FEA Model Updating Using SDM

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In recent years, a variety of numerical approaches have been proposed for modifying a finite-element-analysis (FEA) model so that its modal parameters more closely match those obtained from experiment. Such factors as real-world boundary conditions and joint stiffness are often difficult to model correctly in an FEA model, and damping is usually left out of the model all together. A method called structural dynamics modification (SDM) was commercialized back in the 1980s as a method for predicting the effects of structural modifications on the modes of a structure. In its more recent implementation, it utilizes the same finite elements to model structural modifications as those used in FEA modeling. SDM is a fast and efficient algorithm that can be used for updating FEA models using experimental results. In this article, we show in several example cases how SDM can be used together with a search procedure to yield a list of the "10 Best" FEA model changes that cause its modes to more closely match a set of experimental modes. Some FEA model changes are always more physically attainable than others, and by providing a list of the 10 best solutions instead of just one solution, a realistic model updating solution can be chosen.

Today, most companies that manufacture mechanical products, or products with mechanical parts in them, are relying more and more on computer modeling and simulation, called finite-element analysis (FEA) to develop products more quickly. In the automobile industry, for example, most companies are using FEA modeling heavily and simulation tools to help bring new car models to market in less time to gain a competitive edge.

FEA models are usually built in the early stages of product development to get a preliminary understanding of the static and dynamic behavior of the mechanical structures involved in the design. FEA models have been used since the 1950s for performing static-load analyses of structures. Static loads are applied to the model to locate the areas of high stress and strain, where the structural material is most likely to fail.

More recently, FEA models are being used to simulate the dynamic responses of structures under a variety of operating conditions. Dynamic loads can often exceed static loads by orders of magnitude, causing unacceptable levels of noise and vibration and perhaps unexpected structural failures.

Before using an FEA model for simulation work, it should be correlated with experimental data to ensure that it models the dynamics of the real structure. If that's not the case, then it must be updated so that its dynamic responses more closely match the real structure dynamics.

Experimental modal analysis (EMA), also called *modal testing* or a *modal survey*, is performed on a real structure to characterize its dynamic behavior in terms of its modes of vibration. Each mode is defined by its modal *frequency*, modal *damping*, and a *mode shape*.

An FEA model also provides the modes of vibration of the structure. FEA is analytical (using a computer model), and EMA is experimental (requiring the testing of a real structure). Modes are the common ground by which these two engineering activities are compared for accuracy.

If both an EMA and FEA are done correctly, then both should yield the same modes of vibration. In practice, however, this rarely occurs, even for the simplest of structures. Since EMA produces a set of modes for a real structure, these modes can be used for updating an FEA model so that its modes more closely match the modes of the real structure.

In recent years, a variety of numerical approaches have been proposed for modifying an FEA model so that its modal parameters more closely match those obtained from experiment. This is called *FEA model updating*.

Advantages of EMA and FEA. Fortunately, EMA and FEA are complementary and each has advantages over the other. EMA can accurately measure the modal frequency and damping of the modes of a real structure. But for practical reasons, EMA mode shapes typically have far fewer DOFs (a degree of freedom is motion at a point in a direction) than FEA mode shapes. FEA mode shapes also contain rotational DOFs, which are usually not measured experimentally.

Even though FEA mode shapes may have many thousands of DOFs, their associated modal frequencies are usually less accurate than experimental frequencies. In addition, modal damping is typically not modeled at all but can always be obtained experimentally.

To summarize:

- EMA is good for obtaining accurate modal frequency and damping.
- FEA is good for obtaining mode shapes with thousands of DOFs, including rotational DOFs.

Modal Model. Although mode shapes are eigenvectors and have no unique values, a set of properly scaled mode shapes preserves the mass (inertia), stiffness (elastic) and optionally the damping properties of a structure. This set of modes is called a *modal model*.

Modes are solutions to the homogeneous equations of motion for a structure:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = 0 \qquad (1)$$

Equation 1 is a set of **n** simultaneous, second-order, linear-differential equations in the time domain, where **x(t)** is the displacement vector, and the dots above **x(t)** denote differentiation with respect to time. **M**, **C** and **K** are the (**n** by **n**) real symmetric mass, damping and stiffness matrices respectively.

Orthogonality. Equation 1 is a *force balance* between the internal forces within a structure after all external forces have been removed, but it still undergoes resonant vibration. If the damping forces, represented by $\mathbf{C}\dot{\mathbf{x}}(t)$, are assumed to be *insignificant* compared to the inertia, $\mathbf{M}\ddot{\mathbf{x}}(t)$, and stiffness, **Kx(t)**, forces (or if **C** is assumed to be proportional to the mass **M** and stiffness **K)**, then the mode shapes are calculated in a manner that *simultaneously diagonalizes* both the mass and the stiffness matrices. This is the so-called *orthogonality* property.

When the mass matrix is postmultiplied by the mode shape matrix and premultiplied by its transpose matrix, the result is a diagonal matrix:

$$[\phi]^{\mathbf{t}}[\mathbf{M}][\phi] = \left\lfloor \ddots \hat{\mathbf{m}} \ddots \right\rfloor \qquad (2)$$

where:

[M] = (**n** by **n**) mass matrix
$[\phi]^{\mathbf{t}}$ = [{u_1} {u_2}...{u_m}] = (**n** by **m**).mode shape matrix
t = the transpose
m = number of modes in the model

This diagonal matrix is called the *modal mass matrix*. Modal masses, like mode shapes, are *arbitrary in value*. One of the common ways of scaling mode shapes is so that the modal masses are one (unity). This is called *unit modal mass* (UMM) scaling. The modal mass matrix then becomes an *identity matrix*, with diagonal elements equal to one and zeros elsewhere.

Based on the paper "FEA Model Updating Using SDM," presented at IMAC XXV, the 25th International Modal Analysis Conference, Orlando, FL, February 19-22, 2007.

When the mode shapes are scaled to UMM, the orthogonality property for the stiffness matrix becomes.

$$[\phi]^t[\mathbf{K}][\phi] = \lfloor \ddots \Omega^2 \ddots \rfloor \tag{3}$$

where $\lfloor \ddots \Omega^2 \ddots \rfloor$ = (**m** by **m**) modal stiffness matrix
Each diagonal term in the modal stiffness matrix is the *undamped natural frequency* squared of a mode.

Computing the Modes of an FEA Model. An FEA dynamic model is essentially a set of differential equations that describes the dynamic behavior of a mechanical structure. An FEA model will often contain thousands, sometimes millions, of differential equations. Each equation describes motion for a single DOF. Consequently, the mass and stiffness matrices of an FEA model are typically very large.

Modes of vibration are computed for and FEA model by solving a so-called eigensolution problem. That is, modal frequencies are computed as eigenvalues, and mode shapes are computed as eigenvectors of its differential equations of motion.

Very large numbers of equations are usually required to obtain sufficient accuracy with an FEA model. This means that the mass and stiffness matrices are very large. Therefore, solving for an FEA eigensolution requires a large computer with lots of memory.

Structural Dynamics Modification (SDM). The SDM method, also called eigenvalue modification or diakoptics, was originally developed as a way to more quickly calculate the new modes of an FEA model[1-3] when localized changes were made to it.

SDM was first commercialized in 1980 as a method for predicting the effects of structural modifications (changes in its mass, stiffness, and damping properties) on the modes of a structure. Structural Measurement Systems, Inc. (SMS), a Santa Clara, CA, engineering software company, was the first company to commercialize the use of the SDM method for use with experimental modal data.[4]

Since FEA models typically have no damping, for FEA model updating, the damping term in the equations of motion (Eq.1) will be ignored. With only mass and stiffness modifications, the equations of motion become:

$$[\mathbf{M} + \Delta\mathbf{M}]\ddot{\mathbf{x}}(t) + [\mathrm{K} + \Delta\mathbf{K}]\mathbf{x}(t) = 0 \tag{4}$$

where:
$[\Delta\mathbf{M}]$ = mass modification matrix (**n** by **n**)
$[\Delta\mathbf{K}]$ = stiffness modification matrix (**n** by **n**)
While Equation 4 is a set of time domain differential equations, its eigenvalues (modal frequencies) and eigenvectors (mode shapes) are found as solutions to the equivalent set of algebraic equations in the frequency domain:

$$\lfloor -[\mathbf{M} + \Delta\mathbf{M}]\omega^2 + \mathbf{K} + \Delta\mathbf{K} \rfloor \mathbf{X}(\omega) = 0 \tag{5}$$

where:
$\mathbf{X}(\omega)$ = Fourier transform of displacement
ω = frequency variable
A typical FEA model will easily create thousands of equations (Eq. 5), and solving them for the new modes due to mass and stiffness modifications is still very time consuming.

However, the SDM method transforms Equation 5 into the *modal domain* by taking advantage of the orthogonality property in Equations 2 and 3 of the mode shapes of the unmodified structure. Using orthogonality, Equation 5 becomes:

$$\lfloor -\hat{\mathbf{M}}\omega^2 + \hat{\mathbf{K}} \rfloor \mathbf{Y}(\omega) = 0 \tag{6}$$

where:

$$\hat{\mathbf{M}} = \lceil \ddots \mathbf{I} \ddots \rceil + [\phi]^t[\Delta\mathbf{M}][\phi]$$

$$\hat{\mathbf{K}} = \lfloor \ddots \Omega^2 \ddots \rfloor + [\phi]^t[\Delta\mathbf{K}][\phi]$$

SDM solves for the eigenvalues of Equation 6. This equation contains (**m** by **m**) matrices instead of (**n** by **n**) matrices as in Equation 5, and **m** (the number of modes) is usually much smaller than **n** (the number of physical DOFs). Therefore, literally thousands of SDM solutions to Equation 6 can be found in the same time that it takes to calculate one solution to Equation 5.

Computing Modes Using SDM. While an FEA eigensolution requires very large matrices with thousands to millions of DOFs, SDM typically solves for an eigensolution using matrices with less than 100 DOFs. Since its eigensolution problem size is much smaller, SDM can solve for the modes due to *thousands* of potential mass and stiffness modifications in the same time that it takes to solve for *one* FEA eigensolution. Furthermore, SDM can be implemented in software running on a desktop or laptop PC with the two advantages of computational speed and the need for less computer memory. This makes SDM ideal as a practical tool for FEA model updating.

FEA Model Updating Method

Our new FEA model updating method (called SDM targeted model updating) uses the SDM method along with a search procedure to yield a list of the "10 Best" FEA model updates that cause its modes to more closely match a set of experimental modes. Some FEA model changes are more physically attainable than others, so by providing a list of the 10 Best solutions instead of just one, a more realistic model updating solution can be chosen from the list.

Changes to Finite-Element Properties. It is usually too difficult to make changes directly to components of the mass and stiffness matrices, and more importantly, those changes may correspond to structural changes that are not physically attainable. A more practical approach is to change the physical properties of the finite elements themselves in the FEA model and translate those changes into mass and stiffness changes.

Typical finite-element property changes that are physically attainable are:
- Point translational and rotational *masses*
- Translational and rotational spring *stiffnesses*
- Translational and rotational *damping*
- Rod-element, cross-sectional *areas*
- Beam-element, cross-sectional *areas and inertias*
- Plate-element *thicknesses*
- Rod-, beam-, plate- or solid-element *elasticity*, *Poissons ratio*, and *density* material properties

A typical FEA model may contain several different kinds of elements, each having the above properties. Spring elements, plate elements, and solid brick elements are used in the examples that follow.

Cost Function. *FEA model updating is concerned with changing the physical properties of the finite elements of an FEA model so that its modes more closely match a set of experimental modes.*

To be closely matched, either the modal frequencies and/or the mode shapes of the updated FEA model should be as 'close' as possible to the experimental modal parameters. Therefore, a numerical measure of the 'closeness' of modal parameters is required.

The following cost function quantifies errors in both modal frequencies and mode shapes:

$$\text{Cost} = \sum_{k=1}^{m} \frac{\|\Omega_A(k) - \Omega_E(k)\|}{(\Omega_E(k) * MAC(k))} \tag{7}$$

where:
$\Omega_A(k)$ = analytical modal frequency for mode (k)
$\Omega_E(k)$ = experimental modal frequency for mode (k)
$MAC(k)$ = modal assurance criterion between analytical and experimental mode shapes for mode (k)
The modal assurance criterion (MAC) is essentially a normalized *dot* or *scalar product* between a pair of mode shapes. Its values range between *1* (meaning that the two shapes are alike) and *0* (meaning that they are different).

The 10 Best Solutions. To find the 10 Best solutions, we use an exhaustive search using a prescribed number of steps between prescribed lower and upper bounds for each physical parameter to be updated in the FEA model. The 10 solutions that have the 10 lowest cost function values are saved. This approach has several advantages:
- The speed of the SDM algorithm allows an exhaustive search of the entire solution space. That is, all combinations of parameter values stepped between the lower and upper limits of each parameter are evaluated.
- The exhaustive search finds the solution with the true minimum cost, thus avoiding the potential problem of getting 'stuck' at

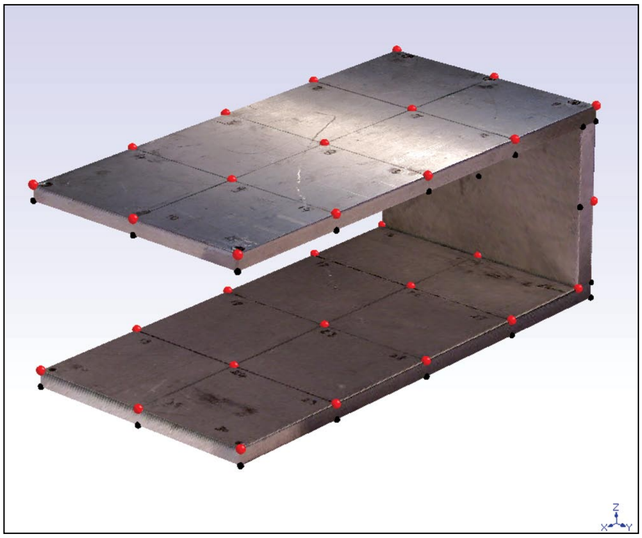

Figure 1. Beam structure showing 33 test points.

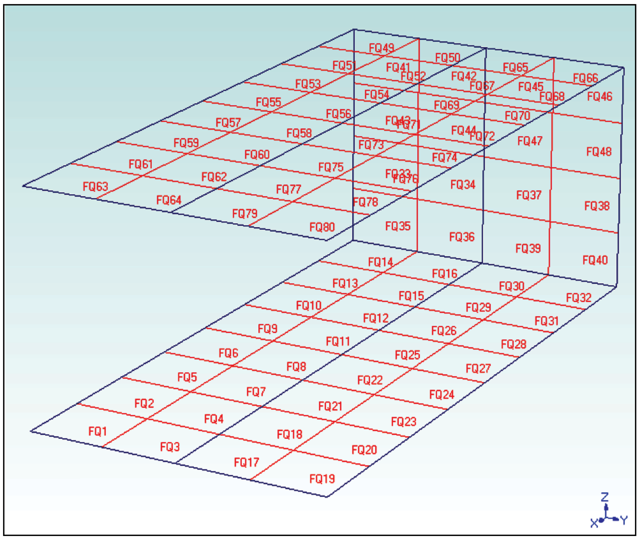

Figure 2. FEA model with 80 quad plate elements.

local minimum values of the cost function.
- The 10 Best solutions provide a choice of modifications, some of which may be more physically attainable than others.
- The 10 Best solutions show the sensitivity of the structure to different potential modifications.

This 10-Best procedure does not require eigensolutions of the original FEA equations. Therefore, it can be implemented on a desktop or laptop PC using the FEA model, its analytical mode shapes and a set of experimental mode shapes.

Example 1 – Updating Plate Thicknesses

In this example, the experimental modes of the beam structure shown in Figure 1 will be used to update the thickness of some of the plate elements in the FEA model shown in Figure 2. The beam was constructed using three, 3/8-inch-thick aluminum plates fastened together with cap screws. The overall dimensions of the structure are 12 in. long by 6 in. wide by 4.5 in. high.

The experimental modes were obtained from a set of 99 Frequency Response Functions (FRFs) that were acquired during an impact test of the beam structure. During the test, the structure was impacted at the same DOF (a corner of the top plate), and a roving tri-axial accelerometer was used to measure the beam's 3-D response at 33 points. The resulting experimental mode shapes had three DOFs per point, for a total of 99 DOFs each.

An FEA model of the beam structure was built using 80 quad-plate elements, as shown in Figure 2. The plate elements had the following properties:
- Thickness = 0.375 in
- Elasticity = 1×10^7 lbf/in^2
- Poissons ratio = 0.33
- Density = 0.101 lbm/in^3

The FEA model was solved for its first 20 (lowest frequency) modes. The FEA mode shapes had three translational and three rotational DOFs at 105 points, for a total on 640 DOFs each.

FEA Versus EMA Shapes. Ten FEA mode shapes matched with 10 of the experimental mode shapes. This was verified by animated display of the mode shapes and by their MAC values. Only translational DOFs of the analytical shapes at the same 33 points as the experimental shapes were used for the MAC calculations. Table 1 lists the analytical and experimental modal frequencies and MAC values between the paired shapes.

FEA Model Updating Results. Table 1 clearly shows that the analytical mode shapes match the experimental mode shapes quite well (indicated by MAC values above 0.80). However, the FEA model is not as stiff as the real structure, since each FEA frequency is less than its corresponding EMA frequency. In this model updating example, thickness values of the elements on the back (vertical) plate were allowed to vary in an attempt to make the FEA modes more closely match the EMA modes. The search for the 10 Best solutions was done over a range (0.2 to 0.7 in.) using 50 different thickness values on either side of the original thickness (0.375 in.).

The 10 Best thicknesses for updating the back plate are shown in Table 2. The 10 Best cost function values indicate that all of the 10 best solutions yield similar overall errors between the modal parameters. There is only a *1% increase* in the cost between Solution 1 and Solution 10.

This small change indicates that the cost function 'surface' is *very flat* in the region of the optimum solution. With a flat cost function like this, it would be difficult to find the optimum solution using derivatives of the cost function (variational calculus) as part of a search method.

Table 3 contains the modal properties of the beam structure after the back plate thickness was changed to 0.421 inches. There is a clear improvement in the FEA modal frequencies, and the MAC values indicate a negligible change in the mode shapes.

Example 2 – Updating Boundary Conditions

One of the most challenging problems in FEA modeling is constraining the model with boundary conditions that match real-

Table 1. Mode shapes before model updating.

Mode	FEA Frequency, Hz	EMA Frequency, Hz	MAC
1	149	165	0.957
2	211	225	0.963
3	311	348	0.948
4	417	460	0.925
5	451	494	0.950
6	590	635	0.935
7	1000	1110	0.902
8	1100	1210	0.892
9	1180	1322	0.848
10	1400	1560	0.830

Table 2. 10 Best solutions.

Solution	Back Plate Thickness, in.	Cost Function
1	0.421	0.819
2	0.427	0.821
3	0.433	0.822
4	0.440	0.823
5	0.446	0.824
6	0.453	0.825
7	0.459	0.826
8	0.466	0.827
9	0.472	0.827
10	0.479	0.828

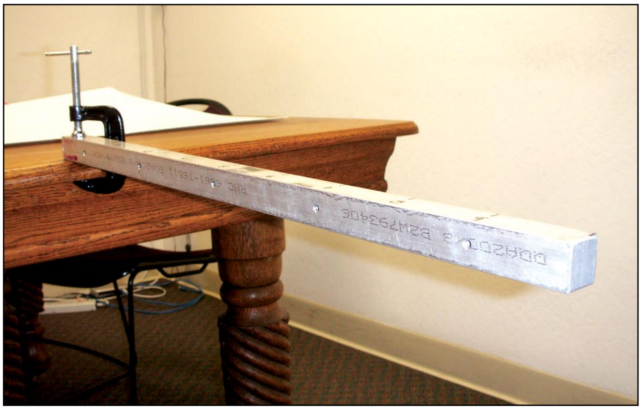

Figure 3. Aluminum bar clamped to table top.

world boundary conditions. In this example, we update an FEA model of a cantilever beam so that its FEA modes more closely match its EMA modes.

The aluminum beam shown in Figure 3 is made from 1-in.-square aluminum bar stock and is 25 in. long. To approximate a cantilever beam, the aluminum bar was clamped to a table top using a C clamp.

It is easy to convert an FEA model of a free-free beam into a cantilever beam, simply by *rigidly constraining* one of its ends. In the real world, however, there is no such thing as a rigid constraint; and certainly not in this case, where a C clamp was used to constrain one end of the beam.

FEA Cantilever Beam Model. First an FEA model of the cantilever beam was built using 20 brick elements with the following material properties for aluminum (see Figure 4a):

- Elasticity = 1×10^7 lbf/in^2
- Poissons ratio = 0.33
- Density = 0.101 lbm/in^3

To model the attachment of the beam to the table, several springs were attached between the beam and ground (fixed points) in the vertical direction (Z-axis) and axial direction (X-axis), as shown in Figure 4b. These springs were given nominal values of 100,000 lbf/in. to simulate the stiffness of the C clamp. During model updating, these stiffnesses were allowed to vary to obtain a better match between the analytical and experimental modal frequencies and mode shapes.

The first seven vertical modes of the FEA beam model are listed in Table 4. The frequencies of the first six EMA modes obtained by impact testing the cantilever beam are also shown in Table 4, along with the MAC values between the FEA and EMA shape pairs.

Table 5 contains the model updating results. It shows that both the modal frequencies and shapes are more closely matched following model updating. Solution 1 of the 10 Best solutions was:
S1 = 10 lbf/in
S2 = 1×10^9 lbf/in
S3 = 10 lbf/in
S4 = 10 lbf/in
It had a cost function value of 0.9206. Solution 10 of the 10 Best solutions was:
S1 = 10 lbf/in

Table 3. Shapes after model updating (back plate thickness = 0.421 in.).

Mode	Updated FEA Frequency, Hz	EMA Frequency, Hz	MAC
1	166	165	0.954
2	213	225	0.961
3	314	348	0.948
4	429	460	0.925
5	454	494	0.948
6	593	635	0.932
7	1010	1110	0.901
8	1100	1210	0.891
9	1200	1322	0.851
10	1400	1560	0.829

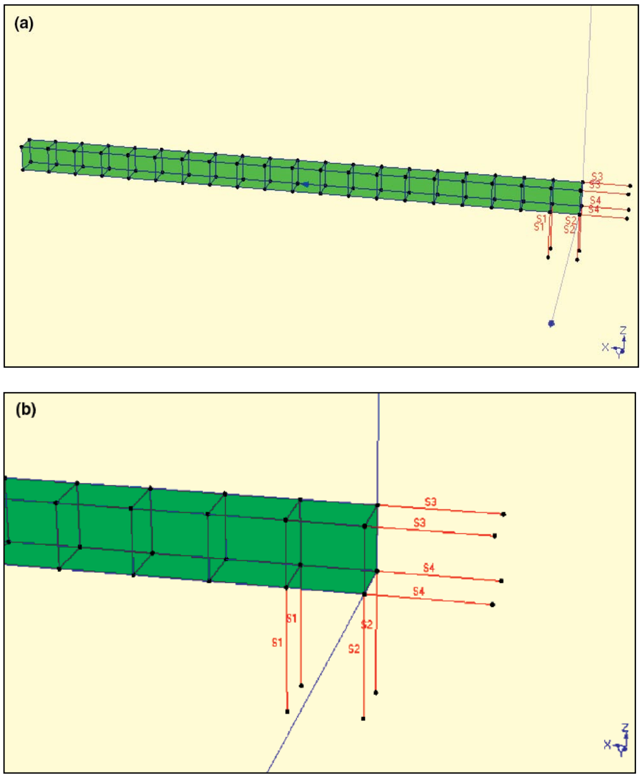

Figure 4. a) Cantilever beam model; b) Close-up view of cantilever beam model.

S2 = 2×10^8 lbf/in
S3 = 1000 lbf/in
S4 = 10 lbf/in

It had a cost function value of 0.9289.

The updated stiffnesses show that the table top and clamp provided plenty of stiffness in the vertical direction but only a negligible amount of torsional stiffness to the beam. In other words, the table top itself was undergoing local bending to be compliant with the much stiffer beam.

Conclusions

A new FEA model updating method based on the SDM (structural dynamics modification) algorithm was introduced. Since this method allows targeting of small areas (such as joint stiffnesses) of a structure for updating, it has been called the SDM targeted model updating, or STMU, method.

The speed of the SDM algorithm allows an exhaustive search for the 10 Best finite-element property changes that minimize

Table 4. Cantilever beam mode shapes before model updating.

Mode	FEA Frequency, Hz	EMA Frequency, Hz	MAC
1	43.8	12.5	0.974
2	292	250	0.963
3	830	800	0.952
4	1620	1580	0.958
5	2600	2610	0.939
6	3700	3800	0.699
7	4700	–	–

Table 5. Cantilever beam shapes after model updating.

Mode	Updated FEA Frequency, Hz	EMA Frequency, Hz	MAC
1	22	12.5	0.993
2	243	250	0.943
3	756	800	0.923
4	1544	1580	0.948
5	2590	2610	0.975
6	3886	3800	0.925

the difference between the modes of an FEA model and a set of experimental modes. Not only is this search procedure fast, intuitive, and easy to use, but it always finds the true optimum solution, together with alternatives from which to pick the best physically attainable solution.

This model updating method was used on two common applications, updating the thicknesses of plate elements of an FEA model, and determining realistic boundary conditions (mounting stiffnesses), for a cantilever beam FEA model. This approach doesn't necessarily require the entire FEA model but only its mode shapes and the elements to be modified. The effects of translational and rotational mass and stiffness changes are easily modeled using the SDM method. In Example 1, only the plate elements of the back plate were required for modal updating. In Example 2, spring and brick elements were used to generate the original FEA shapes, but only the spring elements were required for modal updating.

This tool shows much promise for "closing the gap" between FEA models and EMA results. Model updating not only provides more understanding of how structures behave dynamically, but it also improves the accuracy of FEA models so that they can be reliably used for further modeling and simulation work.

References

1. Hallquist, J. O., "Modification and Synthesis of Large Dynamic Structural Systems," Ph.D. dissertation, Michigan Technological University, Houghton, MI, 1974.
2. Avitabile, P. "Twenty Years of Structural Modification, A Review" *J. of Sound and Vibration*, 37 (1) pp. 14-27, 2003.
3. Allemang, R. J., and Brown, D. L., "A Correlation Coefficient for Modal Vector Analysis," *Proceedings of the International Modal Analysis Conference*, pp.110-116, 1982.
4. Formenti, D., and Welaratna, S., "Structural Dynamics Modification – An Extension to Modal Analysis," SAE Paper No. 811043, Oct. 1980.
5. Schwarz, B., and Richardson, M., "Scaling Mode Shapes Obtained from Operating Data," International Modal Analysis Conf. (IMAC XXIV), February 3-6. 2003.

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