Manufacturing Tolerances and Axle System NVH Performance

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A study of axle system noise, vibration, and harshness (NVH) performance using design for six sigma (DFSS) methods is presented with a focus on system robustness to typical product variations (manufacturing-based tolerances). Instead of using finite-element analysis (FEA) as the simulation tool, a lumped-parameter system dynamics model developed in MATLAB/Simulink is used in the study. This provides an efficient way to conduct large analytical design of experiment (DOE) and stochastic studies. The model's capability to predict both nominal and variance performance is validated with vehicle test data using statistical hypothesis test methods. Major driveline system variables that contribute to axle gear noise are identified and their variation distributions in production are obtained through sampling techniques. Through analytical DOE and analysis of variance (ANOVA) analyses, the critical design parameters that control system NVH variations with respect to product variations and variations due to operating conditions are covered. Design criteria of axle systems with respect to robust NVH performance are also discussed.

Axle gear whine is a major NVH concern in vehicle driveline systems. The noise source is typically the hypoid-gear-mesh, first-harmonic transmission error, which further transforms into dynamic gear mesh force under operation. The dynamic characteristics of the mesh force are controlled by the gear train torsional dynamics. The driveline system translational dynamics and the impedance characteristics at the axle interface with the vehicle are also important in defining the final vehicle interior gear-mesh noise level.

Considerable progress has been achieved in recent years in understanding the physics of axle system gear noise. Using computer-aided engineering (CAE) tools such as FEA to analyze system dynamics in regard to gear noise have been found to be extremely successful.[1-5] In most applications, linear system assumption is satisfied, and a modeling approach called the building-block method has proven to be efficient and accurate. In dealing with gear mesh geometry, a simplified linear gear-mesh model developed by Steyer *et al.*[4,5] is widely used. Figure 1 shows a typical rear-beam axle driveline FEA model and Figure 2 gives its accuracy with respect to vehicle test data.

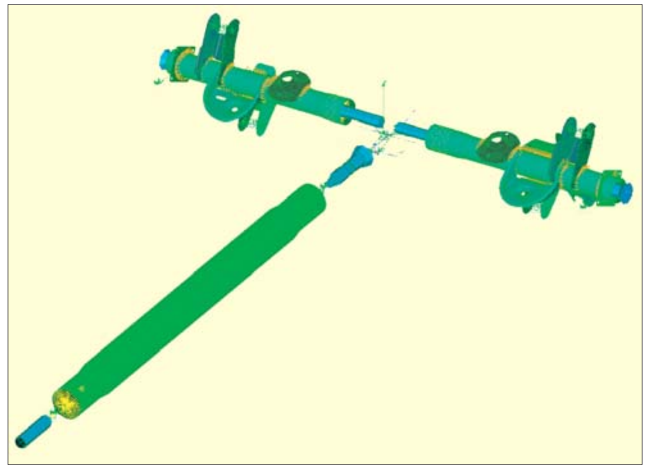


Figure 1. FEA model of rear-beam axle driveline.

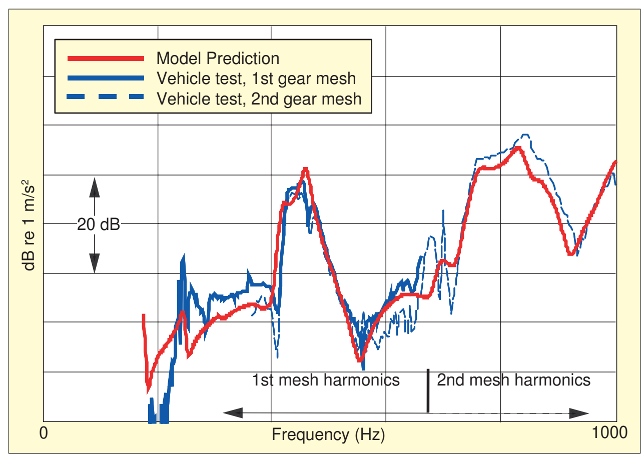


Figure 2. Example of FEA model correlation to test data.

Despite the success of applying simulation tools in design/tuning of system dynamics to achieve a quiet system for a particular vehicle platform, it is even more important that the product has robustness in NVH performance in the following aspects:

- The system meets the NVH requirements in the presence of 'variability' due to both manufacturing process capability and environmental/operating conditions.
- The axle subsystem exhibits minimized system sensitivity to different vehicle platforms.

In general, the robustness of a system can be established through two aspects of the design: *parametric design* and *tolerance design*.[7] The parametric design deals with nominal system design through design parameter settings to achieve reduced system sensitivity to variations. Current CAE tools can be effectively used to achieve a viable nominal design. A good example of parametric design are the results presented by Sun *et al.*,[1] where systems are desensitized to variations through enforcing NVH design specifications that are rolled down from analytical studies. Monte Carlo-type optimization methods for parametric design are being developed.

The tolerance design, on the other hand, deals with manufacturing process capability. This is to identify critical tolerances that contribute to system NVH performance variability and tighten specifications in an effective and economic fashion to minimize system variation. A tolerance optimization is most effectively done after a parameter optimization.

Figure 3 illustrates the concept of both parametric and tolerance designs. Literature on parametric design studies and successful implementation can be found in recent years. However, there are few studies on tolerance designs regarding axle system NVH. There are several reasons behind this:

- It requires an integrated analytical approach that combines statistical and analytical DOE (design of experiment) techniques with simulation tools. However, traditional simulation tools such as FEA are still time consuming for this kind of task. For instance, if one would conduct a three-level full factorial DOE with six parameters, 729 runs are needed.

Based on Paper #2005-01-2309, "Variation Reduction of Axle System NVH," © 2005 SAE International, presented at the Noise & Vibration Conference, May 16-19, Traverse City, MI 2005.

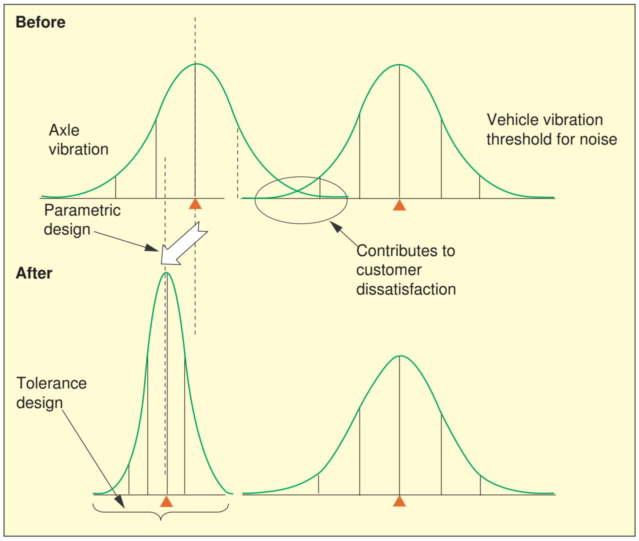


Figure 3. Axle subsystem robust design.

- It requires a tremendous effort to obtain the statistical data for the component-level parameters as inputs to the analytical models. In many cases, special tests have to be designed and conducted on large sample sizes to obtain meaningful results.

The work here focuses on the tolerance design. To facilitate the analytical study, a stand-alone, lumped-parameter axle system dynamic model built in MATLAB/Simulink is developed and used instead of traditional FEA models. This model, as the system solver, enables large analytical DOEs and statistical analyses. HyperStudy from Altair Engineering, Inc. is used as the analytical tool. Validation of the approach is presented and shows satisfactory accuracies in predicting both system nominal and variance performance.

Examples of probability distribution functions of modal parameters are also presented. In most of the cases, they are obtained through testing on large populations of production samples. Under certain circumstances, special analytical tools have to be used to obtain reasonable data on particular parameters; for instance, the hypoid-gear-mesh, line-of-action variation.

The validated models are then used to conduct DOE studies and ANOVA analyses. Factors critical to quality are identified and discussed.

Scope of Study

This study focuses on the axle carrier assembly (or third member) regarding axle system gear noise variation reduction. The axle carrier assembly includes the following major components: axle carrier, differential assembly (with hypoid ring gear bolted on the flange), pinion shaft and gear, pinion flange, differential bearings and pinion bearings (see Figure 4).

For axle first-gear-mesh harmonic noise, the dynamics of differential case assembly and axle carrier typically have little effect and can be considered as lumped masses since their resonant frequencies are well above the gear-mesh frequency range (typically 200-700Hz). Their mass and mass moment of inertia will affect the system gear-mesh force or vibration responses but are secondary compared with other variables. The axle carrier stiffness at bearing bores is important and can be included into final effective bearing stiffness in supporting the gearing system. The pinion shaft is relatively more compliant in both torsional and lateral directions; typically it is modeled with beam elements in FEA models.

With the knowledge base of the axle system gear noise mechanism, the following lumped parameters will be studied in DOE and ANOVA analysis:

- Drive pinion tail bearing radial stiffness
- Drive pinion head bearing radial stiffness
- Differential bearing radial and axial stiffness

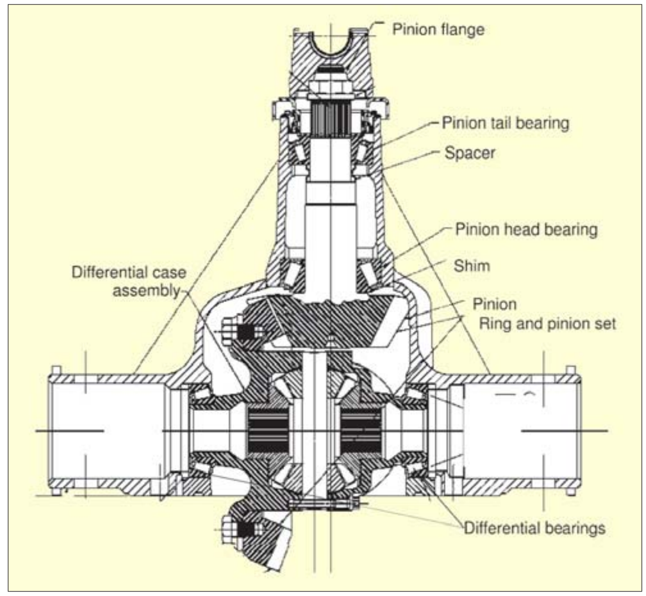


Figure 4. Typical rear axle carrier assembly.

- Gear mesh line of action (LOA)
- Gear pitch point location
- Effective prop torsional stiffness
- Effective prop bending stiffness
- Gear mesh transmission error (MTE)

Notice from the above that the propeller shaft torsional and bending dynamic stiffnesses are included in the study. This is because the axle gear mesh dynamic force and system vibration responses are strongly controlled by the propeller shaft, as previously discussed.[1,2] Including the propeller shaft dynamics in the analysis is essential in studying the interaction between axle carrier assembly variations and the system gear-mesh dynamics. Modal models of the propeller shaft for bending and torsion are used in the lumped-parameter model, which is described further in the following section.

In a linear system as in this study, MTE is actually a linear scaling factor. However, it is important to include it in the study to evaluate its contributions to system NVH variation and comparison with other parameters. The evaluated system response in the study is the axle pinion nose vertical vibration, since it is the most appropriate indicator for axle gear mesh-induced pitch and bounce motion.

Analytical Approach

HyperStudy/MATLAB Approach. To study the system variations, an analytical tool had to be used that is capable of doing variation simulation (stochastic study), analytical DOE, and ANOVA analysis. Optimization capability is also a necessity. Instead of developing our own analytical toolbox in performing this task, this paper uses HyperStudy provided by Altair, Inc., which is part of the HyperWorks package. Using this commercially available software enabled us to focus our effort and time on problem solving instead of tool development.

Two options were evaluated regarding axle system gear-mesh dynamics modeling: FEA and stand-alone, lumped-parameter models. The later was chosen because, although FEA models are convenient and readily available, conducting analytical DOE with these models would be time consuming. Stochastic studies with a large number of runs (say, >2000) would be impractical with FEA models.

Instead, the authors decided to develop a new lumped parameter model using MATLAB/Simulink from The MathWorks, Inc. The axle carrier assembly model (together with propeller shaft) is built in Simulink, while the model parameters and intermediate variable calculations are given in the MATLAB program developed by the authors.

The concept of the analytical approach to this study is shown by the flow chart shown in Figure 5. HyperStudy is customized to interface with the MATLAB program.

MATLAB/Simulink Program. The model is illustrated in Figures 6 and 7, where the system is divided into subsystems and each subsystem is represented by lumped parameters. That is, lumped masses are connected by springs and dampers with space transformations of the motion. Modal models are used for the propeller shaft bending and torsional dynamics, and the modal parameters are determined in a separate MATLAB program.

The MATLAB program was developed to perform the following tasks: read in the system configuration data; calculate and set the parameter values used in the Simulink model; and obtain the outputs from Simulink to conduct further mathematical processing to obtain the system gear mesh force and vibration responses. As illustrated in Figure 5, HyperStudy is interfaced with the MATLAB program so that the model parameters can be easily replaced to conduct a large number of runs.

Determination of Model Parameters

Determination of PDFs. To perform the variation simulations of the parameters affecting NVH variation, the probability distribution functions (PDFs) must be estimated for each of the main parameters to be studied. The PDFs can be estimated by testing actual parts or purely analytically. The preference for this study has been to use the test method as much as possible.

PDFs from Testing. The first step in establishing the test-derived PDFs is to obtain a representative sample of production parts. Care must be taken to randomly select the samples over a long enough time and large enough group of production machines to adequately represent typical manufacturing variations. The size of the sample helps determine the confidence intervals (error estimates) of the estimated 'true' population's PDF parameters (μ,σ, etc.), with a larger sample size yielding a more precise estimate. There are obvious trade-offs between sample size and its associated costs (parts, testing, etc.) and the precision of the estimate.

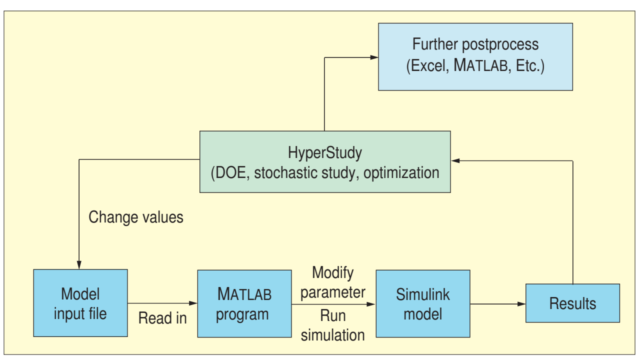


Figure 5. Flow chart of analytical model.

The next step is to acquire the test data for the parameters of interest. For example, the bearing stiffnesses may be derived from appropriate frequency response function (FRF) measurements on the axle assembly. From the measurement of multiple production assemblies the bearing stiffness PDFs can be determined. Prop bending and torsional stiffnesses can also be derived from FRF measurements. PDFs can be estimated for all key NVH factors as (see Figure 8).

In the case of characterizing the PDF for gearset MTE, it becomes a little more complicated. A sample of gearsets can be measured on a single flank tester at various buildable gearset pattern positions. These data combined with the percentages of axles built at each pattern position can yield an overall PDF of the "axle assembly" MTE delivered to the customer (see Figure 9). A Weibull PDF typically fits this data best with the MTE values highly skewed to the low end.

PDFs from Analysis. Determining PDFs that are related to gear-mesh geometry parameters is a challenging task. The parameters include gear mesh point location, which can be expressed as (x_m, y_m, z_m) in a Cartesian coordinate system; and the gear mesh line of action (LOA), which can be expressed as (n_x, n_y, n_z). Notice that there are only two independent parameters in the LOA vector. In defining the variation of the LOA vector, a variation cone surface is used, as illustrated in Figure 10, where the apex angle α and cone angle γ are defined.

If the vector $\mathbf{r_a}$ is defined as being the rotation of nominal LOA vector $\mathbf{r_n} = (n_x, n_y, n_z)$ around the global x-axis by apex angle α, then any vector $\mathbf{r_b}$ on the cone surface defined by angle a can be expressed as:[8]

$$\mathbf{r_b} = \mathbf{L_a r_a} = [\mathbf{I} + (1-\cos\gamma)\mathbf{C}^2 + \sin\gamma\mathbf{C}]\mathbf{r_a} \tag{1}$$

where $\mathbf{I}$ is the 3 × 3 identity matrix, and

$$C = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_z \\ -n_y & n_x & 0 \end{pmatrix} \tag{2}$$

Similarly, the relationship between $\mathbf{r_b}$ and $\mathbf{r_n}$ can be expressed as:

$$\mathbf{r_b} = (\mathbf{L_a L_n})\mathbf{r_n} \tag{3}$$

where $\mathbf{L_n}$ is the matrix defined similarly as $\mathbf{L_a}$ for the rotation from $\mathbf{r_n}$ to $\mathbf{r_a}$.

Physically measuring the variations of the gear-mesh point location and LOA vector is currently not available in practice. A test rig to conduct this kind of measurement is underway in the author's organization.

Therefore, theoretical studies of the gear-mesh variation through special gear analysis tools were conducted, and PDFs for mesh point (x_m, y_m, z_m) and LOA variables (α,γ) were obtained through this study. The details of gear mesh geometry analytical studies are not be explicitly described here.

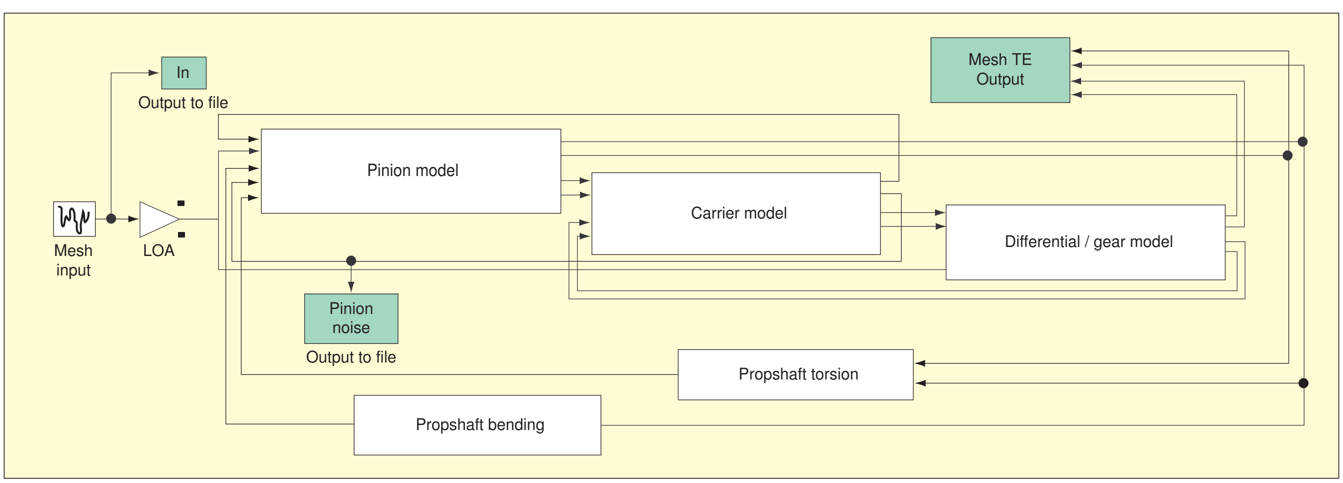


Figure 6. Axle carrier assembly system gear mesh model in MATLAB/Simulink.

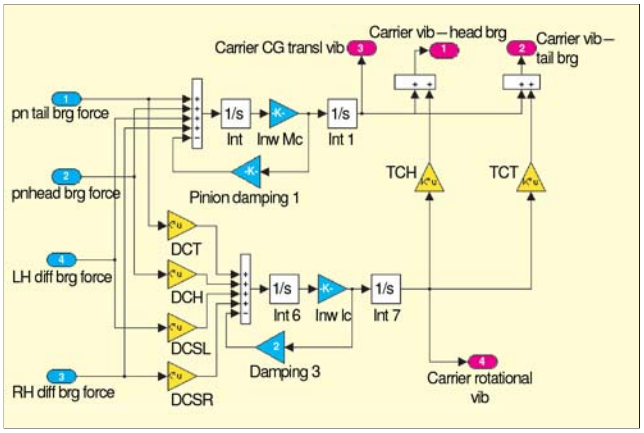


Figure 7. The carrier model in MATLAB /Simulink.

Model Validation

To obtain maximum confidence in model results, they should be validated in regard to their ability to predict both system mean and the variation of NVH performance. The model predictions were compared to experimental vehicle test results and hypothetical tests were conducted to evaluate differences in the mean and deviation.

Validation of Model's Nominal Prediction. The model's prediction of nominal axle vibration is compared to the experimental average of 20 axles randomly selected from production. Hypothesis tests for 'means' are conducted at two key frequencies; prop second bending resonance and the gear-mesh resonance at an $\alpha = 0.05$ level of significance. As shown in Figure 11, there is no statistically significant difference between the predicted model and experimental means for the drive-side results. Similar results were obtained for the coast side, and consequently the model was validated for predicting nominal vibration performance.

Validation of Model's Variation Prediction. The model's prediction of variation in axle vibration was compared to the experimental variation of 20 axles randomly selected from production. The 20 axles had key NVH parameters measured or estimated so that the PDFs could be generated. The estimated PDFs were inputs to the stochastic analysis module of HyperStudy so that axle NVH performance variations could be generated according to the PDFs. The process is illustrated in Figure 12. Figure 13 shows a comparison of the actual experimental variation compared to the analytically predicted variation. The data were reasonably Gaussian, so hypothesis tests for 'sigma' were conducted at the same two key frequencies referenced above prop second bending resonance and the gear mesh resonance at an $\alpha = 0.05$ level of significance. As shown in Figure 13 there is no statistically significant difference between the predicted model and experimental sigmas. Similar results were obtained for the coast side, and consequently the model was considered validated for predicting the variation in vibration performance.

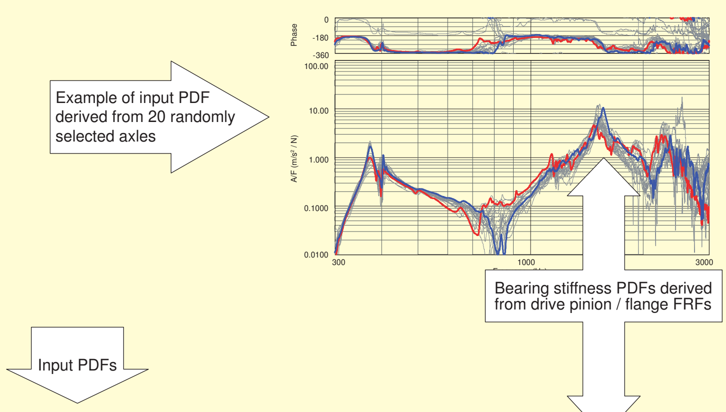
Analytical DOE

Time and other constraints necessitated a strategy of several exploratory screening DOEs to precede the main DOE. The purpose of screening is to allow subsequent detailed analysis to focus on the most important factors and interactions.

Screening DOE. Figure 14 shows the main effects of a screening 12-factor, two-level DOE for the drive side. The two levels chosen represent the anticipated maximum and minimum or "tolerance range" of each of the factors. It is clear from Figure 14 that the drive-side main effects are dominated by one factor (Factor I). Coast-side results were similar.

Figure 15 shows the typical low level of interactions that were evident in the analysis. This is a desirable situation in a driveline assembly, since it means that the system is predictable and robust against normal manufacturing and product variations.

Three-Level, Full-Factorial DOE. Based on the screening DOE results, several significant factors were selected for further detailed study. A three-level DOE was conducted on the six most significant factors and the main effects are shown in Figure 16. Only minor nonlinear effects are noted. An evaluation of interactions in the three-level DOE again shows that they are of negligible magnitude. Typical three-level interactions are shown in Figure 17.



Factors		Distribution	Mean	Standard Dev	Variance
Pinion tail bearing radial stiffness		Normal	5[illegible]3 (N/m)	[illegible].65E+07	[illegible]3E+[illegible]
Pinion head bearing radial stiffness		Normal	[illegible]8 (N/m)	[illegible]0E+07	[illegible]03E+[illegible]
Diff bearing stiffness	radial	Normal	4[illegible] (N/m)	[illegible]7E+07	[illegible]3E+1[illegible]
	axial	Normal	[illegible]e7 (N/m)	[illegible]13E+06	[illegible]54[illegible]2
Mesh LOA angle change		Normal	0	[illegible]67	[illegible]78E+0[illegible]
Mesh LOA angle cone		Uniform	0-360 (degree)		
Gearset TE		Weibull	Shape = [illegible]45 Scale = [illegible]35		
	X	Normal	[illegible].289 (mm)	[illegible]3	[illegible]E+0[illegible]
Pitch Point	Y	Normal	-[illegible]2 (mm)	[illegible]0	[illegible]0E-0[illegible]
	Z	Normal	[illegible].53 (mm)	[illegible]57	[illegible]6E+0[illegible]
Test Measurement Variation		Normal	1	[illegible].05	[illegible]50[illegible]1

Figure 8. Example of input PDF derived from FRFs of 20 randomly selected axles.

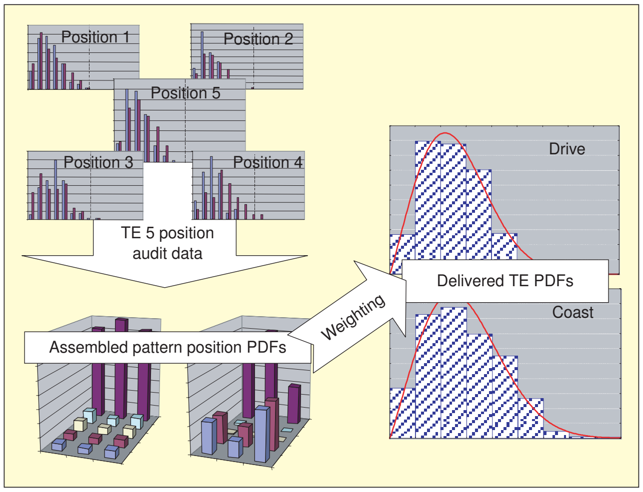


Figure 9. Schematic diagram for obtaining PDFs of MTE.

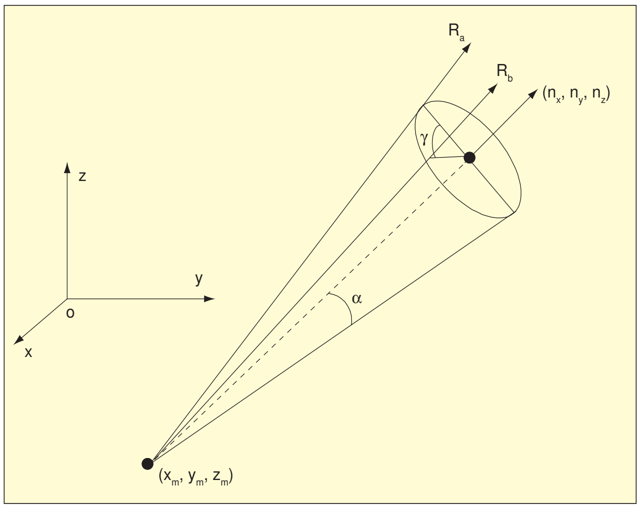


Figure 10. Vector rotation on a cone surface.

It is shown from Figure 16 that Factors B and K have the most significant contributions to the gear-mesh vibration variation among the six factors. To better rank the sources of system NVH variation, an ANOVA analysis was conducted.

ANOVA. An analysis of variance (ANOVA) was performed to establish a rank of the sources of axle NVH variation. These results are shown in Figure 18. It is shown that Factor I is responsible for the vast majority of normal product NVH variation (93% and 74% for drive and coast, respectively). The results also demonstrated that the rank of the factors and their contribution percentages are different under different vehicle/axle load conditions. This is due to the change of axle gear mesh kinematics and its interaction with system dynamics. Therefore both vehicle drive and coast conditions should be studied to capture all the significant factors.

These results clearly establish what factors should be the primary focus of variation reduction efforts. Potential changes in factor tolerances can be confirmed or evaluated in a variation simulation by modifying a factor's PDF to reflect the proposed tolerance. Running the model multiple times with the new tolerances and comparing the results with the original tolerances will quantify the improvement in variation reduction.

Conclusions

It is possible to analytically estimate the variation in driveline NVH performance due to typical product tolerances and manufacturing variations. The analytically derived variation correlated well to the experimentally measured variations. Various stochastic studies, DOEs, etc., can be performed to try to analyze and optimize product and manufacturing process

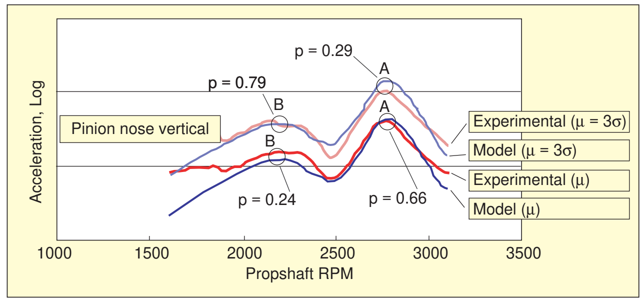


Figure 11. Hypothesis test results on model accuracy with respect to system nominal and variations.

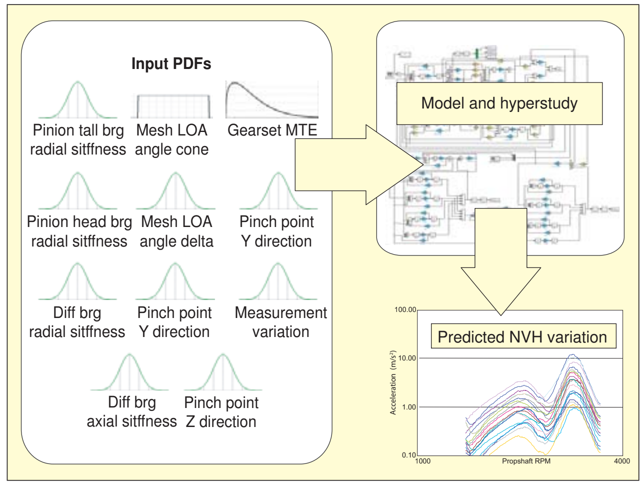


Figure 12. Flow chart of model prediction on system variations.

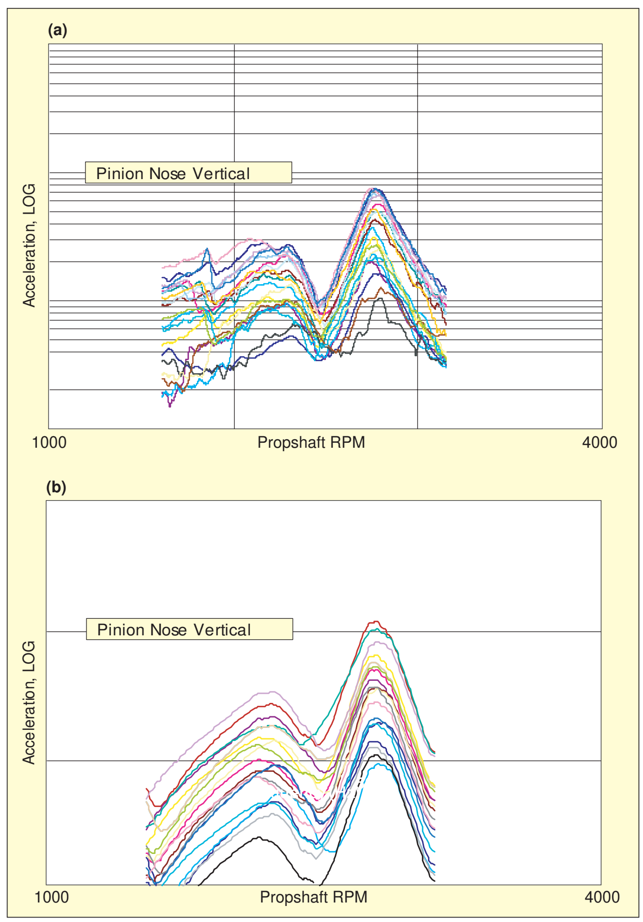


Figure 13. Model correlation to vehicle data related to system variations. a) Experimental variation of 20 randomly selected production axles. b) Model predicted variation of 20 randomly selected simulated axles (model runs).

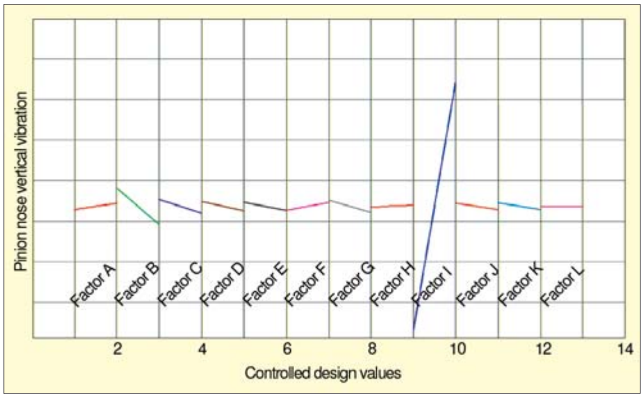


Figure 14. Main effects from two-level, 12-factor screening DOE; driveside Altair hyperstudy analyses.

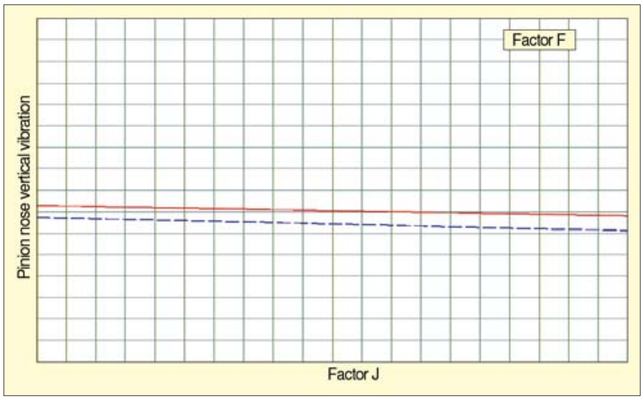


Figure 15. Example of typical variable interactions.

tolerances to achieve the least NVH variation for given manufacturing and cost constraints.

A possible scenario for obtaining maximum benefit of these new tools would be to conduct a parameter (nominal design) optimization first to obtain several of the lowest nominal NVH parameter combinations in the "design window." Then each of these 'good' candidate nominal designs can be evaluated for tolerance sensitivity. The final design will be the best combination of low nominal NVH, low sensitivity to tolerances/manufacturing variation, within existing cost/manufacturing/etc. constraints.

Acknowledgments

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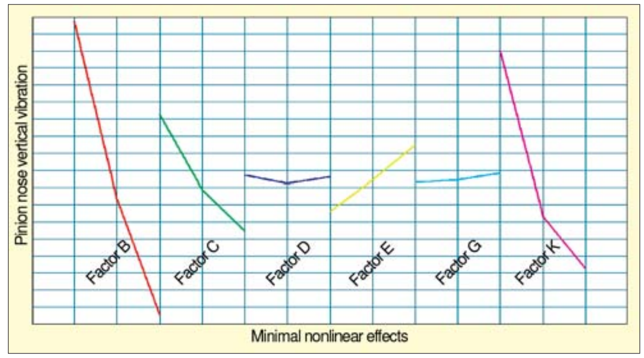

Figure 16. Main effects from three-level, 10-factor DOE; Altair hyperstudy analyses.

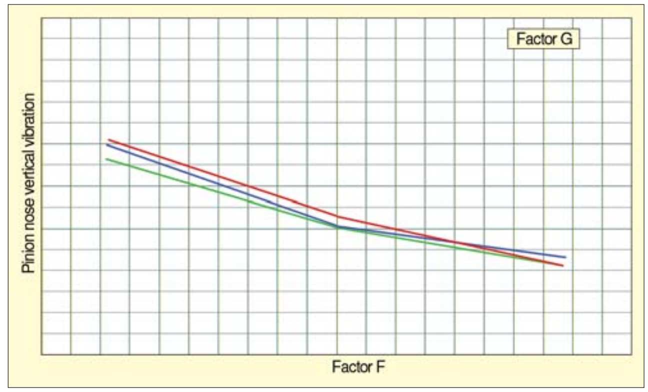

Figure 17. Typical variable interactions in three-level DOE.

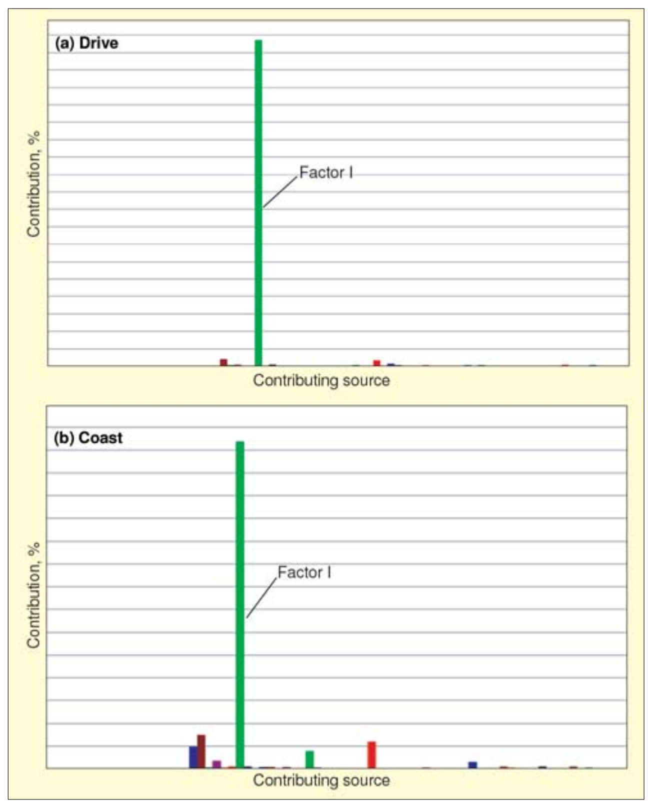

Figure 18. Driveside (a) and coastside (b) results of ANOVA analysis; two-level, 10-factor Altair hyperstudy analysis.

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