The problem is that more realistic simulations do not ensure the accuracy of the model. Furthermore, even if the analytical model accurately captures the physics of the structure, material uncertainties can result in differences between the FE model and the real structure. Testing of the physical structure is critical to validate the analytical model and the analysis results.

For the development of aircraft, for example, months and years are often spent analyzing the full vehicle. The FE model is used to determine loads, strength and margins of safety for the structure. All of the design and analysis is performed with a model that has not been verified to be true and accurate. For this reason, every aircraft undergoes a ground vibration test (GVT, see Figure 1) before any flight occurs. The GVT measures the dynamic properties of the vehicle by applying an excitation force to the structure using electro-mechanical shakers and measuring the response of the structure by placing accelerometers throughout the vehicle. After the test occurs, the accuracy of the analytical model is evaluated by comparing the test measured response of the aircraft to the analytical FE model.

While the analytical model is often close to representing the physical structure, it is quite common for key mode frequencies to be different by 15 percent and the shape of the mode to be in error by 20 to 30 percent. Rarely does the analytical model match the physical article before some physical testing has occurred to check the model against. With the aircraft built and ready for flight, the engineering team must quickly understand why the analytical model differs from the physical product and if those differences are going to affect the previous analysis that was performed; this includes coupled-loads analysis, flutter, control-system development, and other advanced analyses that were the basis for the design of the aircraft. Years of analysis need to be validated in a very short amount of time. For a typical aircraft development program, the GVT is performed only a few weeks before the first flight. The challenge is to quickly assess the differences between the real aircraft and the analytical model, understand why there are differences, and update the FE model so that it accurately represents the real structure. This process is called test-analysis correlation and model updating.

Test-Analysis Correlation

The process for test-analysis correlation begins with the comparison between the analytical model and the test-measured structure. This comparison is made based on the mode shapes and frequencies. The frequencies are simply compared by examining the difference between the analytical frequency and the measured frequency. The mode shapes are compared using primarily two different methods. The first shape comparison is the calculation of the cross-orthogonality (XO) of the two shapes. Cross-orthogonality is a mass-weighted measure of the linear dependence of the test mode shapes on the analytical mode shapes (Eq. 1). The other shape comparison sometimes used is the cross-modal assurance criteria (cross-MAC, Eq. 2). The cross-MAC is a comparison of the shapes without the mass weighting. For both criteria, a perfect match between the analytical and test shapes would result in an identity matrix, i.e., ones on the diagonal and zeros on all of the off-diagonals. A typical goal for correlation is for diagonal terms to be above 0.9 for primary modes and the off-diagonal values to be less than 0.1. Both the cross-orthogonality and the cross-MAC are used by industry. The advantage of the cross-orthogonality is that it puts a higher weighting on deflections associated with large masses.

$$X_{ORTHO} = [\Phi_{FEM}]^T [M_{reducedFEM}][\Phi_{TEST}] \tag{1}$$

$$X_{MAC} = \frac{\left(\Phi_{TEST\,i}{}^T \Phi_{FEM_j}\right)^2}{\left(\Phi_{TEST_i}{}^T \Phi_{TEST_i}\right)\left(\Phi_{FEM_j}{}^T \Phi_{FEM_j}\right)} \tag{2}$$

After the comparison of the model is made to test, a sensitivity analysis is performed. All of the model-update calculations that are part of the correlation are based on the sensitivity of some system parameter (e.g., stiffness, frequency, cross-orthogonality, cross-MAC) with respect to design variables. The design variables are the physical or material properties in the model that the engineer is willing to change to get the analytical model to match the test measured modes. Often these design variables include shell element thicknesses, spring stiffness and material properties. The sensitivities are computed in an FEA code using a finite difference approach. In a finite difference method, the design sensitivity is calculated by making a small variation in each design variable and recalculating the required system parameter. The resultant design sensitivity coefficients define the relationship between the design variables and system parameters.

The analytical model is then updated utilizing the design sensitivity coefficients and optimization methods to identify the model changes that minimize the difference between test and analysis results.

Attune

Attune is a program for test-analysis correlation and model updating (Figure 2). It uses design sensitivity and optimization methods to align the mode shapes and frequencies of an analytical model to test measurements. It is a stand-alone tool that combines more than 25 years of experience into a user-friendly, flexible toolkit. By automating the correlation process and providing a suite of leading edge optimization tools, Attune allows

Figure 1. Ground vibration test of Predator aircraft.

the engineer to meet the challenges associated with assessing, updating, and verifying the FE model in a very short amount of time.

The two main criteria of test-analysis correlation are to achieve an analytical model that matches the test measured structure, and to do so quickly. The development of Attune focused on these two aspects, accuracy and efficiency. The goal of accuracy was accomplished through new technology development. Attune approaches the correlation and model updating from a different methodology than previous test-analysis correlation tools. The goal of efficiency was addressed through technology development, but also through ease-of-use features. Correlation of the analytical model takes many iterations. If each iteration can be performed in a shorter amount of time, the total time needed to attain a test-verified FE model can be dramatically reduced.

Technology

Unlike other correlation tools that use modal parameter sensitivities (mode shapes and frequencies) in the optimization process, Attune uses the sensitivities of the reduced model matrices in its calculations. This not only decreases the solution time for each iteration, but the variation in the reduced model matrices is much more linear with respect to the design sensitivities. This results in greater accuracy in the optimization process and a reduced number of iterations needed to converge to an optimized solution.

Several different optimization algorithms have been implemented in Attune to drive the model updating. The optimization routines are used to determine, based on the design variable sensitivities, the minimum number of variable changes that need to be made to achieve a correlated model. The objective of the optimization is to minimize the following objective function:

$$J = \{y\}^T \left[W_y\right]\{y\} + \{x\}^T \left[W_x\right]\{x\}, \ \{x_{lb}\} \le \{x\} \le \{x_{ub}\} \qquad (3)$$

where:

$\{y\}$ = Vector of state variable errors (frequency, cross-orthogonality).

$[W_y]$ = Diagonal weighting matrix for state variables (frequency, cross-orthogonality).

$\{x\}$ = Vector of design variables.

$[W_x]$ = Diagonal weighting matrix for design variables.

Attune uses Monte-Carlo, gradient-based and genetic algorithms.

The Monte Carlo algorithm is a "brute force" technique wherein a user-defined number of designs are randomly generated. The design with the minimum objective function value is chosen as optimal. The benefit of this algorithm is that it cannot be trapped by a local minima. The downside is that because each design is chosen at random, there is no guarantee that the global minimum will be an element of the design population.

Figure 2. Attune software for test-analysis correlation and model updating.

The gradient algorithm uses design parameter sensitivities to compute approximate model parameter sensitivities. The derivatives of the model parameters are linearized approximations about the current design. Because the equation includes sensitivities of state variables with respect to design variables, the derivatives of the modal parameters discussed in previous sections must be approximated based on the modal matrix sensitivities.

The gradient-based optimization must solve the following equation:

$$2[DS]^T[A]\{\Delta y_0\} + \Big[[DS]^T[A][DS] + B\Big]\{\Delta\alpha\} = 0 \qquad (4)$$

subject to the constraints.

$$\{\alpha_{LB}\} \le \{\alpha\} \le \{\alpha_{UB}\}$$

One method for solving the constrained optimization problem is called MINQ. MINQ uses a combination of coordinate search and subspace minimization. Since the derivatives of the modal parameters are approximate, the gradient optimization process is iterative. The steps are as follows:

1. Compute approximate modal parameter sensitivities.
2. Minimize the objective function subject to constraints.
3. Repeat the first two steps until convergence.

The strength of this method is computational efficiency. Depending on the method for computing the approximate sensitivities, this optimization scheme could have the lowest computational cost per iteration. The main weakness of this method is in the linear approximation of the sensitivities. This approximation can lead this method to get trapped in local minima. Also, to make sure that the predicted change in state variables matches the change achieved by a given change in design, the design change per iteration must be kept small.

The last method, the genetic algorithm (GA), seeks to minimize the same objective function as in the gradient optimization. However, the approximate sensitivities of the modal parameters are not necessary. The GA evaluates a set of randomly chosen designs against the objective function, then tries to systematically improve the designs through iteration.

The first step in a GA is to discretize the design space. For this optimization scheme, the design variables must be bounded. Each design variable is allowed to take on a discrete number of values within the specified range. The designs are encoded as binary numbers. The number of discrete values that a design variable can take on is determined by the number of bits, N, used for that variable ($2N$ values). For example, a de-

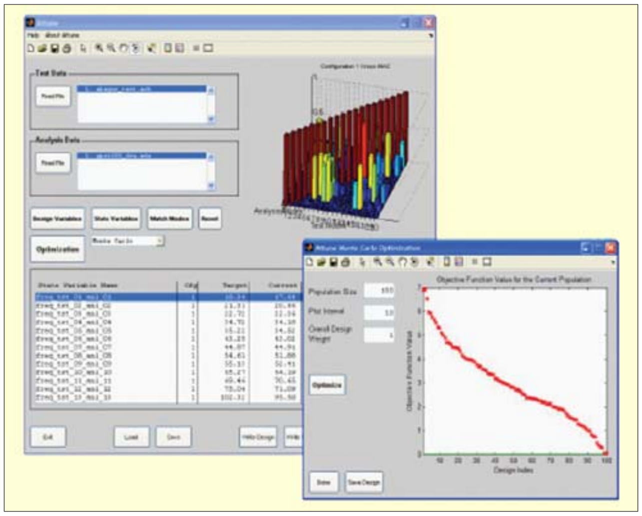

Figure 3. Attune user interface.

sign variable discretized using 4 bits can take on 16 values. The resolution – the distance between discrete points – is given by

$$r(N) = \frac{UB - LB}{2^N - 1} \tag{5}$$

where UB is the upper bound and LB is the lower bound.

The next step in the GA is to create an initial population and evaluate the population against the objective function. In general, the computational cost and quality of the result both increase as a function of the population size. If L is the total number of bits used to discretize the design, an empirical study showed that a population size of $4L$ would be effective.

Once the population has been evaluated, the designs are made to compete against one another in a two round tournament (tournament selection). In each round, two designs are chosen at random and their objective function values compared. The design with the lower objective function value is added to the 'winner' population. This type of selection process has two certain outcomes. The design with the minimum overall objective function value will always be chosen and the design with the maximum overall objective function value will never be chosen.

After selection, the GA performs 'crossover.' There are several forms of crossover, including two popular forms referred to as uniform crossover and single-point crossover. For uniform crossover, two designs are chosen at random from the 'winner' population. From these two designs, two 'children' designs will be generated. For each bit of the design, it is randomly determined which child will receive the bit from which parent. For example, there is a 50% chance that parent 1's first bit will go to child 1. If it does, child 2 gets its first bit from parent 2. If it does not, child 1 gets its first bit from parent 2 and child 2 gets its first bit from parent 1. This is repeated for all bits.

The process for single-point crossover is similar except that the designs of the children are not randomly chosen bit-by-bit. Instead, a single random number determines the point at which crossover occurs. In other words, if the design is made up of 10 bits and the crossover point is 4, child 1 would be a copy of parent 1 for bits 1-4 and a copy of parent 2 for bits 5-10. Similarly, child 2 would be a copy of parent 2 for bits 1-4 and a copy of parent 1 for bits 5-10. Because single-point crossover is more likely to leave large portions of the design intact, it is considered a more aggressive method, whereas uniform crossover is considered more exploratory.

There is one further step before the next generation is complete – mutation. Mutation is the process by which new "genetic material" is added to the population, which is useful in case the initial population did not contain the part of the design space that included the global minimum. For uniform mutation, there is a specified probability that any bit in the design population could flip. The default mutation probability

Figure 4. Aircraft model used in Attune example.

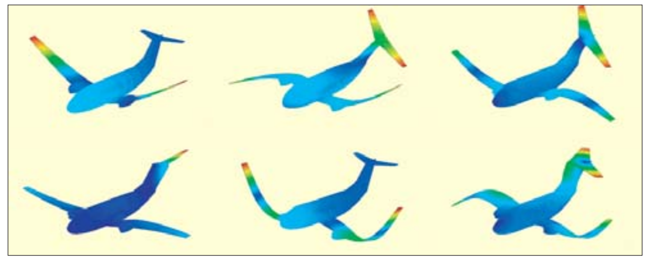

Figure 5. First six mode shapes of the generic aircraft analytical model.

is $(L + 1)/(2LN_{POP})$, where L is the total number of bits used in the design and N_{POP} is the size of the population. A larger mutation rate would be more exploratory, less would be more aggressive.

An often used option with GA is called elitism, which allows the best design(s) to propagate forward to the next generation without crossover or mutation. This guarantees that the GA will have monotonic convergence; however, it also effectively diminishes the population size by the number of elite members. The disadvantage of the genetic algorithm is that each iteration is typically more computationally intensive than the other optimization methods.

Utilization of all of these different algorithms during the correlation process results in obtaining an updated, test-verified model faster and often more accurately then previous capabilities.

Usability

A significant amount of attention was paid to the overall usability of the Attune software package (Figure 3). The design in the user interface was driven by ease of learning and also efficiency of use. Very little training is needed to learn Attune, typically a one hour on-line training seminar for most users.

The main interface has five distinct areas that reflect the process flow for correlation. At the very top of the window is the file input region. Correlation starts with the user inputting the test shapes and the analytical shapes they want to use for the correlation. Once the data are imported, the status of the correlation (the user can choose either the cross-orthogonality or the cross-MAC) is automatically summarized graphically and textually in a summary table. Underneath the input file region is the action region. The action region has buttons for the user to examine the design variables and the state variables, to examine and redefine the matches between test and analysis shapes, and to perform the analysis. The design variables are the model variables that the user defines in the FEA sensitivity analysis. The state variables are the model frequencies and shapes that the user is trying to align between test and analysis. Attune automatically matches the modes between test and analysis by examining the cross-MAC or cross-orthogonality matrix and selecting the modes that have the strongest correlation. The user can both review and manually override this matching by selecting the match modes button. The last action button is for optimization. A simple pull-down window selects the optimization algorithm and the optimization button then

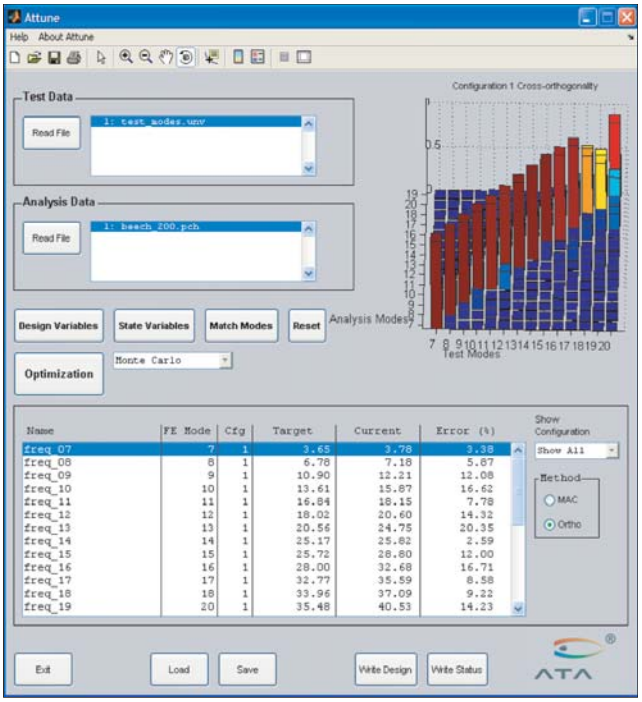

Figure 6. Initial correlation between test and analysis.

performs the optimization.

After the optimization is performed, the graphical display of the correlation and the table summary are automatically updated to reflect the new design. The user can also save the Attune session, write a status file to MS-Excel to summarize the correlation and keep an external record of the design variables that were changed, and write the design. The write design button tells Attune to write the updated design variable values back to the FEA model input file.

Besides being easy to use, Attune is also very fast and efficient. When the test and analysis data are imported, Attune automatically matches the modes based on their orthogonality and displays the results to the user. Significant time is saved by performing the mode matching automatically and not relying on the engineer to define it. Attune also allows users to perform multiple optimizations within one correlation session, and also gives them the ability to use different optimization algorithms within one correlation session.

Attune also allows multiple configurations of the structure to be correlated at the same time. It is quite common for testing of an aircraft in full fuel, half fuel and empty states. The physical structure is not changing between these configurations. Attune allows modes for multiple configurations to be imported into the software and the software automatically recognizes that the design variables effect all of the configurations.

The other significant time saving capability is the "write design" action. Attune actually modifies the FEA sensitivity analysis input deck and writes the new design variable values into the deck. The user does not have to directly edit the analysis bulk data deck, and thus can very quickly perform correlation iterations.

Example

To demonstrate capabilities, an example aircraft model was created, representative test data were developed, and the model was correlated using the Attune software program. The aircraft finite element model is shown in the Figure 4, a generic business aircraft. The FEM incorporates internal structures that cannot be seen in this image of bulkheads, spars and ribs.

The fundamental modes of the structure were solved for using Nastran and are shown below, with illustrations in Figure 5. The first few modes are 1st wing bending, wing antisymmetic bending with vertical tail out-of-phase and in-phase, wing torsion, and wing 2nd bending. These are going to be the target modes for our correlation effort. The frequencies for these modes are listed below:

- Mode 1 – 3.8 Hz, 1st wing bending.
- Mode 2 – 7.2 Hz, anti-symmetric wing bending with vertical tail out-of-phase.
- Mode 3 – 12.2 Hz, anti-symmetric wing bending with vertical tail in-phase.
- Mode 4 – 15.9 Hz, vertical tail roll.
- Mode 5 – 18.1 Hz, wing 2nd bending.
- Mode 6 – 20.6 Hz, vertical tail torsion.

The test frequencies and shapes were developed by modifying the analytical model properties and rerunning the modes. This modification represents the differences in mode frequency and shape between test and analysis for typical aircraft. The test shapes look similar to the analysis shapes. The test frequencies are listed below:

- Mode 1 – 3.6 Hz, 1st wing bending.
- Mode 2 – 6.8 Hz, anti-symmetric wing bending with vertical tail out-of-phase.
- Mode 3 – 10.9 Hz, anti-symmetric wing bending with vertical tail in-phase.
- Mode 4 – 13.6 Hz, vertical tail roll.
- Mode 5 – 16.8 Hz, wing 2nd bending.
- Mode 6 – 18.0 Hz, vertical tail torsion.

The next step in the correlation process is to choose design variables and run a sensitivity analysis to determine the sensitivity coefficients. The design variables are chosen by the user and are any physical property that the engineer is willing to change in the model. For this example case, the design variables included bulkhead shell thickness, thickness of the aircraft skin, thickness of the tail spar, thickness of the ribs, and thickness of the forward and aft wing spars. These design variables will be utilized by Attune during the optimization and model updating tasks. A provided alter in Nastran is used for the calculation of the sensitivities to the modal parameters.

After the sensitivity analysis is performed, the correlation can be performed using Attune. The first step is to read in both the test measurements and the analytical model. Once the test and analysis data are imported into Attune, the program automatically performs the cross-orthogonality or cross-MAC calculation and displays the results to the user. Figure 6 shows the initial correlation between test and analysis. The shapes for the lower frequency modes match quite well, but the frequencies are in error by up to 20 percent.

The "Design Variables" button (Figure 7, top) allows the user

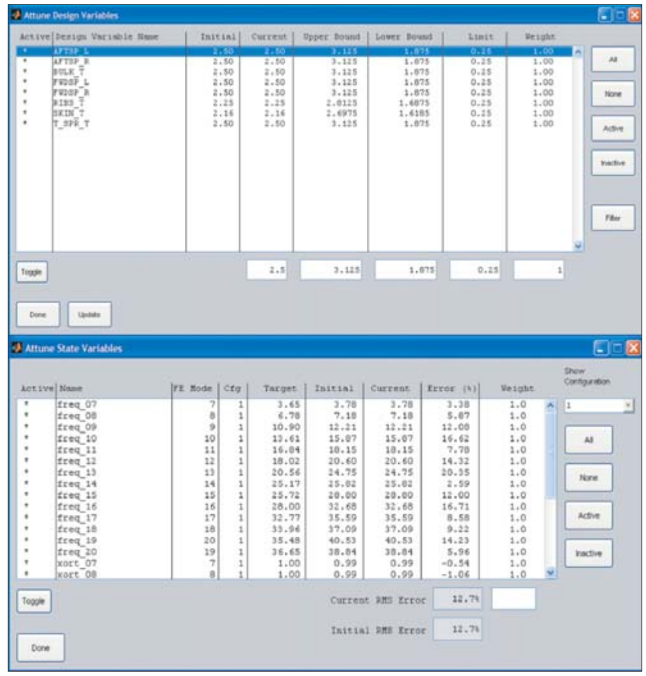

Figure 7. Top – Attune design variables, Bottom – Attune state variables.

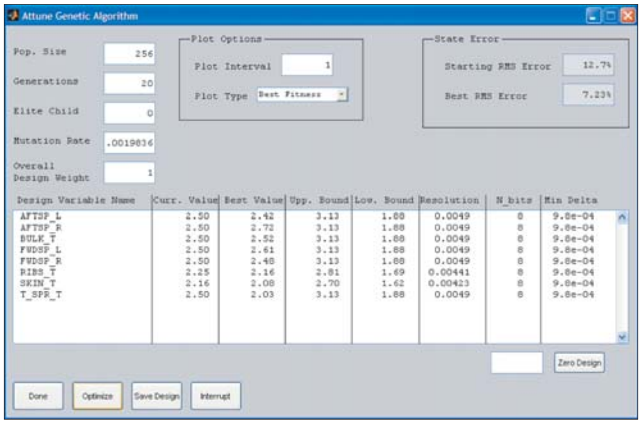

Figure 8. Attune genetic algorithms.

to set the upper and lower bounds for changes in the variable as well as setting weighting factors. The weighting factors are penalty factors that are used in the objective function during the optimization routines. The "State Variables" button (Figure 7, bottom) shows the parameters that are being correlated; frequencies and cross-orthogonalities. The user can also set a factor on state variables to weight key modes higher than secondary modes.

The different optimization algorithms were described in the technology section. For this problem, the genetic algorithm is going to be utilized (Figure 8). By selecting genetic algorithm optimization, a sub-window appears to set the genetic algorithm parameters, including the population size, number of generations, whether an elite child is passed from one generation to the next, and the mutation rate. The optimization window also shows the current value for the design variables and the optimized best values.

After the optimization is performed, the Attune main window shows the updated correlation between the analytical model with modified design variables and the test measurements (Figure 9). Attune also exports a formatted spreadsheet to view the current correlation and the modifications that were made (Tables 1 and 2).

Table 1. State variables. Green boxes indicate variables within goal criteria.

Config.	Label	Original	Target	Modified	Orig_Err	Mod_Err
	freq_07	3.78	3.65	3.50	−3.38%	4.26%
	freq_08	7.18	6.78	6.54	−5.87%	3.52%
	freq_09	12.21	10.90	10.83	−12.08%	0.60%
	freq_10	15.87	13.61	13.80	−16.62%	−1.39%
	freq_11	18.15	16.84	16.44	−7.78%	2.35%
	freq_12	20.60	18.02	18.23	−14.32%	−1.15%
	freq_13	24.75	20.56	21.82	−20.35%	−6.12%
	freq_14	25.82	25.17	24.14	−2.59%	4.09%
	freq_15	28.80	25.72	25.33	−12.00%	1.33%
	freq_16	32.68	28.00	28.80	−16.71%	−2.86%
	freq_17	35.59	32.73	32.32	−8.58%	1.39%
	freq_18	37.09	33.96	33.54	−9.22%	1.22%
	freq_19	40.53	35.48	35.81	−14.23%	−0.94%
1	freq_20	38.84	36.65	35.20	−5.96%	3.96%
	xort_07	0.99	1.00	0.99	−0.54%	0.71%
	xort_08	0.99	1.00	0.99	−1.06%	0.63%
	xort_09	0.98	1.00	0.99	−1.81%	0.62%
	xort_10	0.99	1.00	1.00	−1.10%	0.39%
	xort_11	0.97	1.00	0.96	−3.02%	4.07%
	xort_12	0.97	1.00	0.97	−2.77%	3.37%
	xort_13	0.99	1.00	0.99	−1.09%	1.23%
	xort_14	0.99	1.00	0.98	−1.49%	1.56%
	xort_15	0.99	1.00	0.99	−1.05%	1.01%
	xort_16	0.96	1.00	0.98	−3.70%	1.56%
	xort_17	0.95	1.00	0.94	−4.63%	5.89%
	xort_18	0.72	1.00	0.76	−27.64%	23.78%
	xort_19	0.61	1.00	0.76	−39.35%	24.49%
	xort_20	0.88	1.00	0.95	−12.14%	4.83%
				RMS error	12.67%	7.03%

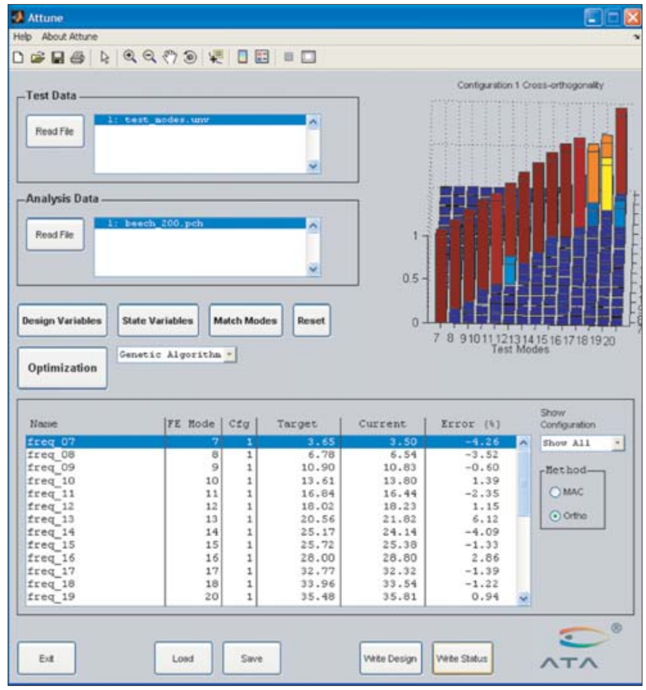

Figure 9. Updated correlation between the analyticsl model with modified design variables and the test measurements.

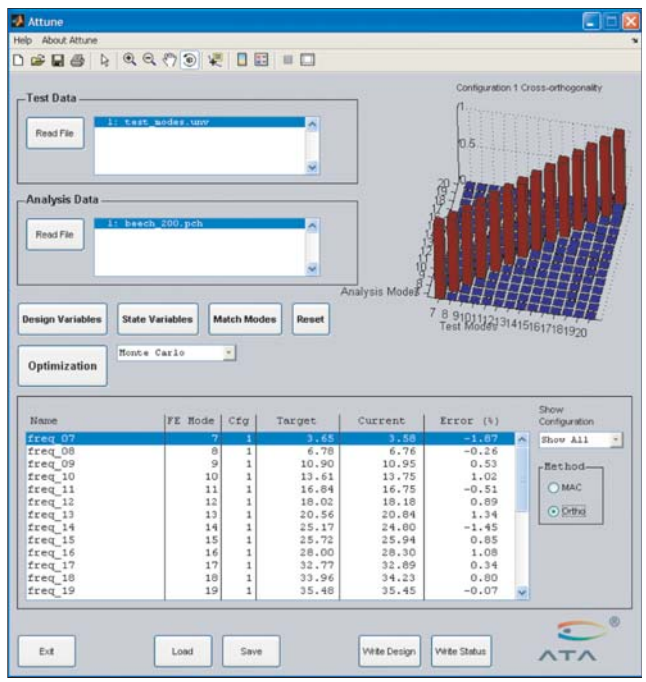

Figure 10. Test-verified analytical model with frequency errors less than two percent.

The optimized model appears to be significantly closer to the test measurements. To verify that these design changes are valid and to calculate new sensitivities about this new base-state, the design is written back to Nastran. Attune automatically modi-

Table 2. Design variables.

ID	Label	Original	Modified	Upper Limit	Lower Limit
151	AFTSP_L	2.50	2.30	3.75	1.25
171	AFTSP_R	2.50	2.81	3.75	1.25
101	BULK_T	2.50	2.50	3.75	1.25
141	FWDSP_L	2.50	2.53	3.75	1.25
161	FWDSP_R	2.50	2.56	3.75	1.25
131	RIBS_T	2.25	2.07	3.38	1.13
111	SKIN_T	2.16	2.09	3.24	1.08
121	T_SPR_T	2.50	2.06	3.75	1.25

fies the Nastran input file, removing this burden from the engineer.

The correlation process typically includes many iterations of solving for the analytical sensitivities, updating the model using Attune, writing out the new design, and resolving for the updated modes and sensitivities. After several iterations the analytical model does a very good job of matching the test data. The cross-orthogonalities are all above 0.95 and the frequency errors are all below 2 percent (Figure 10). The result is a test-verified analytical model that can be used for full vehicle analysis.

Summary

Regardless of the improvements in FEA simulation of "real world" events, testing will always be a critical and necessary component to the vehicle development process. For aircraft, the full vehicle GVT is the final validation of the FE model and the FE analysis that was performed during the design phase of the development. It is critical that in a very short amount of time the analytical model can be compared to the test data, the differences can be understood, and the analytical model validated.

Attune represents the latest technology for test-analysis correlation and was written based on extensive experience in both modal testing and test-analysis correlation. Attune combines the very latest in technology with an interface that is easy to learn and very efficient when performing correlation. At the same time, Attune allows the engineer complete flexibility and control to drive the correlation and optimization. SV

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