

Comparison of Test Specifications and Measured Field Data

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It is not always obvious how to compare a given test specification with measured field data. This article presents a systematic method based on the shock response spectrum and the fatigue damage spectrum. A random-on-random test specification for tracked vehicles, AECTP 400, is used as an example.

A common situation for the test engineer is comparing test specifications with measured field data. In many cases, the test specification is intended to mimic the real environment, but usually there is no way to directly make the comparison. A typical test specification for munitions transported as secure cargo in tracked vehicles, AECTP 400, is shown in Figure 1.

The bandwidth of each narrow band is 10 Hz. The frequencies of the band should be harmonically related and swept, so that:

- $15 \text{ Hz} < f_1 < 150 \text{ Hz}$
- $30 \text{ Hz} < f_2 < 300 \text{ Hz}$
- $45 \text{ Hz} < f_3 < 450 \text{ Hz}$

There are versions of the test specification where the bandwidths of the frequency bands are 5, 10, and 15 Hz, respectively. All the levels in the spectrum are defined and the sweep rate should not exceed 1 octave per minute. The test time is 1 hour per axis. The test specification is meant to mimic the background vibration and track laying frequency with two harmonics.

Comparison Methods

There are basically two different ways to do a comparison between the environment defined by the test specification and the environment measured in the field. The first way looks at the damage to the transported goods or equipment from a maximum point of view. The method tries to find the maximum response of the equipment in the two cases. The natural tool in this case is to use the shock response spectrum (SRS). The ISO 18431-4 standard defines how to calculate the SRS for the field data. The catch here is how to calculate the SRS for the test specification. How this may be done is explained later. The other way looks at the damage from a dose point of view. The proposed tool in this case is the fatigue damage spectrum (FDS), which may be regarded as the SRS with dose. FDS is defined below and a method to estimate the FDS for the test specification is given.

Comparison Using the Shock Response Spectrum. A typical model for the SRS calculation, shown in Figure 2, assumes that the signal to be analyzed is applied to an array of independent SDOF systems with a common base. The peak response of each of the masses is of interest. In ISO 18431-4, the SRS is calculated by the use of digital filters, one filter for each SDOF system in Figure 2. The filter coefficients are calculated using the ramp-invariant method.¹ The SRS for absolute acceleration response may be calculated for the field data using a simple MATLAB[®] script:

```
function [y,f] = srs(x,fs,fmin,fmax,fno,Q)
%SRS Shock Response Spectrum Ramp Invariant Version
Maximax
% [y,f] = srs(x,fs,fmin,fmax,fno,Q)
% y maximax is calculated
% f frequency vector Hz
% x data vector
```

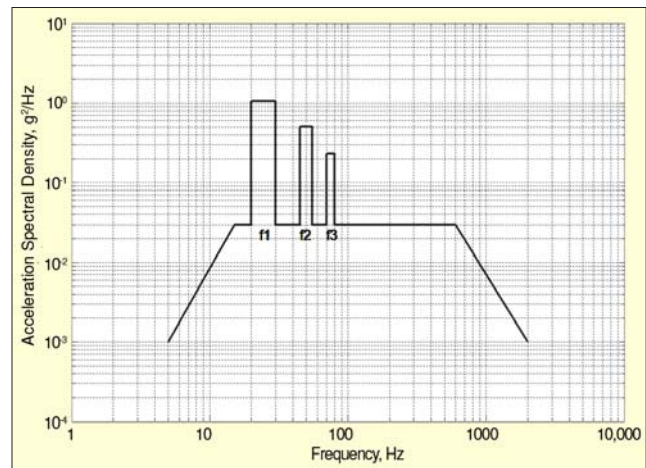


Figure 1. AECTP 400, method 401; test specification for munitions transported as secure cargo in tracked vehicles.

```
% fs sampling frequency Hz
% fmin low frequency Hz
% fmax high frequency Hz
% fno number of frequencies
% Q Q value
y = zeros(fno,1);
f = zeros(fno,1);
%
k1 = log(fmax/fmin)/(fno-1);
k2 = pi/Q/fs;
k3 = 2*pi/fs*sqrt(1-1/(4*Q*Q));
%
for n = 1:fno;
    f0 = fmin*exp(k1*(n-1));
    A = k2*f0;
    B = k3*f0;
    a = [1, -2*cos(B)*exp(-A), exp(-2*A)];
    b = [1-exp(-A)*sin(B)/B, 2*exp(-A)*(sin(B)/B-cos(B)), exp(-2*A)-exp(-A)*sin(B)/B];
%
    z = filter(b,a,x);
y(n,1) = max(abs(z));
f(n,1) = f0;
end
```

So, the calculation of the SRS for the measured field data is quite straightforward. To get the SRS to correspond to the test specification, we first have to consider how to estimate the SRS of a vibration with a given power spectral density PSD_{in} . For each filter $H_n(f)$ in the SRS filter bank, we can calculate the output PSD_{out} :

$$PSD_{out}(f) = PSD_{in}(f) \cdot |H_n(f)|^2 \quad (1)$$

We get the RMS value s_n of the filter output by integrating the output PSD:

$$\sigma_n = \sqrt{\int PSD_{in}(f) \cdot |H_n(f)|^2 df} \quad (2)$$

which is easily done, since we know the filter functions. This is a better estimate than the one usually given, which assumes that the input PSD is constant with the level given for the resonance frequency under consideration. Then the PSD level is multiplied with the filter bandwidth. With a given RMS value, the n th maximum value, max_n , can be estimated by the follow-

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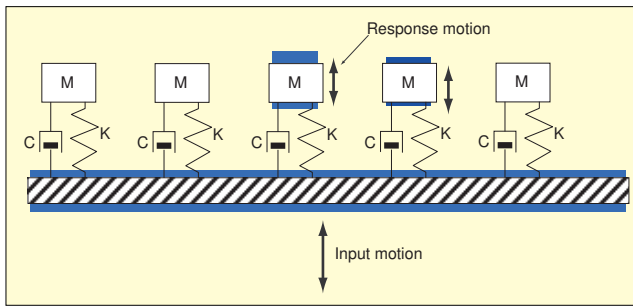


Figure 2. Typical model used for calculating the shock response spectrum (from Strether Smith). The motion is assumed to be applied to a common base of an array of damped and independent SDOF systems.

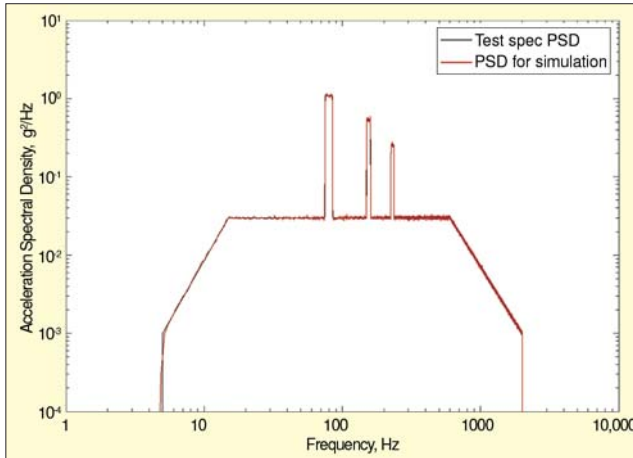


Figure 3. Comparison of specified and simulated power spectral density.

ing equations:

$$\begin{aligned}
 c_n &= \sqrt{2 \ln(f_n T)} \\
 C_n &= c_n + \frac{0.5772}{c_n} \\
 \max_n &= C_n \cdot \sigma_n
 \end{aligned}
 \quad (3)$$

where f_n is the resonance frequency for the n th filter, and T is the total time under consideration. The equations can be found in Reference 2.

To show how the method works with a simulation, we consider the case in the test specification when f_1 is at 75 Hz. We create a time signal of a certain time length with the given PSD by adding a random phase to the given Fourier amplitude and perform an inverse Fourier transform. The given PSD and the calculated spectrum for the simulation are given in Figure 3. The PSD is calculated using a Hanning window and averages with overlap, which is the reason for the randomness in the spectrum.

We can now use the simulated time signal to calculate the SRS and compare this with the estimated SRS from the specification PSD. The result is shown in Figure 4 and shows good agreement. This method can be used to estimate all SRS for the test specification by stepping the gliding frequencies 1 Hz at a time. We calculate the time spent in each position to mimic a logarithmic sweep, adding up to 1 hour. The result is shown in Figure 5. The maximum envelope for all SRS in Figure 5 is our final result. This is used for comparison with the field data SRS.

Comparison Using the Fatigue Damage Spectrum. The fatigue damage spectrum (FDS) is used for comparison of a test specification and measured field data from a dose point of view. The FDS may be regarded as an SRS with dose (see Figure 6). As for the SRS calculation, the vibration signal to be investigated is applied to a set of SDOF systems. For each SDOF system, the damage according to Miner's rule is estimated using a simplified S/N curve defined by the exponent. To make a proper damage estimate, a lot of information has to be entered

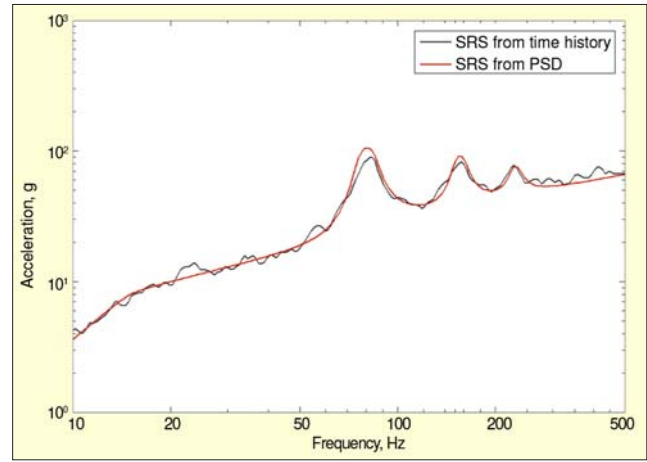


Figure 4. Comparison of calculated SRS for a simulated signal and estimated SRS from the specification PSD.

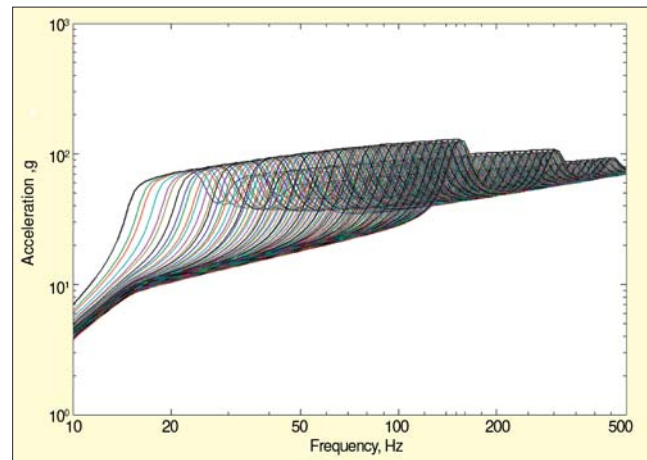


Figure 5. Estimated SRS for the test specification, corresponding to a 1-hour test time.

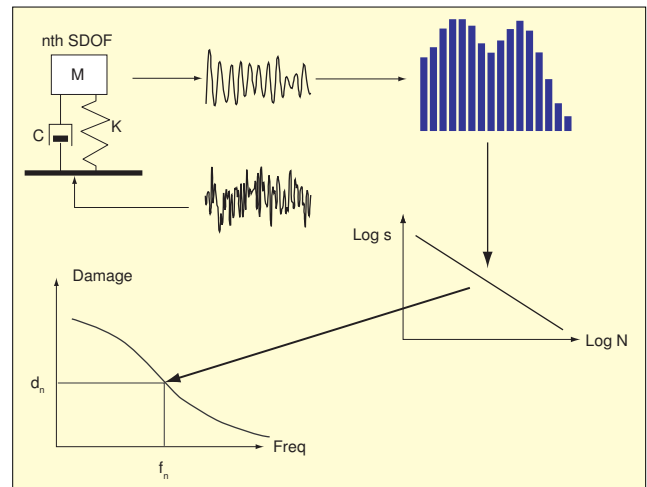


Figure 6. Principle for fatigue damage spectrum (FDS) calculation.

into the calculations, such as geometry of the object in question, the constants defining the proper S/N curve, etc. But as the FDS is only used to make a comparison, we can forget all this and just set all unknown constants to 1.

The steps in Figure 6 are as follows:

1. The response to the given input vibration in the n th SDOF system, usually as relative displacement, is calculated.
2. For the SDOF system narrowband output signal, a histogram, showing the number of peaks per peak amplitude interval is calculated.
3. The histogram is applied to an S/N curve, and Miner's rule is applied to calculate the damage.

4. The resulting damage represents one point in the FDS corresponding to the resonance frequency of the n th SDOF system.

5. As for the SRS, the procedure is repeated for all significant resonance frequencies.

The parameters in the calculation are the Q-value of the SDOF systems and the S/N curve exponent. Since the response of the SDOF system is a narrowband signal, the histogram may be replaced with a list of the peak values. A simple MATLAB script to calculate the FDS is given here:

```
function [y,f] = fdsdispl(x,fs,fmin,fmax,N,Q,b)
%FDSDISPL  Fatigue Damage Spectrum, relative displacement
% [y,f] = fdsdispl(x,fs,fmin,fmax,N,Q,b)
% y        FDS output
% f        frequency axis for FDS
% x        input data vector, acceleration
% fs       sampling frequency in Hz
% fmin     low frequency for calculation
% fmax     high frequency for calculation
% N        number of frequency points for calculation
% Q        Q value for calculation
% b        Wöhler S/N exponent

k1 = log(fmax/fmin)/(N-1);
%
for n = 1:N;
    fn = fmin*exp(k1*(n-1));
    [B,A] = dispfilt(fs,fn,Q);
    d = filter(B,A,x);
    de = abs(extr(d));
    y(n) = sum(de.^b)/2;
    f(n) = fn;
end
```

The function *dispfilt* calculates the filter coefficients for a relative displacement output for an SDOF system. The function *extr* extracts the peak values in the response signal.

As for the SRS case shown above, there is a simple way to estimate the FDS when the input PSD is given. The principle is the same, but the estimate of the maximum is replaced with an estimate of the damage when the RMS value of the SDOF output (and the S/N exponent b) is given. This is simple, providing the assumption of a Gaussian output is valid. A comparison (like the one in Figure 4) between the FDS calculated from a simulated time history and the estimated FDS based on the PSD, is shown in Figure 7. Since the procedure works well, we can estimate the full FDS for a 1-hour sweep as we did for the SRS. The result is shown in Figure 8. The difference to the SRS case is that the curve we have to use for comparison with field data is the sum per frequency as we deal with dose.

Conclusions

A method to compare test specifications with measured field data has been proposed in this article. The comparison is made

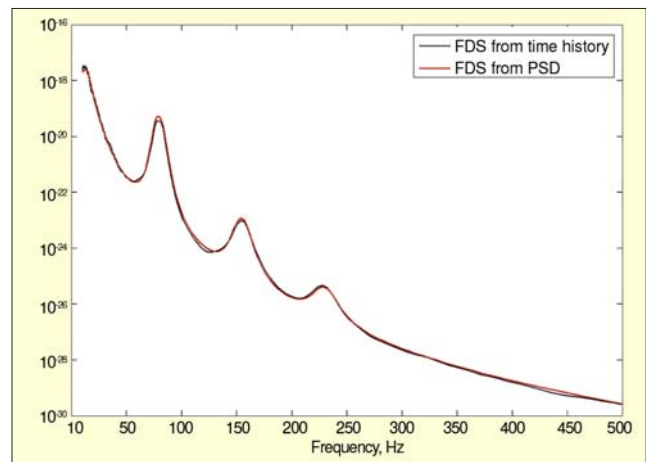


Figure 7. Comparison of FDS calculated from the simulated time history and estimated from the given PSD.

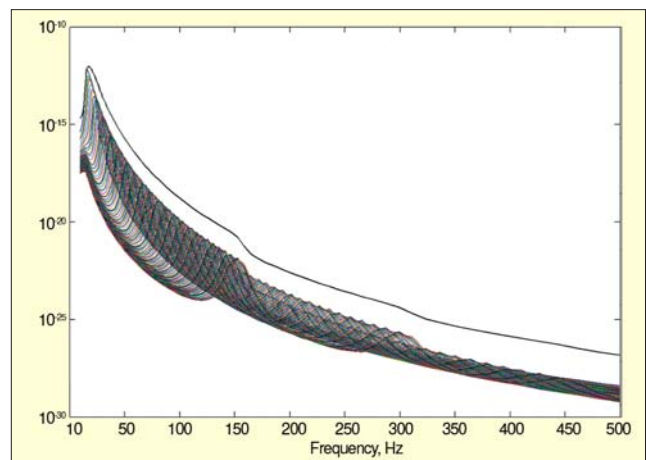


Figure 8. Resulting FDS corresponding to 1-hour test time; solid curve can be used for comparison with FDS for measured field data.

in two ways, with a maximum approach using the Shock Response Spectrum and with a dose approach using the Fatigue Damage Spectrum. The key feature of the method is the introduction of a simple way to estimate the Shock Response Spectrum and Fatigue Damage Spectrum from a given test specification.

References

1. Smallwood, D. O., "An Improved Recursive Formula for Calculating Shock Response Spectra." 51st Shock and Vibration Bulletin, 1980.
2. Nigam, N. C., *Introduction to Random Vibration*, MIT Press, Cambridge, MA, 1983.

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