# Modal Correction Methods in Acoustic Reliability Analysis of Vehicles

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**This article discusses some of the vibro-acoustic design methods available that include the effects of model parameter uncertainty in the low-frequency response analysis process. The emphasis is on uncertain model parameters that are represented probabilistically. The so-called "modal correction method" is reviewed, where the effects of physical structural modifications are estimated by rapid resolution of the modal equations of motion. The accuracy of the modal correction method in the context of uncertainty analysis is discussed as well. The application of CDH/VAO software for the analysis of uncertain structures is discussed. These methods include Monte Carlo analysis, first-order sensitivity (FOSM), and first-order reliability analysis (FOR). An approach to stochastic optimization and its application to a representative vibro-acoustic design using several different methods is presented.**

Vehicle vibro-acoustic prediction analysis is usually performed using finite-element models in which the properties are assumed to be known with certainty. Owing to variation in the actual physical components of a real structure, many model parameters are random and not deterministic variables. For example, nominally identical vehicle body and engine mounts can exhibit large variations from specification values due to the effects of manufacturing tolerances and scatter in polymer material properties. These effects result in significant scatter in the acoustic and tactile response of nominally identical vehicles. Conventional deterministic analysis does not provide information about the scatter of the responses. In the presence of random variable model parameters, it is not possible to make reliable design decisions based entirely on deterministic analyses.

Figure 1 shows a typical histogram of engine idle-load acoustic response obtained from nominally identical vehicles. Deterministic analysis carried out using an accurate model would predict the expected response to be around 50 decibels. If customers were to complain about noise in the vehicle at 54 decibels, the vehicle structural design may not be acceptable, since a considerable proportion of vehicles would involve complaints from customers. Similarly, a nominally optimum structural design may be unreliable because of scatter manifest in the acoustic and tactile response of individual vehicles. There is currently considerable interest in analysis methods that provide information about the scatter in responses. This article discusses some of the methods available in CDH/VAO, a commercially available vibro-acoustic analysis program. A review of the theoretical basis of these methods is followed by applications to representative design problems.

## Reliability Methods for Response Prediction

Methods for predicting the effects on structural response of uncertainty in structural parameters belong to the theory of structural reliability. The methods may be divided into two categories; those that employ parametric models of structural uncertainty and methods that employ nonparametric models.[1-4] For methods that use parametric models, the equations of motion are solved for some prescribed values of the uncertain parameters. This process can be conducted only if a de-

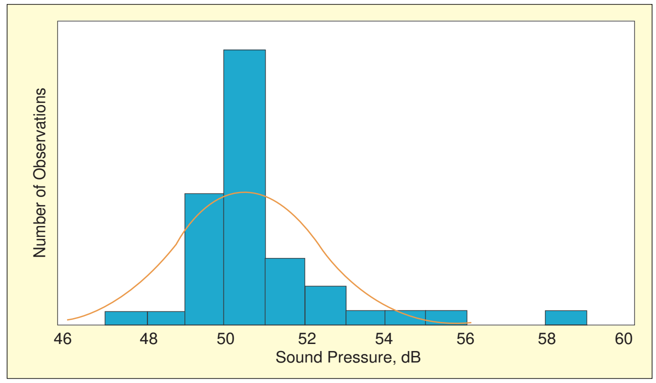


*Figure 1. Histogram of sound pressure at driver ear.*

scription of the uncertainty and an appropriate analysis procedure is available. Parametric model uncertainty may be described probabilistically by means of a probability density function. Where distributions are not known, possibilistic analysis provides an alternative for reliability analysis.[5-7] In structural reliability, the term 'failure' is used to describe a condition where the response exceeds some limiting condition. In the context of vehicle NVH analysis, for example, the limit condition could be the maximum allowable acoustic response in decibels at a driver ear position for some specified engine excitation frequency. A probability density function is a model that describes the probability that an uncertain variable will have a particular value. Gaussian, lognormal, and uniform distributions are commonly adopted. The probability of failure can be expressed[8] in general terms as:

$$p_{\text{failure}} = P[G(X) \le 0] = \int \dots \int_{G(X)\le 0} p(X)dX \tag{1}$$

where $p/(X)$ is the joint probability density function of the $n$ dimensional vector $x$ of model parameters. The region of integration $G(X) < 0$ denotes the space of limit state violation. Except for very special cases, the above integral cannot be performed analytically. Some of the application methods are described in the following section.

**First-Order Sensitivity Method**. The first-order sensitivity method (FOSM) is based on a Taylor expansion for mean and variance of a function $z$ of random variables $x$. By considering only the first-order terms, an approximation to the actual mean and variance can be obtained as follows:

$$E(z) = h\left[E(x_1), E(x_2), \dots E(X_n)\right] \tag{2}$$

The expression for variance of the function is:

$$\operatorname{var}(z) = \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x}\right)^2 \operatorname{var} x_i \tag{3}$$

The mean and variance values for the design variables are available from input data; the derivatives can be calculated as in design sensitivity. In frequency response analysis, the response and derivatives are complex quantities. It has been established that for modest deviations and many random variables, the first-order approximation can give excellent results.

**Stochastic Finite-Element Method**. In the stochastic finite–element method, the structural uncertainties are used to find the uncertainties in the properties of each element in the finite-element model. The uncertainties in the assembled finite-element system matrices are considered to be small, and a first-order perturbation technique is used to estimate the mean and vari-

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ance of the response vector. A discussion of the theory of stochastic finite elements is presented in References 9 and 10.

**Monte Carlo Method**. The Monte Carlo method evaluates the function at a series of samples generated in accordance with the probability density function of the individual uncertainty parameters. Using the probability density distributions of the basic variables, the sampling process and series of response evaluations allows the numerical estimation of the failure probability. In this case, it follows directly from sample statistics that the probability of failure is given by:

$$p_{\text{failure}} = \frac{1}{N}\sum_{j=1}^{j=N} I\left[G(x_j) \le 0\right] \tag{4}$$

where the value $I$ is unity when the inequality is satisfied. The Monte Carlo method in its crude form is computationally expensive. By means of importance sampling and other related techniques, improvements in efficiency can be obtained. For recent developments on advanced Monte Carlo methods see References 11-16.

**First-Order Reliability Method**. The works of numerous authors[17-22] form the foundation of FORM and SORM (second-order reliability method). In the FORM approach, the variables are transformed to uncorrelated Gaussian random variables and then standardized so that the design represented by the mean of the variables is located at the origin of an n-dimensional hyperspace. The joint probability density function of the variables is described by a standardized multivariate Gaussian distribution in the hyperspace. The first-order estimate of the probability of failure is then given by:

$$p_{\text{failure}} = \Phi(-\psi) \tag{5}$$

where $\Phi$ is the cumulative normal distribution function and $\psi$ is the minimum distance from the origin of the hyperspace to the limit state surface. If the limit state surface is a hyperplane, the failure probability given by FORM is exact. Depending on the curvature of the limit state surface, the FORM estimate will be either a lower or upper bound. In SORM analysis, the principal curvature information at the design point is used to provide an improved estimate of failure probability. The FORM procedure requires the solution of a constrained minimization problem to determine $\psi$.

## Optimization and Robust Analysis In CDH/VAO

CDH/VAO is an interactive graphics program for performing vehicle noise, vibration, and harshness (NVH) comfort analysis. The modal equations of motion for an acoustic fluid-structure coupled system are developed[23] as follows:

$$\left[-\omega^2\begin{pmatrix} M_S & 0 \\ 0 & M_F\end{pmatrix} + i\omega\begin{pmatrix} D_S & -C\omega_F^{-1} \\ \omega_F^{-1}C^T & D_{FN}+D_{FB}\end{pmatrix} + \begin{pmatrix} K_S & 0 \\ 0 & K_F\end{pmatrix}\right]\begin{Bmatrix} q_S \\ q_F\end{Bmatrix} = \begin{Bmatrix} Q_S \\ Q_F\end{Bmatrix} \tag{6}$$

It is the primary task of the program to assemble and solve this equation for a set of user-defined excitation frequencies and physical loads and to transform the modal coordinates to physical structural responses and acoustic pressures. In Equation 6, the matrices are obtained from a finite-element code. Using NASTRAN, for example, the data required can be generated very efficiently in a single run for both fluid and structure after DMAP (Direct Matrix Abstraction Program) modifications to standard solution sequences.

For fast re-analysis of structural responses, the original system eigenvectors are used to calculate a perturbation of the modal equation of motion. The modified structural stiffness is:

$$\overline{K}_s = K_s + \Phi_s^T \Delta k_s \Phi_s \tag{7}$$

Analog modifications to the modal mass and damping are performed. Modifications to the modal fluid normal incidence and bulk reacting absorption matrices are possible as a consequence of damping modifications. The physical modifications lead to a final perturbation in the modal equation of motion as follows:

$$\left[-\omega^2\begin{pmatrix} \overline{M}_S & 0 \\ 0 & M_F\end{pmatrix} + i\omega\begin{pmatrix} \overline{D}_S & -C\omega_F^{-1} \\ \omega_F^{-1}C^T & \overline{D}_{FN}+\overline{D}_{FB}\end{pmatrix} + \begin{pmatrix} \overline{K}_S & 0 \\ 0 & K_F\end{pmatrix}\right]\begin{Bmatrix} q_S \\ q_F\end{Bmatrix} = \begin{Bmatrix} Q_S \\ Q_F\end{Bmatrix} \tag{8}$$

VAO allows the user to define grouped design variables for modification and optimization studies. Appropriate scaling of a nominal perturbation reduces the variable to a parameter. Numerical optimization enables the user to study the effects of proposed design changes and to predict a set of modifications to provide optimum dynamic behavior of the structural system. The modal correction approach is the key to a drastic reduction in the computational effort for repeated dynamic response analyses as required in numerical optimization calculations or Monte Carlo analysis. In essence, it avoids the necessity to recalculate the system eigenvectors at each realization of the design variables. The modal correction method is an approximation performed at the level of modal equations. These equations are then solved 'exactly.' In effect, it is assumed that the new solution can be expressed as a linear summation of the eigenvectors of the original structure. Therefore, engineering accuracy is only achieved if the number of basis eigenvectors is sufficient. For optimization and robust analysis CDH/VAO includes Monte Carlo sampling, FORM (first-order reliability analysis), FOSM (first-order sensitivity analysis) and stochastic optimization. Gaussian and uniformly distributed variables can be defined. For Monte Carlo analysis, any distribution can be used by direct input of data sets of realizations of variables.

## Approach to Stochastic Optimization

Stochastic optimization is a branch of operational research and is a subject of active development.[24,25] It involves the search for robust solutions to problems with random variables. The approach to stochastic optimization in CDH/VAO is based on a FOSM approximation to the variance of the response. Expected maximum frequency response amplitudes are expressed in a statistical way by adding to the mean response some value that represents, in a probabilistic sense, the likely maximum deviation from the mean response.

In addition to the mean values of the model parameters, the variance for each parameter (expressed as a standard deviation or coefficient of variance) is itself considered to be an independent design variable for the purposes of optimization. These variables are termed "quality variables."

The optimization problem can then be posed to minimize:

$$F = f(X, CoV_x) \tag{9}$$

with:

$$U \le U_{\max}$$
$$X_{\text{lower}} \le X \le X_{\text{upper}}$$
$$CoV_{X\text{lower}} \le CoV_X \le CoV_{X\text{upper}}$$

Using Equations 2 and 3 and assuming Gaussian distribution of response, at probability level $p$ we have:

$$U = E(U) + Z \cdot \left(\sum_{i=1}^{n} \frac{\partial U^2}{\partial X} x_x^2\right)^{1/2} \tag{10}$$

or

$$U = E(U) + Z \cdot \left(\sum_{i=1}^{n} \frac{\partial U^2}{\partial X_1} E(X_i)^2 CoV_{Xi}^2\right)^{1/2} \tag{11}$$

where

$$Z(p) = \Phi^{-1}(p)$$

In CDH/VAO, the so-called "beta method" has been used to formulate the optimization problem. Briefly, this is a min-max strategy that minimizes a dummy variable 'beta' in the optimization process, and sets up constraint equations given by:

$$\frac{R^f - R_0^f}{C} + 2 - \beta \le \frac{S}{C}, \quad \beta \le 2, C \ge 0 \text{ for all } f \tag{12}$$

The values $R^f$, $R_0{}^f$ are the maximum and target responses, respectively. $S$ and $C$ are problem-dependent shift and scale constants. In CDH/VAO, the target response is entered interactively after the frequency response plots from Equation 10 or 11 have been displayed.

Where "quality variables" are defined, peak response can be

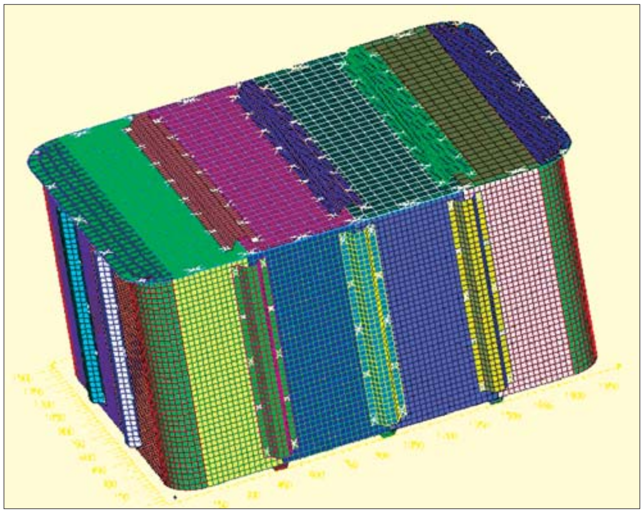

Figure 2. Reverberant acoustic box structure.

reduced in optimization simply by setting the variance to its lower bound. But in reality, model parameters with low variance are likely to result from an expensive manufacturing process. If it is possible to assign a numerical cost penalty to the coefficient of variance for a particular parameter, the minimization of an objective function that includes a cost penalty for variance model parameters will lead to an optimum selection of variables for both mean and coefficient of variance of model parameters.

CDH/VAO stochastic optimization differs from conventional optimization in that it includes:

- Parameter mean and parameter variance (standard deviation or CoV) design variables.
- The definition of cost penalty constraint as function of mean and variance design variables, responses, and constants.

The idea of including "quality variables" and "quality costs" in a structural optimization may provide insight into new solutions. For example, consider the simple cost relationship in Equation 13, where it is assumed that nominal quality corresponds to a coefficient of variance of 5%,

$$C = RC_{\text{prod}}(0.95 + \frac{0.0025}{COV_X} + (1-R)C_{\text{mass}} \qquad (13)$$

where $C_{\text{prod}}$ and $C_{\text{mass}}$ are production quality costs and mass related costs respectively, and $R$ is a cost-weighting factor variable from 0 to 1.0. The results of stochastic analysis on simple structures illustrate that where structure mass-related costs are less important to the designer than manufacturing costs, the cost-weighting factor approaches unity (in shipbuilding for example). A high coefficient of variance in components can be consistent with an optimum design. Alternatively, where the costs associated with quality are less important to the designer than achieving low mass costs with $R$ approaching zero (such as in aerospace design), then the optimum design is achieved by a high-quality, low-variance, structure. It is likely that automobile structures would lie between these extremes.

## Structural Dynamics Application

The methods available in CDH/VAO described in the preceding section have been applied to illustrate an engineering problem that is typically encountered during the NVH development of new vehicles. Figure 2 shows a stiffened, thin-walled steel box structure. Although this structure is clearly not an automobile body (it has no windows), it exhibits many of the dynamic characteristics of a vehicle structure enclosing a passenger compartment air cavity. The box structure comprises some 45 plate components connected by spot and seam welding.

In this example, it is assumed that the 45 plate thicknesses are samples from a Gaussian distribution with a 5% coefficient of variance. Therefore, the plate thicknesses are not known exactly but are scattered about a nominal mean value. The box

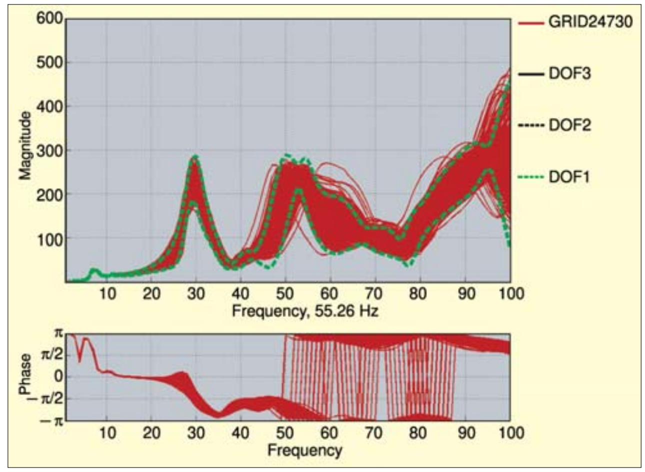


Figure 3. Comparison Monte-Carlo and FOSM analyses.

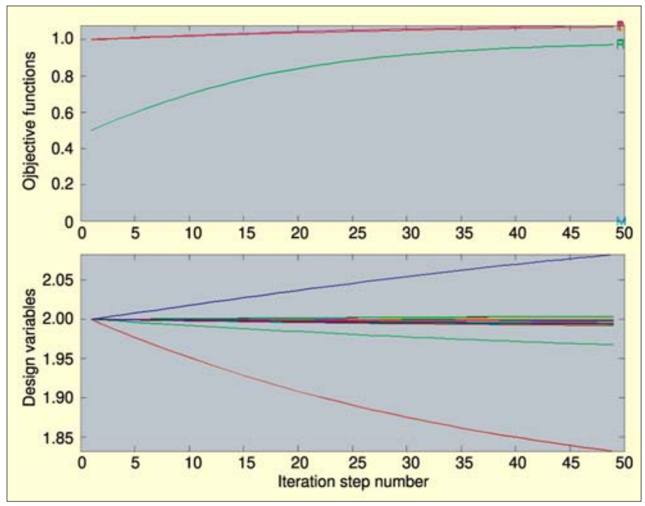


Figure 4. Convergence of FORM algorithm.

is supported on four spring-dampers at the base. Excitation is defined by applying a unit harmonic force in 1-Hz steps up to 100 Hz at one of the supports. Arbitrarily, tactile structural response points have been selected on the box top. This assumes that the designer wants to understand the scatter in the frequency response function at the selected tactile response points.

In preparation of the data, NASTRAN was used to generate the finite-element air and structural matrices and to calculate approximately 500 structural modes to 200 Hz and approximately 80 air modes to 600 Hz. The modal data from NASTRAN was efficiently converted to VAO format by a utility.

**Monte Carlo and FOSM Analysis**. Figure 3 compares the results of the Monte Carlo simulation with the FOSM output. In Monte Carlo analysis, 500 samples were generated from an uncorrelated multivariate normal distribution for the 45 input parameters. The output of these analyses is shown in Figure 3. The results of FOSM analyses are also shown in Figure 3. The FOSM calculation is essentially carried out per Equations 2 and 3. By default, mean ±2 sigma values are used to calculate bounds on responses predicted by FOSM. These bounds are shown by the dashed curves in Figure 3. Assuming a normal distribution of response amplitudes (almost certainly not the case), we can expect that approximately five of every 100 Monte Carlo analyses lie outside the FOSM bounds. Figure 3 shows that the Monte Carlo results correspond very well to the bounds resulting from the FOSM approach.

A variance contribution analysis may be generated to provide information about the contribution of each random parameter (the thickness of each plate) to the response. Figure 3 shows a selected tactile structural response. Similar results can be obtained for acoustic pressures. Note that a standard frequency response analysis provides the design engineer with nominal deterministic results only. The results presented in Figure 3 illustrate that considerable scatter will be present in the real world for nominally identical designs. Statistical analysis of the results in Figure 3 provides the design engineer with the probability that an actual design will in practice exceed some response threshold. The design engineer must then decide whether this probability is acceptable

Note that FOSM calculations can be performed very quickly in CDH/VAO. The FOSM result forms the basis of the stochastic optimization tool described subsequently.

**Robust Analysis Using FORM**. FORM analysis in CDH/VAO is essentially an implementation of Equation 5. The limit state is defined as a scalar function of response. In fact, the limit-state function is a user-specified multiple of the objective function used for optimization. The objective in optimization is a weighted function of tactile and acoustic responses over some frequency range and structural mass. Therefore, FORM in CDH/VAO may be used to study the reliability or robustness of an 'optimum' design. To illustrate the use of FORM, an objective function was defined as the sum (after scaling) of an acoustic and tactile response of interest in the peak region 50-55 Hz in Figure 3.

The limit state was defined to correspond to a 10% increase in the objective function. Figure 4 shows the progress of the FORM algorithm to convergence. Using this limit-state, the reliability $R$ shown in the upper plot of Figure 4, was calculated to be 97%. The lower plot in Figure 4 illustrates the minimum changes in design variables required to deteriorate the selected response objective by 10%. In itself, this information provides important information to the designer. By controlling the quality (scatter in thickness) of a few critical variables, the reliability of the design may be improved.

**Example of Stochastic Optimization**. To illustrate the stochastic optimization capability, 20 plate thickness properties were selected as design variables as shown in Table 1. Corresponding to each of these mean variables, a coefficient of variance "quality variable" was defined. A quality cost function similar to Equation 13 was used. Tactile and acoustic response peaks were selected for reduction as shown in Figures 5 and 6. These illustrate the plots generated at the end of the optimization. The initial and final values of response with their upper- and lower-mean $\pm Z$-sigma bounds are shown. Also shown is the user defined 'target' line entered interactively during the analysis. The upper bound of response has been reduced to or below the target response. The cost function was reduced from 6300 cost units to 5810 during optimization.

The values of mean variables and coefficient of variance "quality variables" at the conclusion of the analysis are shown in Table 2. Note that some of the 'quality' variables increased and others decreased. A minimum cost solution may require that the quality of critical components be improved, while lower standards may be acceptable elsewhere.

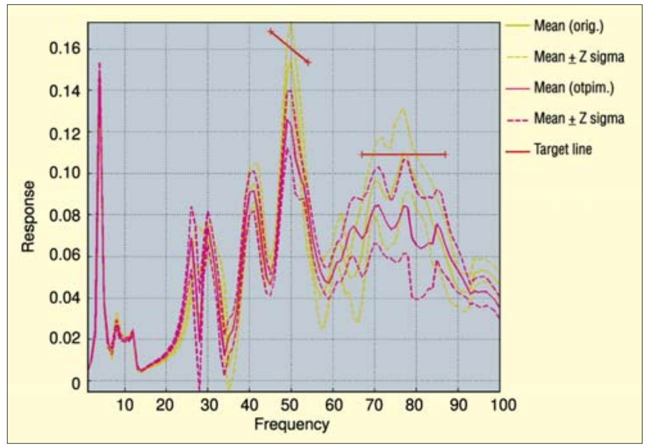


*Figure 5. Optimized velocity response.*

## Conclusions

With particular reference to vibro-acoustic design applications, we have discussed some of the methods available to include the effects of model parameter uncertainty in the analysis process. We have emphasized uncertain model parameters that are represented probabilistically. The theoretical background to methods available in CDH/VAO, a special-purpose vibro-acoustic dynamics program, has been described. The program was applied to a representative design problem. The work confirmed the importance of including parameter uncertainty in the design process. CDH/VAO proved to be a useful tool for frequency response analysis and optimization of vibro-acoustic structural systems containing model uncertainty.


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*Table 1. Initial design parameters.*

| | Design Variable | | | Coefficient of Variance | | |
|---|---|---|---|---|---|---|
| Name | Value | Min. | Max | Value | Min | Max |
| D_1 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_2 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_3 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_4 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_5 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_6 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_7 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_8 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_9 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_10 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_11 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_12 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_13 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_14 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_15 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_16 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_17 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_18 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_19 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |
| D_20 | 2.0 | 1.5 | 2.5 | 0.05 | 0.0 | 0.2 |

*Table 2. Optimized design parameters.*

| | Design Variable | | | Coefficient of Variance | | |
|---|---|---|---|---|---|---|
| Name | Value | Min. | Max | Value | Min | Max |
| D_1 | 2.1017 | 1.5 | 2.5 | 0.0706 | 0.0 | 0.2 |
| D_2 | 1.5000 | 1.5 | 2.5 | 0.0590 | 0.0 | 0.2 |
| D_3 | 2.1341 | 1.5 | 2.5 | 0.0582 | 0.0 | 0.2 |
| D_4 | 2.0446 | 1.5 | 2.5 | 0.0533 | 0.0 | 0.2 |
| D_5 | 2.0507 | 1.5 | 2.5 | 0.0513 | 0.0 | 0.2 |
| D_6 | 1.5000 | 1.5 | 2.5 | 0.0511 | 0.0 | 0.2 |
| D_7 | 1.9468 | 1.5 | 2.5 | 0.0592 | 0.0 | 0.2 |
| D_8 | 2.2331 | 1.5 | 2.5 | 0.0563 | 0.0 | 0.2 |
| D_9 | 2.0939 | 1.5 | 2.5 | 0.0552 | 0.0 | 0.2 |
| D_10 | 2.1221 | 1.5 | 2.5 | 0.0553 | 0.0 | 0.2 |
| D_11 | 2.3242 | 1.5 | 2.5 | 0.0590 | 0.0 | 0.2 |
| D_12 | 2.4451 | 1.5 | 2.5 | 0.0535 | 0.0 | 0.2 |
| D_13 | 2.4023 | 1.5 | 2.5 | 0.0506 | 0.0 | 0.2 |
| D_14 | 2.2840 | 1.5 | 2.5 | 0.0568 | 0.0 | 0.2 |
| D_15 | 1.7898 | 1.5 | 2.5 | 0.0512 | 0.0 | 0.2 |
| D_16 | 2.0275 | 1.5 | 2.5 | 0.0533 | 0.0 | 0.2 |
| D_17 | 2.5000 | 1.5 | 2.5 | 0.0460 | 0.0 | 0.2 |
| D_18 | 2.0215 | 1.5 | 2.5 | 0.0462 | 0.0 | 0.2 |
| D_19 | 1.7239 | 1.5 | 2.5 | 0.0518 | 0.0 | 0.2 |
| D_20 | 1.5000 | 1.5 | 2.5 | 0.0509 | 0.0 | 0.2 |

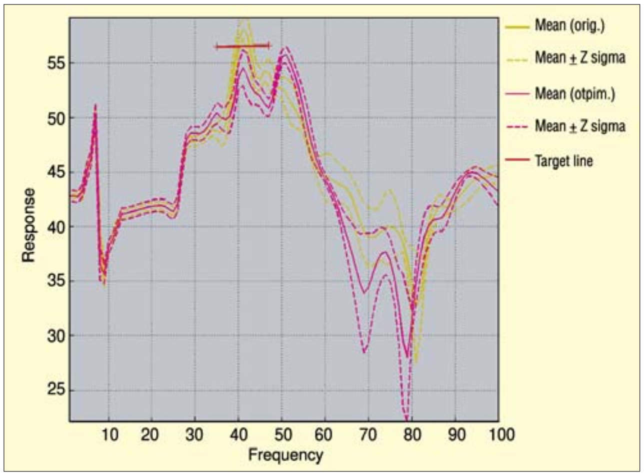


*Figure 6. Optimized acoustic response.*

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