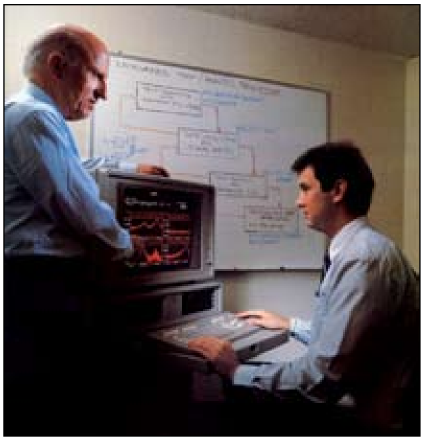
A Personal History of Random Data Analysis

Julius S. Bendat, J. S. Bendat Company, Los Angeles, California



Julius Bendat (left) discussing a test/analysis processor which utilized NASTRAN software

My interest in Applied Mathematics started during the second World War when I was an undergraduate student at the University of California in Berkeley. In my last two years there, I worked as a Physics Research assistant on the Manhattan Project where I learned a little about nuclear energy, the first major development from this war. Upon graduation in 1944, with the war still on in both Europe and the Pacific, I enlisted in the U.S. Navy to become a radar officer where I learned a little about radar, the second important development from this war. Thus, I was always motivated in my later graduate studies and in my following technical work in the field of random data analysis to investigate how to apply theoretical mathematical ideas to solve practical engineering and scientific problems. The books that I have written both alone and with Allan Piersol have been well received throughout the world, leading to pioneering work on many projects during the past 50 years and the presentations of short courses and seminars in 25 countries.

At the International Modal Analysis Conference IMAC XXIII in Orlando, FL, I gave an invited talk on February 1, 2005 where I spoke about some of my work in the field of random data analysis over the past 50 years. Prof. Al Wicks from Virginia Polytechnic Institute, the Conference Chairman, attended this session and asked me to write a paper to benefit people in the field. Prof. Rune Brincker from Aalborg University in Denmark, the Chairman of the first IOMAC International Operational Modal Analysis Conference, to be held in Copenhagen on April 26-27, 2005, was also present. He invited me to write an introductory paper for the IOMAC Proceedings where I might use a historical perspective. This article was my response to Al Wicks and Rune Brincker.

The IOMAC organizing committee also asked me to give the "Lecture of Honor" at the IOMAC conference dinner in Copenhagen on April 26, 2005. I greatly appreciated this special recognition for the contributions I was able to make with the help of people mentioned in this article.

In particular, I want to acknowledge the close association with Allan Piersol, a mechanical engineer whom I met in 1960 when we did some pioneering research work for the Flight Dynamics Laboratory at Wright-Patterson Air Force Base in Dayton, OH. Allan is the coauthor with me of six textbooks on Random Data Analysis and Engineering Applications, that were published by Wiley, New York, from 1966 to 2000.

I also want to thank Paul Palo, an ocean engineer, who sponsored 10 years of research work from 1985 to 1995 that I did for the U. S. Naval Civil Engineering Laboratory and the Office of Naval Research. This work developed new practical ways to identify and analyze nonlinear system dynamic properties, and to two books on Nonlinear System Techniques and Applications published in 1990 and 1998.

Random Noise Theory Book (1958)

In the late 1950s, time series analysis had not yet matured. The mathematical foundation existed but was available primarily only in research papers. Data had to be collected in analog form, and with the limited memory and speed of computers, digitization and subsequent analysis were awkward and expensive. It was in this period that I wrote my first book in 1958 after working in Los Angeles for five years at Northrop Aircraft Company and the Ramo-Wooldridge Corporation (now TRW Systems) on problems dealing with statistical communication theory and control systems. This book, *Principles and Applications of Random Noise Theory*, was published by Wiley, New York.[1] My objective in writing this book was to explain fundamental concepts from random noise theory in understandable engineering language, with emphasis on physical meanings and mathematical restrictions. I tried to bridge the gap between theoretical research work and the engineering applications that followed. This early 1958 Random Noise Theory book contained ideas on how to define and analyze basic amplitude, time and frequency properties of random data by using probability density functions, correlation functions and spectral density functions. I was influenced by the work of two men: (1) Stephen Rice of the Bell Telephone Labs, author of a monumental paper on "The Mathematical Analysis of Random Noise," published in the Bell System Technical Journal in 1944 and 1945; and (2) Norbert Wiener, author of the hard-to-read book, *The Interpolation, Extrapolation and Smoothing of Stationary Time Series*, published by Wiley, New York in 1949.

During this first five-year period, I learned to appreciate the differences between theoretical work based on ideal assumptions about random data and linear systems and practical engineering work based on actual measured data and real physical systems. I found that there are no sine waves in nature, that measured data can never be truly Gaussian, and physical systems are not ideal. Also, one can never compute the true properties of random data or linear systems with finite amounts of data. One can only obtain estimates with statistical bias and random errors.

In 1958, to celebrate the completion of my book on Random Noise Theory, my wife and I made our first trip to Europe. We flew in a Douglas DC-7 propeller airplane from Los Angeles to Copenhagen. It took 22 hours with stops in Newfoundland and Greenland. We landed at the Copenhagen airport early in the morning on a rainy day and took a taxi to our hotel. I saw more people going to work on bicycles than I had ever seen before. The first night, we went to Tivoli Gardens where we agreed it is appropriate that "I lov it" is Tivoli spelled backwards. We also liked the people we met in Copenhagen, and in subsequent trips to Denmark filled our home with Danish furniture.

In 1965, on my second trip to Europe, I again stopped in Copenhagen. This time I visited Brüel & Kjær to meet Jens Trompe-Broch, who was doing important experimental work on the properties of noise passing through linear and nonlinear systems. Over the next 20 years I did some consulting work for B&K and gave several short courses at their facility in Nærum, Denmark.

WPAFB Contracts

My 1958 book was read by engineers in the Flight Dynamics Laboratory at Wright-Patterson Air Force Base (WPAFB) in Dayton, OH. They felt that some of my ideas, as well as ideas from other people, could be applied to their work. In 1960, they issued a Request for Proposal (RFP) for a one-year study to develop new statistical methods to help solve flight vehicle vibration problems. They sent this RFP to many companies and addressed a copy to me. I submitted a $75,000 proposal to do the desired study and won the competition.

I had never done any work on aircraft vibration or acoustic problems so I looked around Ramo-Wooldridge to find people who could assist me. The first key person I was lucky to meet was Allan Piersol, a mechanical engineer engaged in environmental activities who had previously worked at Douglas Aircraft Company in a flight test group. The second key person was Loren Enochson, who was involved in the design of statistical test programs using early large-scale digital computers. A third person was Harold Klein, a structural dynamics engineer. I was also able to arrange for some outside consulting services from Prof. William Thomson at UCLA who had written a 1953 book on Mechanical Vibrations.

I was the project manager for this WPAFB contract and directed the overall work. Frequent meetings were held with engineers at WPAFB to review results. Everyone was highly motivated and dedicated to doing as comprehensive a study as possible. We completed the contract on time and wrote a 500 page report that was published by WPAFB in 1961.[2]

Analytical results studied in Reference 2 were in three parts:

- Part 1. Development of sampling techniques to decrease the amount of data to be gathered for later analysis.
- Part 2. Development of statistical techniques to test fundamental assumptions in the data of randomness, stationarity and normality.
- Part 3. Detailed examination of measurements needed for different applications using probability density functions, correlation functions and spectral density functions.

Procedures were outlined for carrying out a complete analysis of flight vehicle vibration data when data are obtained from either a single experiment or from a collection of experiments. An instrumentation survey was done of equipment available at that time for making desired measurements. Physical results were discussed on the response of different types of linear structures to periodic and to stationary random excitations.

We made recommendations to extend this WPAFB work by conducting a laboratory program to verify the analytical techniques for: (a) decreasing the amount of data to be gathered; (b) testing fundamental assumptions in the data; and (c) interpreting results from single experiments and from repeated experiments. It was necessary to prove the equivalence of different analog and digital ways to compute random data properties. There was a need to derive statistical error analysis formulas for estimates of these properties. We also recommended further mathematical studies on techniques to analyze nonstationary random data, and to predict the response of nonlinear systems to random excitations. Our recommendations were approved by WPAFB and we received a new sole source one-year contract of $75,000 to do this work. The results from this second contract are discussed fully in another 500-page report we wrote that was published by WPAFB in 1962.[3]

These two reports by us in 1961 and 1962 were major pioneering contributions to this field and were widely circulated by WPAFB. The procedures recommended in these WPAFB reports on ways to analyze and apply random data were studied and adopted by many engineers working in the United States on aerospace and automotive projects. We accepted invitations to discuss our results at various local and national meetings, and we received a number of special contracts from different government agencies and private companies to help them solve engineering problems.

Measurement Analysis Corporation

In 1963, with the encouragement of my wife, I resigned from Ramo-Wooldridge to become an Independent Mathematical Consultant. This was a daring thing to do, leaving a secure job with company benefits for an uncertain future. I didn't have a university position lined up to help support my wife and two children. Anyway, I felt that the timing was right because my 1958 book and the two WPAFB reports opened doors. My first client was Ramo-Wooldridge, the second was MTS Systems Corporation in Minnesota, and the third client was a NASA Center where I had never worked before. Within a few months I had more work than I could do by myself so I formed a California Corporation and called it Measurement Analysis Corporation with the logo MAC. I hoped that this MAC logo would not be confused with the same logo used by others, such as the Middlesex Athletic Club in England.

I asked Allan Piersol and Loren Enochson to join me at MAC where I was the President and they became two Vice Presidents. As such we were equal in title to people at companies that were 100 times our size. MAC was very prosperous from the beginning and over a five-year period from 1963 to 1968 grew from 3 to 25 persons. We established our own laboratory to conduct small test programs and to analyze the measured data. There was work from many engineering companies and NASA centers to help them accomplish their missions. We also assisted manufacturing companies who wanted guidance on what to include in their equipment that would be useful for customers. We had contracts from the Navy Shock and Vibration Information Center to write two monographs on: (1) digital computer techniques to analyze stationary random data; and (2) special ways to analyze shock and transient data.

In order to have a commercial product separate from our consulting services, Loren Enochson with other MAC associates developed one of the first government approved software programs for random data analysis and engineering applications, called the MAC/RAN System. We sold this MAC/RAN System to a number of industrial companies as well as to government centers. Together with other speakers from MAC, I presented short courses at UCLA, at government sites and at different companies to acquaint practicing engineers with our work.

Allan Piersol directed one special MAC project of importance. We obtained contracts from NASA to work on the Apollo project to send men to the moon and return them safely to earth. Allan with others at MAC developed the vibration test specifications for components of the Saturn launch vehicle for the Apollo spacecraft. When this mission was successful, we all looked up to the moon with amazement and felt very proud that we played a part.

In the 1960s, I devoted some of my time to deriving practical statistical error analysis formulas for estimates of random data properties from experimental data. For example, prior to my work, people used a complicated sampling distribution, called the F-distribution, to obtain large uncertainties in their estimates of frequency response functions. I didn't want to know how bad my estimates were. I wanted to know how I could get good estimates. I was able to prove that the only two things required to determine small random errors in frequency response function estimates are: (1) the number of averages used to compute the spectral density functions of the input/output data; and (2) the ordinary coherence function between the input record and the output record. Failure to compute this ordinary coherence function makes it impossible to determine the validity of any frequency response function estimates. All one obtains is a pretty picture. This failure is still a common mistake being made today by many engineers doing modal analysis work. I also showed the importance of coherence functions to predict the spectral properties of output response data.

In 1968, we sold our private company to a public software company called Digitech. Digitech was a forerunner of Microsoft in developing compiler programs for computers. This marriage did not last because Digitech was acquired two years later by a bank to do commercial work. MAC engineers left to join other companies.

ISVR Lectures

In 1965, Allan Piersol and I were invited by the Advanced Group on Aeronautical Research and Development (AGARD) of NATO to deliver a series of lectures at the Institute of Sound and Vibration Research (ISVR) in Southampton, England. 150 to 200 engineers attended this conference from all over Europe. We were the only speakers each morning for five days where we discussed results from our work for WPAFB and from activities at MAC. Our lectures were greatly appreciated.

While preparing these ISVR lectures, Allan and I started writing our first joint book, *Measurement and Analysis of Random Data*, published by Wiley, New York in 1966.[4] In this book we gave equal emphasis to analog and digital methods being used to analyze random data. The analog way to compute spectral density functions was done by filtering, squaring and averaging the data. This procedure is superior to the digital way in providing a proper physical understanding. The digital way to compute spectral density functions was driven by the need to minimize computer time. This was done by computing a single Fourier transform of the correlation function of the data. We showed that these two ways give statistically equivalent results. The Fast Fourier Transform (FFT) Technique, which is really a Fast Fourier Series Technique, to compute spectral density functions was not discussed in our 1966 book because it was not yet an established procedure. Throughout this 1966 book, we did not hide behind mathematical equations but explained ideas and various applications in physical and engineering terms. We reviewed and integrated what each of us wrote so that the final material appeared to come from one author.

J. S. Bendat Company

From 1970 to date, I have been engaged in consulting, short courses and writing books as an Independent Mathematical Consultant under a different California Corporation called the J. S. Bendat Company. This company has only two employees and two directors, namely, my wife and myself. I have had many opportunities in 25 different countries to give invited lectures and short courses, and to do personal technical work with creative people on various engineering projects. I learned about new ways they developed to analyze random data for special applications, and I was able to use some of their ideas to advance my work and to acquaint others with the results. From these activities and travels I made many friends and became a citizen of the world, not just the United States.

One of my projects was with MTS Systems Corporation (MTS) to help design a road simulator system to test the performance of cars in a laboratory by reproducing similar road conditions found in actual service. I developed new procedures to analyze multiple-input/multiple-output (MIMO) linear models by revising ideas published in the Journal of Sound and Vibration (JSV) in 1975 by Colin Dodds and Prof. John Robson from the University of Glasgow. I formulated efficient ways to change MIMO models with correlated inputs into an equivalent set of SISO models with mutually uncorrelated inputs, and I showed it is important to order the inputs based on physical requirements. I published some of my results in two 1976 JSV papers: "Solutions for the Multiple Input/Output Problem," Vol. 44, and "System Identification from Multiple Input/Output Data," Vol. 49.

When I published my recommended procedures in the *Journal of Sound and Vibration* in 1976, I inserted a footnote where I wrote, "I hope that my MIMO techniques will not be confused with the well-known GIGO techniques that are widely used throughout the world." As many of you know, GIGO stands for "garbage-in/garbage-out." An engineer working at the Institute of Sound and Vibration Research in Southampton, England reviewed my paper. He wrote that he appreciated my nuance about MIMO versus GIGO, but he said that in England, MIMO stands for "muck-in/muck-out," so I removed the footnote.

My work for MTS led to my being invited to participate in an engineering marketing trip arranged by the U. S. State Department to help sell American products in Eastern Europe. I gave lectures in Moscow, Warsaw, Prague and Bucharest where I met people familiar with some of my work. I found it interesting to visit these countries during the cold war.

Another activity was with Colin Dodds when he was associated with a Scottish Company called Structural Monitoring Limited (SML). They had a contract with British Petroleum (BP) to monitor the structural integrity of BP oil platforms in the North Sea. To do this work, SML developed special spectral analysis programs to analyze random data collected above the water line at different levels and in different directions. These results were then compared with finite element stiffness properties of the BP platforms below the water line to determine if any damage had occurred as a result of severe wave conditions or accidental ship collisions. This technique, when successful, was much cheaper than sending divers down to very deep depths to visually inspect the legs of the platform. Some of my practical statistical error analysis formulas were used to decide how much data to obtain and to assess the validity of computed estimates. To learn how the data were collected, I flew in a helicopter to a BP platform where I had to have a rope tied around my waist in case of a fall. I also had to bring my passport if an evacuation was necessary to a different country than Scotland. I heard later that a similar trip a week before with 10 men aboard crashed in the ocean.

One of my research contracts of special interest was with engineers at Brüel & Kjær (B&K) in Denmark. They asked me in 1985 what to include in a new 2-channel analyzer they were building. I suggested the Hilbert transform because it yields a 90° phase shifted version of an original signal. This Hilbert signal and the original signal can then give the envelope of the original signal, a useful result for correlation applications. B&K agreed and gave me two study contracts to learn: (1) how to compute the Hilbert transform from the Fourier transform; (2) how to apply the result to correlation measurements; and (3) how to obtain good estimates with limited amounts of data. My report from this work was published in a B&K monograph called "The Hilbert Transform and Applications to Correlation Measurements." The cover of this monograph had a picture of David Hilbert with his face gradually rotated through 90°.

More Books with Allan Piersol

I have continued to write books with Allan Piersol who is now an Independent Engineering Consultant. In 1971, as a result of technical advances in the field, we made an extensive revision of our 1966 book to include new developments. Specifically, we covered the use of FFT techniques to compute spectral density functions, there was a broader treatment on statistical errors that occur in computing various estimates from measured random data, and we discussed more recent applications from aerospace and automotive projects. Our Wiley publisher asked us to change the title so they could sell more books. We went along so our 1971 book became the First Edition of *Random Data: Analysis and Measurement Procedures*. The Second Edition of this book was published by Wiley in 1986 and the Third Addition in 2000.[5] The Second and Third Editions of *Random Data* contain new practical analytical and engineering results obtained by others and us from 1971 to 2000.

We wrote a companion Wiley book in 1980 to the 1971 *Random Data* book that we called *Engineering Applications of Correlation and Spectral Analysis*. The main objectives were to answer the following questions: (1) what data should be collected?; (2) what particular functions should be computed?; (3) how should the data be processed to reduce statistical bias and random errors?; and (4) how should computed estimates be interpreted in physical problems? This book contains a summary of practical random error formulas for estimates in single-input/single-output (SISO) linear models that show the importance of coherence functions to determine the validity of various computed quantities. The Second Edition of this Engineering Applications book was published by Wiley in 1993.[6] Examples in these two application books clarify differences between computed results from engineering practice and expected theoretical results from mathematical models.

Our Random Data and Engineering Applications books have been translated into Russian, Polish, Japanese and Chinese, and have had world sales to date of more than 100,000 copies.

To China and Russia

My 1958 *Random Noise Theory* book and the later books written with Allan Piersol on Random Data Analysis and Engineering Applications were translated into both Russian and Chinese without permission from our publisher. I learned about the Chinese translations in 1979 when my wife and I visited China for the first time. While in Beijing, I was contacted by some engineers from the Chinese Institute of Mechanics who asked me to visit their facility and deliver a lecture. I didn't expect this to happen but was happy to accept. I had no slides with me so I gave the lecture using a blackboard and speaking slowly for an interpreter. After the lecture, they asked me what I thought about people translating books without permission. I replied that if they had translated any of my books, I would be pleased because otherwise many people in China would not be able to read my work. The next morning they came to the hotel where we were staying and gave me copies of the 1958 and 1971 books that had previously been translated into Chinese.

We made a second trip to China in 1985, this time as guests of the Chinese government. I delivered a series of lectures at the Nanjing Aeronautical Institute and afterwards was made an Honorary Professor at a special banquet.

In 1972, I visited Russia for the first time and we went to Moscow, Kiev and Leningrad. In Moscow I met the Russian translator of the 1966 book. He had written to me earlier to write a preface for the unauthorized Russian edition. As a goodwill gesture, he took me to the Russian publishing company called MIR, meaning peace, where the Director gave me a gift of 200 rubles, which they valued at $300. These rubles could not be converted into dollars but were good only for ruble purchases. Unfortunately, anything my wife and I wanted to buy was available only at special foreign currency stores. So we spent the money having meals with the translator and his family including caviar for breakfast.

In 1990, Allan Piersol and I were invited by the Soviet Geophysical Committee of the USSR Academy of Sciences to give a three-day series of educational lectures at Moscow State University. Some 200 engineers from all over Russia paid a fee to participate. Many of the attendees had Russian copies of our books. After the lectures they lined up for us to sign their books making us feel like rock stars.

Nonlinear Work for Shell Oil

A significant turn in my work occurred in 1977 when Allan Piersol and I presented a three-day short course in Amsterdam, Holland that was attended by Dr. Jan Vugts, a senior engineer from Shell Oil Company in The Hague. After hearing about our results dealing with frequency-domain spectral analysis of random data through linear systems, Dr. Vugts wrote to me in Los Angeles and gave me a contract to review his work on time-domain differential equation ways to solve the nonlinear Wave Force Problem. This nonlinear problem is of great concern to oil companies and to ship certification organizations. The problem involves a linear system operation in parallel with a nonlinear system operation of a square-law system with sign. After studying Vugt's work and other papers, I came up with some new ideas on how to tackle this problem in the frequency domain by spectral analysis of a combination of parallel linear, square-law and cubic systems, where the nonlinear systems could be followed or preceded by different linear systems. Input data in this work were assumed to be Gaussian.

I received a second contract from Shell to verify these new ideas by analyzing experimental data to be collected at the Netherlands Ship Model Basin. Allan Piersol supervised the desired experiments and brought the measured data back to Los Angeles. These data were then analyzed using software programs based on my ideas written by John Fitzpatrick, who is now Head of the Mechanical Engineering Department at Trinity College in Dublin, Ireland. The analyzed results were successful in proving the validity of the new spectral ways to solve the Wave Force Problem and were confidential to Shell. It was not until 1986 that Allan and I were allowed to publish a paper in *JSV*, Vol. 106, on the "Decomposition of Wave Forces into Linear and Nonlinear Components."

Nonlinear Work for NCEL

In 1985, Paul Palo, a senior ocean engineer at the Naval Civil Engineering Laboratory (NCEL) in Port Hueneme, CA, issued a RFP for an extensive five-year program to develop new ways to identify and analyze nonlinear system properties from measured ocean engineering data. The challenge of this work was that the subject dynamic systems included amplitude nonlinearities, frequency dependent coefficients and arbitrary spectral and probabilistic excitations. I responded and won the contract. The funding came from both NCEL and the U. S. Office of Naval Research (ONR). During the course of the contract, there was active participation by Paul. I wrote many internal reports for projects of concern to NCEL that extended the scope of the earlier nonlinear work done for Shell.

In 1990, with the encouragement of NCEL and ONR who wanted people to know what they had sponsored, I wrote my first nonlinear book, *Nonlinear System Analysis and Identification from Random Data*.[7] This book showed many examples of zero-memory nonlinear systems that can be detected and identified by computing amplitude probability density functions of input/output data. I wrote for engineers a mathematical discussion on the properties of Volterra nonlinear models of parallel linear, bilinear and trilinear systems that require the computation of multidimensional bispectral and trispectral density functions. Results for simpler square-law and cubic systems were obtained as special cases. I derived some complicated statistical error analysis formulas for nonlinear system estimates.

The last Chapter 7 in this 1990 nonlinear book outlines a general methodology for analyzing SISO nonlinear models consisting of linear systems in parallel with arbitrary nonlinear systems. This is followed by examples on how to solve the Spectral Decomposition Problem and the System Identification Problem for nonlinear Wave Force models and for nonlinear Drift Force models. I developed these ideas in 1993 more fully in Section 13.3.4 of Reference 6 where I recommended a step-by-step practical procedure that should be carried out for linear and nonlinear system identification in different types of SISO nonlinear models.

I was given a continuing sole source five-year contract in 1990 by NCEL to do further work in the nonlinear world. Paul Palo was again the NCEL project manager where he suggested nonlinear ideas. To assist me in verifying the new recommended practical techniques for solving SISO nonlinear models called Direct MI/SO Techniques and Reverse MI/SO Techniques. I gave a subcontract to Robert Coppolino to perform computer simulations using MATLAB®. He was also able to suggest nonlinear ideas from his past experience on modal analysis problems.

Direct and Reverse MISO Techniques are implemented using established procedures and computer programs by changing SISO nonlinear models into equivalent MI/SO linear models. The spectral response properties from each linear and nonlinear path can be evaluated using appropriate ordinary coherence functions. Some of the results obtained by Paul, Bob and me are published in two papers: (1) "A General Identification Technique for Nonlinear Differential Equations of Motion," *Probabilistic Engineering Mechanics*, 1992; and (2) "Identification of Physical Parameters with memory in Nonlinear Systems," *International Journal of Nonlinear Mechanics*, 1995.

As part of this NCEL contract, two separate nonlinear experimental test programs were conducted at the Hydromechanics Laboratory of the U. S. Naval Academy in Annapolis, MD. The first series of tests in June 1994 involved a scale model of a naval frigate that was subjected to beam seas. The next series of tests in July 1995 involved a scale model of a naval barge where the barge was subjected to waves in three different directions. The measured data from both nonlinear test programs were extensively analyzed using the Direct MISO Techniques and the Reverse MISO Techniques to determine the nonlinear system response properties of the frigate and the barge. Results from this work for NCEL proved that these new techniques are accurate practical methods to analyze and identify the dynamic properties of SISO nonlinear physical systems from simulated or from measured random data.

1998 Nonlinear Book

I replaced my 1990 nonlinear book with an expanded nonlinear book in 1998 to include technical advances that took place after 1990 and to reflect a greater awareness of pertinent matters learned from work in the second contract for NCEL. I gave this 1998 nonlinear book a new title, *Nonlinear System Techniques and Applications*.[8] My main reason for writing this book was to acquaint people with important analytical ideas and engineering applications of practical nonlinear system techniques in Chapters 3 to 7 that were not published in the 1990 book. Chapter 3 is based on a keynote lecture that I delivered at the Third International Conference on Stochastic Structural Dynamics held in Puerto Rico in January 1995. Chapter 4 presents a general methodology for analyzing the spectral properties of stationary random data passing through SISO nonlinear models with parallel linear and nonlinear systems. Chapter 5 shows how to determine physical parameters with memory in nonlinear systems. The design and computed results from the two nonlinear experimental test programs conducted in 1994 and 1995 are discussed in detail in Chapters 6 and 7.

Engineering examples in this 1998 nonlinear book show that the Direct and Reverse MISO Techniques can solve SISO nonlinear system problems in many fields because they are valid for random data with arbitrary probability, correlation and spectral properties. These procedures give the correct linear and nonlinear system response properties in SISO nonlinear models with parallel linear and nonlinear paths, unlike current linear modal analysis work that gives the wrong linear system results and fails to identify the parallel nonlinear system results.

In the 1990 nonlinear book, I gave a prominent role to complicated multidimensional methods to analyze and identify bilinear and trilinear systems in Volterra models. These Volterra ideas and techniques are moved to the final Chapters 8 and 9 in the 1998 nonlinear book because the results are difficult to compute and interpret, and are restricted to Gaussian input data. I do not recommend the use of Volterra techniques except as a last resort. Instead, I strongly recommend that engineers use the Direct MI/SO Techniques and the Reverse MISO Techniques in the 1998 nonlinear book whenever suitable.

Future Work Needed

I would like to finish this article with a few suggestions on some future needed work in the field. I hope that graduate students and research engineers will fill these needs:

1. Current linear modal analysis procedures for SISO models give erroneous results when nonlinear effects in measured response data are ignored. These linear procedures produce the wrong linear system modal properties and do not identify any parallel nonlinear system modal properties. The correct results for both the linear and nonlinear operations can now be obtained by using the SISO nonlinear system modeling ideas and techniques in my 1998 nonlinear book. There is a need here and an opportunity to extend ways of performing modal analysis work by converting Direct MISO and Reverse MISO Techniques into "off-the-shelf/easy-to-use" software packages in MATLAB®.

2. Another area that requires future work is the development of practical analytical procedures to identify and analyze the linear and nonlinear system properties in nonlinear MIMO models, where each input record can pass through different parallel linear and nonlinear systems. Solutions for this problem are straightforward when the input records are mutually uncorrelated, for then a nonlinear MIMO model can be replaced by a collection of nonlinear SISO models. The problem is difficult and unsolved when the input records are correlated.

3. There is a need for more analysis of probability density functions of measured response records from each linear and nonlinear path in SISO nonlinear models and in MIMO nonlinear models. These amplitude-domain probability results should be combined with frequency-domain spectral results to give a more complete understanding of what happens when stationary random data passes through each linear and nonlinear path.

4. Techniques are known on ways to analyze the properties of nonstationary random data passing through linear systems, as discussed in Chapter 12 of References 5 and 6. New methods are needed to analyze nonstationary random data passing through nonlinear systems.

5. Another problem area that is open for future practical developments is the prediction of structural failures when stationary random data passes through nonlinear systems.

Conclusion

Since 1999 I have given a one-day short course on "Nonlinear System Techniques and Applications" at the annual meeting of IMAC in the United States. I want to thank Al Wicks for inviting me to present these IMAC courses that are organized by SEM, the Society for Experimental Mechanics. I enjoy discussing my mathematical ideas and practical procedures for analyzing random data with attending engineers who are enthusiastic about their work.

On April 24-25, 2005, at the IOMAC meeting in Copenhagen, I presented with Prof. Kjell Ahlin from the Blekinge Institute of Technology in Sweden, a new two-day short course on "Identification and Simulation of Nonlinear Systems." For this course, I discussed analytical and engineering material from my 1998 nonlinear book. Prof. Ahlin showed many practical examples using MATLAB® to illustrate how to implement the methods described in my book. I want to thank Anders Brandt from Axiom EduTech in Sweden and the IOMAC committee for organizing this new course. I hope that Kjell and I have opportunities to repeat this course at other places in the future.

At this stage of my career, my greatest joy is to pass on to engineers some of the things I have learned from my work. People learn from successful results as well as from failures and I have had both. If you never want anyone to know that you have made mistakes, or if you never want to open yourself up to criticism, then you should never publish anything. But that's not the point. It is important to take risks, to do the analytical work and the practical work, and to share it with others.

Finally, I want to conclude by speaking to some of the younger engineers. The nonlinear world is expanding. There are opportunities for you to engage in exciting work using new nonlinear system techniques that have never been tried on existing modal analysis projects. You are bound to obtain useful results not known before where you can make original and important contributions.

References

1. Bendat, J. S., *Principles and Applications of Random Noise Theory*, Wiley, New York, 1958.
2. Bendat, J. S., Enochson, L. D., Klein, G. H., and Piersol, A. G., "The Application of Statistics to the Flight Vehicle Vibration Problem," ASD TR 61-123, Aeronautical Systems Division, AFSC, Wright-Patterson AFB, Ohio, 1961 (AD 271 913).
3. Bendat, J. S., Enochson, L. D., Klein, G. H., and Piersol, A. G., "Advanced Concepts of Stochastic Processes and Statistics For Flight Vehicle Vibration Estimation and Measurement," ASD TDR 62-973, Aeronautical Systems Division, AFSC, Wright-Patterson AFB, Ohio, 1962 (AD 297 031).
4. Bendat, J. S., and Piersol, A. G., *Measurement and Analysis of Random Data*, Wiley, New York, 1966.
5. Bendat, J. S., and Piersol, A. G., *Random data: Analysis and Measurement Procedures*, 3rd ed., Wiley, New York, 2000. (The 1st ed. was published in 1971, the 2nd ed. in 1986.)
6. Bendat, J. S., and Piersol, A. G., *Engineering Applications of Correlation and Spectral Analysis*, 2nd ed., Wiley, New York, 1993. (The 1st ed. was published in 1980.)
7. Bendat, J. S., *Nonlinear System Analysis and Identification from Random Data*, Wiley, New York, 1990.
8. Bendat, J. S., *Nonlinear System Techniques and Applications*, Wiley, New York, 1998. SV

The author may be reached at: jsbendat@yahoo.com.