Test-Based Computational Model Updating of a Car Body in White

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To validate finite-element models, test data from experimental modal analyses may be utilized. The model data must be highly accurate since they form the basis for subsequent validation efforts. An integrated validation strategy is presented that takes into account the complete process chain from model-based test design through modal testing, data evaluation, test/analysis correlation to computational model updating. By means of a real car body in white, the single steps of the validation strategy are highlighted, and it is shown that very encouraging results can be obtained even for very complex systems.

The fidelity of structural mechanical finite-element analyses (FEA) can be evaluated by using data from static or dynamic tests. Eigen frequencies and eigen vectors are employed that can be identified from vibration tests by means of experimental modal analysis (EMA).[1,2] The deviations between test and analysis allow for evaluating the quality of the target finite-element model. If the deviations between test and analysis are not acceptable, the elastomechanical system needs to be reviewed and eventually updated to validate the finite-element model. It is vital to keep uncertainties from the experimental investigations as small as possible and to generate an optimal data base for subsequent model validation by thorough test planning and test execution.

If the structure of the finite-element model with respect to discretization, chosen element types, etc., is correct,[3] the test/analysis deviations can be minimized by changing appropriate parameters based on the experience of the engineer in charge. However, this procedure has limitations for real elastomechanical systems due to the large number of parameters to be considered. Here,techniques for computational model updating (CMU) need to be applied to allow for simultaneous updating of multiple model parameters.[4,5] These techniques minimize the test/analysis deviations and enable validation of the finite-element model.

In practical applications, the finite-element model structure is often incorrect. Computational model updating techniques can still be utilized, but final parameter changes often do not allow for a physical interpretation. They are mathematical substitutes for simply reducing the deviations between test and analysis. Whether individual parameter changes are acceptable or if the finite-element model is to be revised often depends on the intended application of a particular model. A large and physically uninterpretable increase in shell thickness might be irrelevant for vibration analyses but would be unacceptable for stress analyses.

For the project presented in this article, test planning and computational model updating is supported by ICS.sysval, a special MATLAB® software package for model validation, developed by ICS and Professor Michael Link of the University of Kassel.[6] Among other things, this software tool allows for direct update of large-scale MSC.Nastran™ finite-element models from experimental modal data (eigen frequencies and mode shapes). The MSC.Nastran™ solvers are used for eigen value and eigen vector sensitivity analysis under "Solution 200" (optimization).

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Theory Overview

The foundation for updating physical stiffness, mass, and damping parameters is a parameterization of the system matrices according to Equation 1:[4,5]

$$\mathbf{K} = \mathbf{K}_A + \sum \alpha_i \mathbf{K}_i \ , \quad i = 1 \ldots n_\alpha \qquad (1a)$$

$$\mathbf{M} = \mathbf{M}_A + \sum \beta_j \mathbf{M}_j \ , \quad j = 1 \ldots n_\beta \qquad (1b)$$

$$\mathbf{D} = \mathbf{D}_A + \sum \gamma_k \mathbf{D}_k \ , \quad k = 1 \ldots n_\gamma \qquad (1c)$$

where:

$\mathbf{K}_A,\mathbf{M}_A,\mathbf{D}_A$ = initial analytical stiffness, mass and damping matrices, respectively
$\mathbf{K}_i,\mathbf{M}_j,\mathbf{D}_k$ = given substructure matrices defining location and type of model uncertainties
$[\alpha_i,\beta_j,\gamma_k]$ = unknown design parameters.

This parameterization permits local adjustment of uncertain model regions. By utilizing Equation 1 and appropriate residuals, which consider different test/analysis deviations, the following objective function can be derived:

$$J(\mathbf{p}) = \Delta\mathbf{z}^T \ \mathbf{W} \ \Delta\mathbf{z} + \mathbf{p}^T \ \mathbf{W}_p \ \mathbf{p} \ \rightarrow \ \min \qquad (2)$$

where:

$\mathbf{p}$ = $[\alpha_i,\beta_j,\gamma_k]$ = vector of unknown design parameters
$\Delta\mathbf{z}$ = residual vector
$\mathbf{W},\mathbf{W}_p$ = weighting matrices.

The minimization of the objective function yields the desired design parameters $\mathbf{p}$. The second term on the right-hand side of Equation 2 is used for constraining the parameter variation. The weighting matrix must be carefully selected, as for $\mathbf{W}_p >> 0$ no parameter changes will occur.[4]

The residuals $\Delta\mathbf{z} = \mathbf{z}_T - \mathbf{z}(\mathbf{p})$, ($\mathbf{z}_T$ = test data vector, $\mathbf{z}(\mathbf{p})$ = corresponding analytical data vector) are usually nonlinear functions of the design parameters. Thus, the minimization problem is also nonlinear and has to be solved iteratively. One solution is the application of the classical sensitivity approach.[5] Here, the analytical data vector is linearized at point 0 by means of a Taylor series expansion truncated after the linear term. Proceeding this way leads to:

$$\Delta\mathbf{z} = \Delta\mathbf{z}_0 - \mathbf{G}_0 \ \Delta\mathbf{p} \qquad (3)$$

where:

$\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ = design parameter changes
$\Delta\mathbf{z}_0 = \mathbf{z}_T - \mathbf{z}(\mathbf{p}_0)$ = test/analysis deviations at linearization point 0
$\mathbf{G}_0 = \partial\mathbf{z}/\partial\mathbf{p}\,|_{\mathbf{p}=\mathbf{p}_0}$ = sensitivity matrix at linearization point 0
$\mathbf{p}_0$ = design parameters at linearization point 0

As long as the design parameters are not bounded, the minimization problem (Equation 2) yields the linear problem (Equation 4). The latter is to be solved in each iteration step for the current linearization point:

$$(\mathbf{G}_0^T \mathbf{W} \ \mathbf{G}_0 + \mathbf{W}p) \ \mathbf{\Delta p} = \mathbf{G}_0^T \mathbf{W} \mathbf{\Delta z}_0 \qquad (4)$$

For $\mathbf{W}_p = 0$, Equation 4 represents a standard weighted least-squares approach. Of course, any other mathematical minimi-

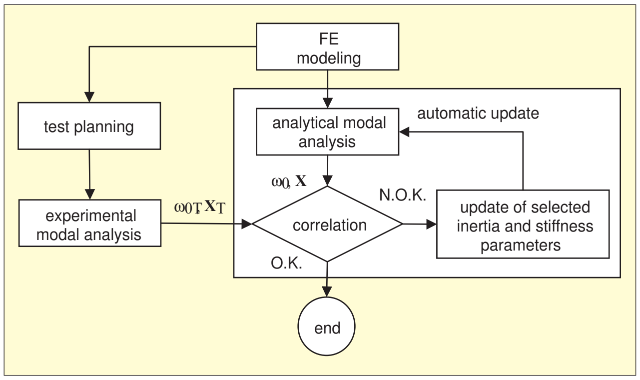

Figure 1. Model validation strategy.

zation technique can be applied for solving Equation 2.

In contrast to the assembly of the analytic stiffness and mass matrix, generating the analytic damping matrix is usually a difficult task. Modal damping parameters can be utilized alternatively for treating system damping in an update process. See the literature for further discussions on this topic.[4,8]

Typically, the eigen value and the eigen vector residuals are employed. Here, the analytical eigen values (squares of the eigen frequencies) and eigen vectors are subtracted from the corresponding experimental results. The residual vector in this case becomes:

$$\Delta \mathbf{z}_0 = \begin{bmatrix} \lambda_{Ti} - \lambda_i \\ \cdots\cdots \\ \mathbf{x}_{Ti} - \mathbf{x}_i \end{bmatrix}_0, \; i = 1,\ldots,n \tag{5}$$

where:
λ_{Ti}, λ_i = test/analysis vectors of eigen values
$\mathbf{x}_{Ti}$, $\mathbf{x}_i$ = test/analysis mode shape vectors

The correlation between analytical data and test data is accomplished by means of the MAC value of the eigen vectors:

$$\text{MAC:} = \frac{\left(\mathbf{x}_T^T\,\mathbf{x}\right)^2}{\left(\mathbf{x}_T^T\,\mathbf{x}_T\right)\left(\mathbf{x}^T\,\mathbf{x}\right)} \tag{6}$$

which states the linear dependency of two vectors $\mathbf{x}_T$, $\mathbf{x}$. A MAC value of 1 denotes that two vectors are collinear; a MAC value of 0 indicates that two vectors are orthogonal.

The sensitivity matrix for the residual vector introduced in Equation 5 is given by Equation 7. The calculation of the partial derivatives can be found in References 4 and 5.

$$\mathbf{G}_0 = \begin{bmatrix} \dfrac{\partial \lambda_i}{\partial \mathbf{p}} \\ \cdots\cdots \\ \dfrac{\partial \mathbf{x}_i}{\partial \mathbf{p}} \end{bmatrix}_0, \; i = 1,\ldots,n \tag{7}$$

If real eigen values and eigen vectors are employed, the adjustment of damping parameters is not possible. The corresponding sensitivities equal zero, since the real eigen values and eigen vectors depend solely on the stiffness and mass parameters of the system.

Model Validation Strategy

The model validation is accomplished through computational model updating of physical parameters (stiffness and inertia parameters) of the finite-element model by minimizing the deviations between the identified and the analytical eigen values and mode shapes. All deviations between test and analysis are presumed to be exclusively based on uncertainties of the finite-element model. An integral part of the validation strategy is keeping the inevitable uncertainties from the test side as small as possible and to create a reliable data base for the following validation tasks by thorough test planning and test execution. Figure 1 shows the principle proceeding.

The test planning utilizes the given finite-element model, which enables not only the test design but also considerably simplifies the later correlation with the analytical results (finite-element model and test model 'match'). Test planning should cover the following aspects:

- Selecting relevant target modes
- Selecting measurement degrees of freedom with respect to:
  - Essential test information
  - Sufficient spatial resolution of target modes
  - Coincidence of measurement and finite-element nodes
  - Accessibility of measurement nodes
  - Redundancy of the measurement degrees of freedom
  - Robustness of the test model
- Selecting exciter positions (if possible, simultaneous excitation of all target modes)
- Sufficient frequency resolution (for following identification methods)

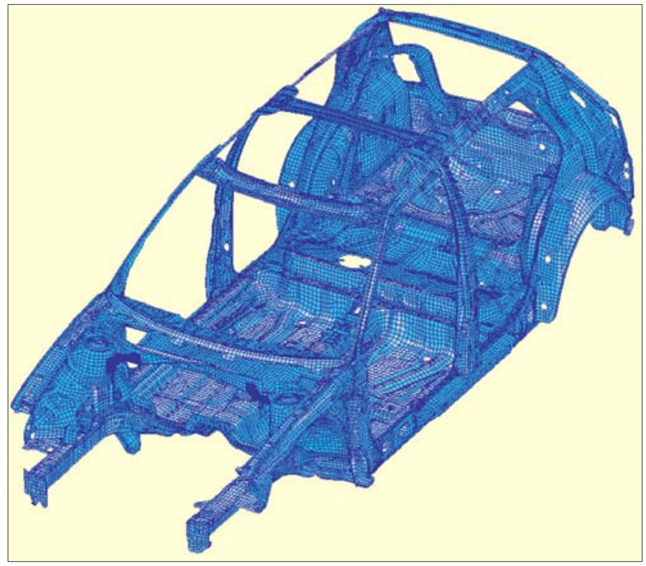

Figure 2. Finite-element model of the body in white.

Test planning and computational model updating was conducted using ICS.sysval. Necessary parameter changes are directly applied to the so called "bulk data" section of the MSC.Nastran input file. Typical parameters are shell thicknesses, beam section properties, Young's moduli, and densities. However, virtually all physical parameters, which can be considered in an eigen value and eigen vector sensitivity analysis by MSC.Nastran, can be used for model updating.

After successfully updating the stiffness and inertia properties (physical parameters), modal damping parameters (modal parameters) can be adjusted subsequently by minimizing the deviations in the resonance regions between measured and simulated frequency response functions.[8]

A common difficulty in computational model updating is the proper choice of appropriate model parameters. Automated methods may be applied[7] in addition to selection based on engineering experience, but currently do not deliver reliable predictions. Another possibility is to select parameters based on a sensitivity analysis. Here the sensitivity matrix according to Equation 7 is computed for a set of suitable parameters. In a subsequent investigation, those parameters that have a significant influence on results are identified. However, the sensitivity analysis does not supply any information on the physical relevance of a particular parameter but merely detects its potential to change analysis results.

Example – Car Body in White

The validation strategy that has been discussed will be demonstrated by means of the car body in white depicted in Figure 2. This is currently investigated under the auspices of the "work group 6.1.19 structure optimization and acoustics" of the German car industry.

The finite-element model consists of about:

- 142,000 nodes
- 130,000 elements
- 3,500 spot weld elements

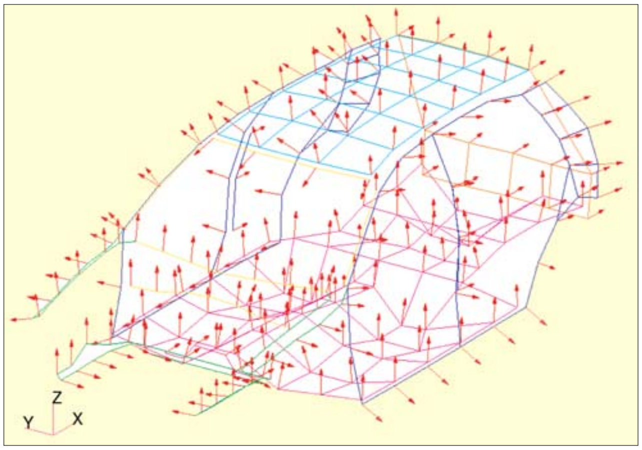

Figure 3. Test model of the body in white with measurement degrees of freedom.

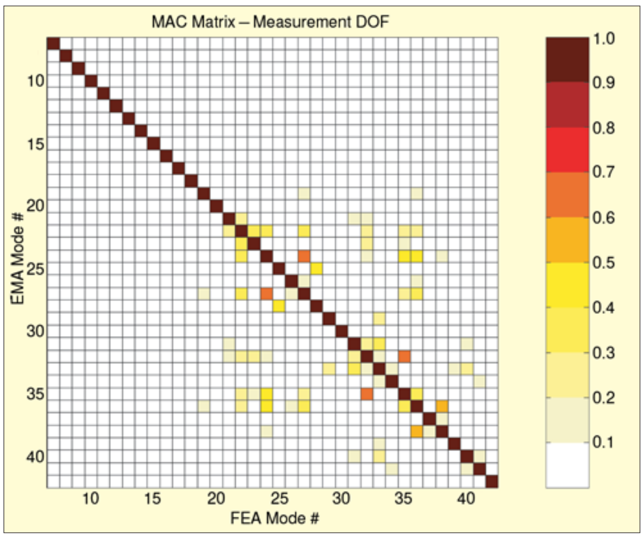

Figure 4. Auto MAC matrix of analytical mode shapes at measurement degrees of freedom.

The spot welds are modeled by means of MSC.Nastran 'CWELD' elements. These elements are currently used by some automotive companies. The goal of the model validation is to correctly predict the structural dynamics of the body in white up to a frequency of about 100 Hz.

As already noted, test planning provides an important basis for all subsequent investigations. It is especially important that all required information be collected during the tests, which is necessary for the following validation steps.

For the analyzed body in white, the following planning steps are performed:

Select Target Mode Shapes. First, the boundary conditions, frequency range and the relevant mode shapes are determined. The body in white will be investigated in a free/free configuration that can be relatively easily realized in the test setup via bungee cords or air springs. Since the goal of the model validation is to predict the structural dynamics of the body in white up to a frequency of about 100 Hz, all analytic mode shapes in this frequency range are to be considered in test planning.

Select Measurement Degrees of Freedom. Since a reliable orientation of the accelerometers on the body in white is difficult due to the curvature of the car body, only measurement degrees of freedom normal to the sheet surfaces are considered in the test design. This provides the possibility to perform a roving-hammer structural excitation. Of course, selection of the measurement degrees of freedom should also ensure the unique classification of individual mode shapes.

All measurement considerations noted under Model Validation Strategy should be taken into account. These include: essential test information; sufficient spatial resolution of the target modes (preferably diagonal shape of the auto MAC matrix); the coincidence of measurement and finite-element nodes (vital for correlation); redundancy of the measurement degrees of freedom (for uncertainties in some measurement degrees of freedom); and the robustness of the test model (due to uncertainties in the finite-element model used for test planning).[9]

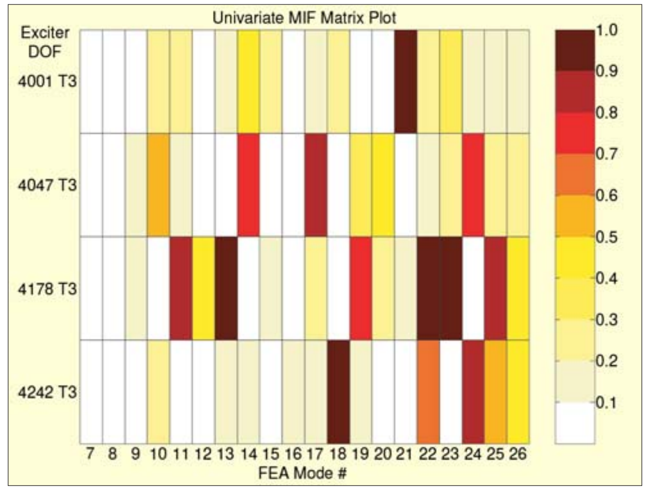

Figure 5. Mode indicator values of the body in white.

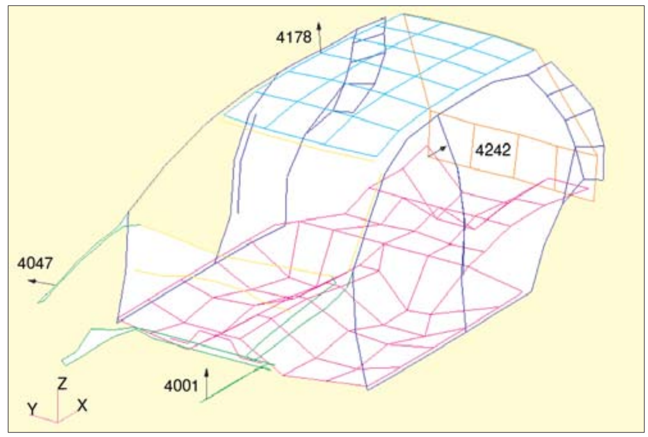

Figure 6. Test model of the body in white with exciter degrees of freedom.

The final test model resulting from test planning is shown in Figure 3, and the corresponding auto MAC matrix of the analytical mode shapes at the measurement degrees of freedom is depicted in Figure 4. The overall spatial resolution is sufficient. Only four mode shapes (No. 24 and 27 as well as No. 32 and 35) exhibit off diagonal coupling larger than 60%. These are relatively local mode shapes that require a considerably higher resolution. But to keep the measurement efforts manageable, a higher resolution of the measurement mesh is not applied.

Select Exciter Degrees of Freedom. Identification of the exciter degrees of freedom is done in two steps. First, a preliminary subset of appropriate exciter degrees of freedom is calculated using a special automatic method.[9] Next, the final exciter degrees of freedom are defined by means of "mode indicator values." For a given exciter position, a mode indicator value of 0 at a certain eigen frequency states that the corresponding mode shape can be fully excited (satisfaction of the so called phase-resonance criteria). In contrast, a value of 1 means that the corresponding mode shape cannot be excited at the chosen exciter position.

The strategy is to select several exciter positions so that each mode shape can be sufficiently excited at least at one exciter position. The accessibility of the exciter position for a modal exciter test is also taken into account. Figure 5 shows the mode indicator values for the first 20 target modes at the four selected references according to Figure 6. Obviously, each mode shape can be excited at least by one of the defined exciter positions.

Select Frequency Resolution. The applied techniques to identify the experimental modal data require a sufficient frequency resolution, especially in the lower frequency range. Therefore, a minimal frequency resolution based on the first elastic eigen frequency and the expected modal damping is determined. Since the real damping behavior is usually not known in advance, these results are to be verified before the final test execution.

Test Setup and Preliminary Investigations

For approximating the free/free boundary conditions of the finite-element model, the car body is mounted on four air springs as shown in Figure 7. If necessary, the suspension can be accommodated in the finite-element model by means of springs. The spring properties can be estimated from the experimental rigid-body modes.

Different preliminary studies with impact-hammer and modal exciters are performed to investigate the real behavior of the body in white. Specifically, the linearity is reviewed by means of modal exciter tests with different levels of excitation and by coherence and reciprocity examinations. Altogether, the investigated car body shows a sufficiently linear behavior.

Since test results with the impact hammer exhibit a very good consistency with the results of the modal exciter tests, the final test was executed as a roving-hammer test with fixed-reference accelerometers. Proceeding this way also avoids mass loading caused by moving accelerometer masses. Mass loading effects easily arise at large surface areas and can generate frequency shifts of the resonance peaks. In regions with large modal density, these effects produce unreliable test results.

Since the measurement degrees of freedom are manually mapped onto the car body, an uncertainty regarding the real position of the measurement points exists. For quantifying possible deviations, the measurement points are digitized using supersonic triangulation. Acquired real measurement point positions agreed well with the finite-element model.

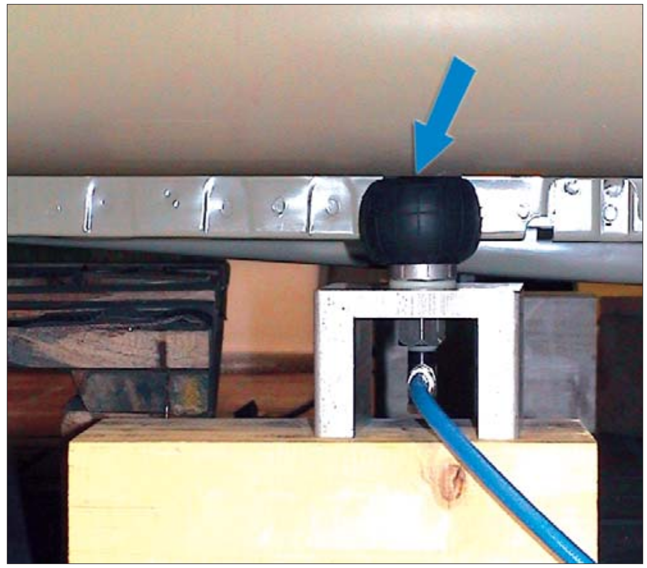

Figure 7. Mounting of the body in white using air springs.

Table 1. Initial correlation.

No.	EMA[1]	FEA	Δf, %	MAC, %	No.	EMA[1]	FEA	Δf	MAC, %
1	1	7	–7.04	98.24	8	12	18	–3.07	66.41
2	2	8	0.19	87.42	9	13	19	–0.02	72.49
3	3	9	–6.17	63.55	10	14	20	–1.85	86.89
4	5	13	0.83	73.20	11	15	22	1.39	78.18
5	6	11	–11.72	61.13	12	17	24	–0.07	6788
6	8	14	–3.39	79.49	13	18	26	–1.07	96.49
7	10	15	–6.55	66.51	14	19	27	–1.49	76.60

1) without rigid body modes

Table 3. Correlation after computational model updating.

No.	EMA[1]	FEA	Δf, %	MAC, %	No.	EMA[1]	FEA	Δf	MAC, %
1	1	7	–4.35	98.69	10	10	16	0.21	82.22
2	2	8	0.75	97.54	11	11	17	0.09	69.57
3	3	9	–1.49	95.49	12	12	18	–0.90	85.80
4	4	10	1.10	94.38	13	13	19	0.15	75.85
5	5	11	–0.50	93.86	14	14	20	–1.35	92.13
6	6	12	–1.83	65.05	15	15	22	1.49	63.63
7	7	13	–2.67	90.49	16	17	24	0.85	85.47
8	8	14	–1.26	95.39	17	18	25	–0.71	98.09
9	9	15	–1.55	80.52					

1) without rigid body modes

Table 2. Correlation after remodeling.

No.	EMA[1]	FEA	Δf, %	MAC, %	No.	EMA[1]	FEA	Δf	MAC, %
1	1	7	–4.51	98.62	10	11	17	–0.14	68.37
2	2	8	3.01	97.51	11	12	18	–0.57	88.51
3	3	9	–1.49	96.22	12	13	19	0.19	72.25
4	4	10	–0.16	90.12	13	14	20	–1.31	89.43
5	5	11	1.26	90.92	14	15	21	0.83	65.52
6	6	12	0.65	68.55	15	16	22	0.18	66.29
7	7	13	–0.71	59.68	16	17	24	0.48	86.01
8	8	15	0.22	94.88	17	18	25	–0.98	98.19
9	10	14	–6.35	52.61					

1) without rigid body modes

Experimental Modal Analysis

The analysis of the acquired test data is conducted by two completely different identification algorithms, one working in the time domain (polyreference) and one working in the frequency domain (direct parameter estimation). The identified sets of modal data are subsequently correlated, allowing for an assessment of the individual quality of the measured eigen frequencies, mode shapes, etc. This is of particular importance for subsequent model validation, since only experimental results with sufficient quality should be used.

Initial Correlation

For evaluating model quality, frequency deviations between test and analysis as well as the MAC values of the corresponding mode shapes according to Equation 6 are employed. Besides the correlation table, the MAC matrix will also be posted for assessing the quality of the correlation.

The initial correlation for the investigated body in white is given by Table 1 and Figure 8 and consider only mode shapes with a frequency deviation less than 30% and a MAC value larger than 50%. Usually, a tolerance limit of 70% is acceptable to ensure a consistent correlation of the mode shapes used for updating. In this case, however, a decrease is necessary for considering all relevant mode shapes in the initial correlation and especially in subsequent computational model updating.

Even by reducing the tolerance limit to 50%, not all mode shapes can be correlated ('gaps' on the main diagonal of the MAC matrix). Moreover, some couples show frequency deviations larger then 10%.

Sensitivity Analysis

For reducing the multitude of potential parameters to a subset of parameters, which have a significant influence on the model behavior, a sensitivity analysis is performed by computing the eigen value and eigen vector sensitivities for all potential parameters. Subsequently, the most promising parameters for each mode shape are determined using the sensitivity module of ICS.sysval. These parameters constitute the basis for the following computational model updating.

Remodeling and Computational Model Updating

In the investigated example, dividing the updating task into two individual steps proves very efficient. In both cases Young's moduli are used for updating (for preserving the total mass of the car body).

In the first step, areas with large, physically not interpret-

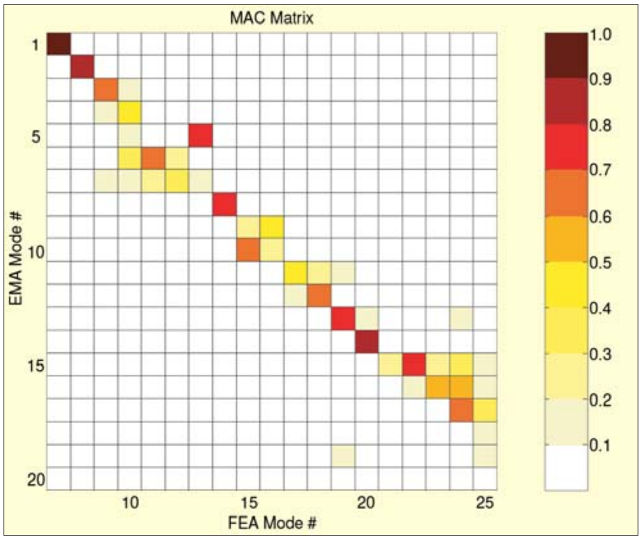

Figure 8. MAC matrix, initial correlation.

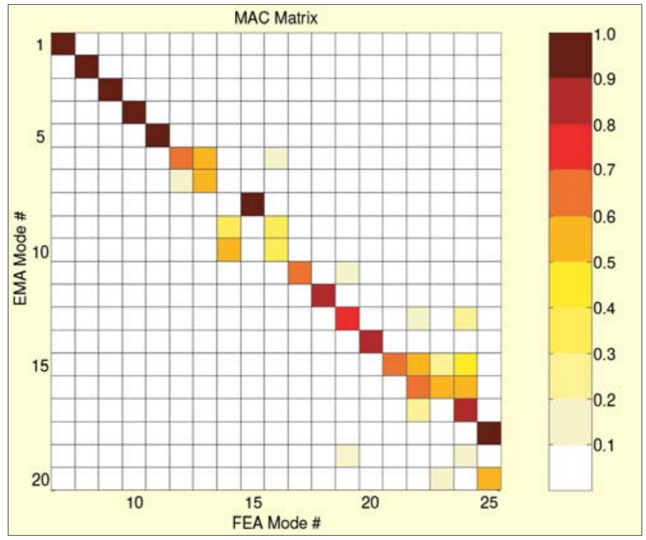

Figure 9. MAC matrix after remodeling.

able, parameter changes are identified by applying computational model updating methods. These areas are then remodeled using the CAD data of the geometry, which already leads to an improvement of model quality. The results after remodeling are collected in Table 2 and Figure 9, respectively.

The remodeling already yields a significant increase of model quality. On the one hand, the MAC values can be increased to more than 90%, especially in the lower frequency range. On the other hand, frequency deviations are noticeably reduced. However, the correlation in the range of the sixth, seventh, ninth, and tenth measured mode is still not fully satisfactory.

For further increasing the model quality, multiple updating runs are performed in a second step using the remodeled finite-element model. The final results are summarized in Table 3 and Figure 10.

Comparing these results with Table 2 and Figure 9 shows an additional increase in the model quality achieved by subsequent computational model updating. Both the mentioned gaps on the main diagonal of the MAC matrix can be filled, and the frequency deviations can be further reduced. Besides the first eigen frequency, all frequency deviations are now below 3%. However, note that the total deviation for the first eigen frequency is less than 1 Hz.

Summary

We examined validation of the finite-element model of a body in white using a special software package for computational model updating. It enables direct updating of large-scale MSC.Nastran finite-element models.

For keeping the inevitable uncertainties from the test side as small as possible, thorough test planning is essential. By using the initial finite-element model, both measurement degrees of freedom and exciter positions are virtually defined and then transferred onto the body in white. For avoiding mass loading effects, data acquisition is performed using a roving-hammer excitation with fixed accelerometer positions.

Besides the direct increase of model quality, the computational model updating technique provides the additional opportunity to identify regions where remodeling already yields an improvement of model quality. Test/analysis correlation before and after computational model updating as well as remodeling exhibits a noticeable reduction of frequency deviations along with an increase of MAC values over a broad frequency range.

These investigations are currently expanded towards attachment parts like doors and subassemblies under the auspices of the "work group 6.1.19 structure optimization and acoustics" of the German car industry. A first goal is to sufficiently represent the structural dynamics as a thorough basis for subsequent investigations on acoustic phenomena.

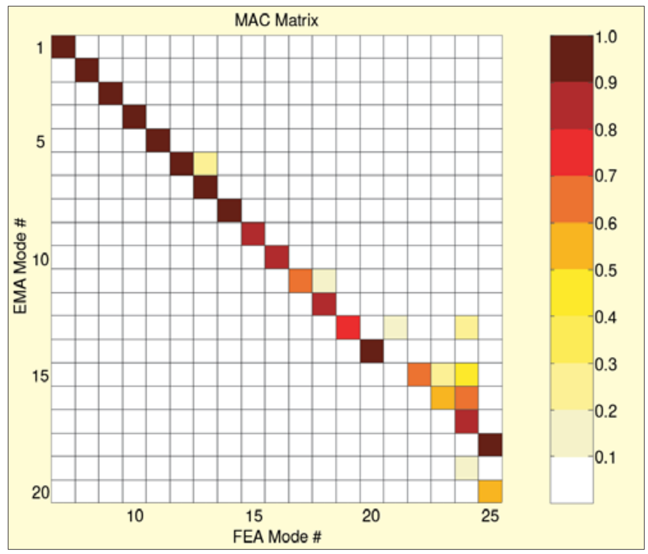

Figure 10. MAC matrix after computational model updating.

Acknowledgments

This paper is dedicated to Mr. Jörn Frappier, who died in a tragic accident at the beginning of 2004.

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