Tricks and Tools for Solving Abnormal Combustion Noise Problems

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Effective Methods for solving combustion noise problems in boilers are reviewed. System modeling and diagnostic testing procedures are presented as well. Part one of this two-part series was presented in the July 2004 issue of S&V.

This is a sequel to my 2004 article[1] titled "How to Solve Abnormal Combustion Noise Problems." I must confess that this title may have created the impression that the article provided all one needs to know for solving these noise problems. Actually, what it did was demonstrate how unacceptably loud noise can be produced by flame oscillations that result in pressure oscillations in the combustion chamber and are radiated as a very loud tonal noise.

While this understanding is the first step for any rational effort to solve such problems when discovered during development of a new high-efficiency, low-NO_X boiler, a timely solution requires an assortment of tools for diagnosing how in that particular boiler the pressure oscillations feed back on the flame via the mixture supply system, causing the flame to oscillate. This is a vicious circle involving fairly intricate interactions of the boiler, burner, and flame.[2] The key for diagnosing these interactions is to represent the vicious circle as a chain of "black boxes" forming a feedback loop[3] that under certain conditions becomes unstable. In my experience, the cause of this instability can best be tracked with the help of a simple computer-based model of the components of the loop that are described here.

Whenever I have been retained in recent years by a boiler manufacturer as a consultant to help solve a problem of combustion oscillation, I have found that my clients have not been very eager to deal with the complexities of feedback loops and modeling. What they really wanted was some magic solution. I am not Harry Potter, but I happen to know three tricks that may seem like magic. To start with, I will describe them and show that they are not magic at all. They all have perfectly logical explanations.

Trick No. 1 – Add Damping

This is a trick that can be demonstrated quite spectacularly. When adding a porous sleeve to the demo rig described in my 2004 article and sliding it up so that the sleeve projects about 1 inch beyond the top of the flame tube, the loud noise will stop abruptly. Sliding it down again will bring the noise back. A folded handkerchief can be used for the sleeve and can then be removed and unfolded to show that there is indeed no magic hidden in that sleeve. The reason why the sleeve kills the noise is that its porosity absorbs some of sound emanating from top of the flame tube. If enough acoustic energy is absorbed, the abnormal noise will stop.

This is in keeping with the so called "Rayleigh criterion" for self-excited thermo-acoustic oscillations. Putnam[2] has expressed Rayleigh's verbal criterion in the following mathematical form:

$$\oint (h \times p)dt > 0 \tag{1}$$

where:
h = fluctuating heat release of an oscillating flame
p = fluctuating pressure in the combustion chamber
and the product of $h \times p$ is integrated over one complete cycle.

The left-hand side of Eq. 1 is a measure of the acoustic energy generated by the flame during one cycle of oscillation. Obviously, no sound can be produced if this is less than zero. Therefore, Eq. 1 is a *necessary* condition for the oscillations that cause the unacceptable noise. It is not a *sufficient* condition, however. The rate at which the flame generates acoustic energy must not only be greater than zero, but it must also be greater than the rate at which acoustic energy is dissipated from the system, partly by radiation and partly by damping. Putmam[2] stresses the importance of damping in his book over and over.

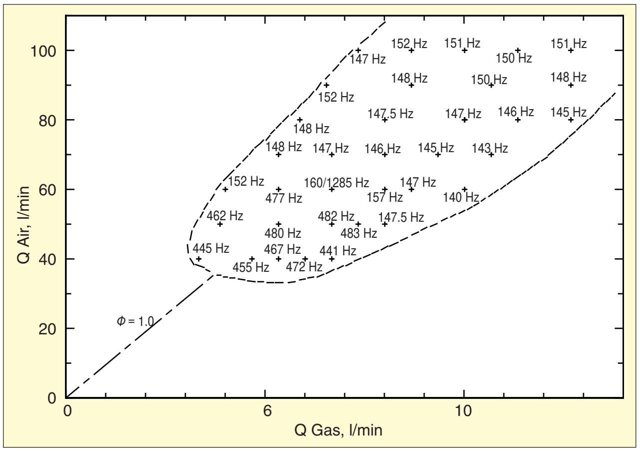

Figure 1. Map of instability range of typical boiler.[4]

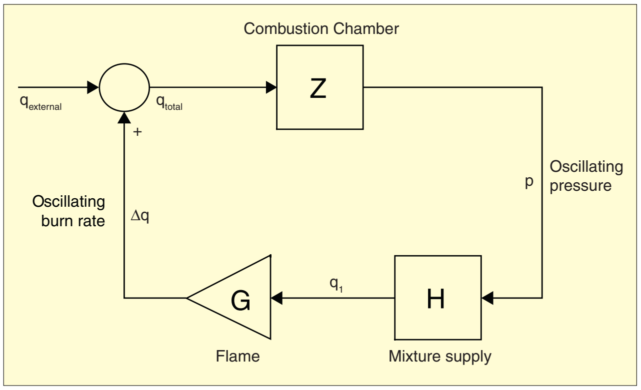

Figure 2. Block diagram of the feedback loop.[3]

Increasing damping is the surest way to solve any combustion oscillation problem, but it is often difficult to find a practical way to add *enough* damping. Anything less than the required amount will have very little effect on the noise level at any given operating condition.

Elsari and Cummings[4] have explored a number of ways for adding damping to a gas-fired boiler. They found that the easiest way was to increase the pressure drop through the burner. This is a brute-force approach, but it can be very useful provided that the blower has enough reserve capacity to handle the flow required at full load. They have also shown that any benefits of adding lesser amounts of damping can best be measured by mapping the range of flow rates and fuel/air ratios over which oscillations did occur, as shown in Figure 1. Any reductions in this range resulting from added damping are a measure of how far that damping has moved the system toward the goal of stability. It is interesting to note in Figure 1 that the frequency of oscillation for low firing rates was much different than for high firing rates. This is not uncommon.

Trick No. 2 – Stretch the Flame

The properties of the flame have a major effect on its tendency to generate acoustic energy. This is also demonstrated easily with the rig described in Reference 1. All one has to do is to slip a 1-inch-long sleeve of tubing over the burner so that it will partly cover the air holes near the bottom of the vertical part of the burner. This will kill the noise. The reason is that this reduces the air/fuel ratio of the flame, which reduces the burning velocity and lengthens the flame.

The length effect is twofold:

- The longer the flame, the longer it takes before the heat release of the flame responds to a change in the mixture flow through the burner ports. This time delay amounts to a phase shift between pressure and heat release. For given amplitudes of h and p on the left-hand side of Eq. 1, the magnitude of this term depends on the cosine of the phase angle between h and p. Therefore, a phase angle of about 0° produces a maximum of acoustic power.
- Longer flames are less sensitive to being acoustically driven, particularly at higher frequencies. This effect has been found very useful for solving many oscillation problems.

Figure 1 shows that, in the test boiler, oscillations ceased when the fuel/air ratio was changed further away from stochiometric, not only toward the rich but also toward the lean side.[4] However, Vanoverberghe reported cases of boilers using lean premix gas burners in which increasing the excess air had the opposite effect.[5]

Changing the fuel/air ratio is the simplest way to stretch the flame, but it is likely that it will be more practical to do this by increasing the port size.[6] Putnam reported that there are many cases where this has solved an oscillation problem.[2]

These various trends are very difficult to sort out by using only the Rayleigh criterion of Eq. 1. Therefore, it is helpful to use a stability criterion for the feedback loop based on the well known Nyquist criterion widely used in the design of electronic control systems.

In the diagram of Figure 2, the symbols G, H, and Z represent complex functions of frequency, which means that they each have a magnitude and a phase, both of which are functions of frequency. The stability criterion for the feedback loop is that instabilities (i.e. self-excited oscillations) will occur only if the product of the magnitudes, $|Z| \times |H| \times |G|$, is larger than 1.0 at a frequency at which the sum of the phase angles is zero. Reference 3 also shows that this condition is essentially an extension of the Raleigh criterion and fully compatible with it.

Richards, *et al.*,[7] have used a feedback loop that is very similar, except that the critical sum of the phase angles is 180°. This results from their using a different sign convention. Either convention is OK as long as one is consistent.

In the model of Figure 2, G stands for the frequency response of the flame commonly called the transfer function, which is discussed later. What complicates the use of the feedback model is that there are two entirely different mechanisms that can drive the flame, each having a different transfer function G.[8] These mechanisms are:

- Modulations of the *velocity* of the mixture flow
- Modulations of the *composition* of the mixture

Of these, mixture velocity modulation usually dominates in heating boilers but, in cases where the frequency of oscillation was very low, I have also encountered the other mechanism that is quite common in gas turbines and in the large process heaters used in petroleum refineries.[9] Stretching the flame will work only if mixture velocity modulation dominates. If the flame is driven by mixture composition modulations, different measures have to be taken to solve the oscillation problem.

So, one has to identify early on which of these mechanisms dominates in a particular problem. A case history is given later showing how a model of the feedback loop can be used to do that and how this identification led to a simple solution.

Trick No. 3 – Drill a Hole

This is an old trick that sometimes works like magic and sometimes does not work at all. Putnam[2] paraphrases an article in the trade literature saying, "to stop oscillations, drill a hole in the front of the furnace. If that doesn't work, drill two holes." He then describes how in a simple system changing the size of the hole and its location relative to the pressure field caused the oscillations to stop. He also found that the ratio of the hole diameter to the wall thickness and size of the duct in which the hole was drilled were important variables.

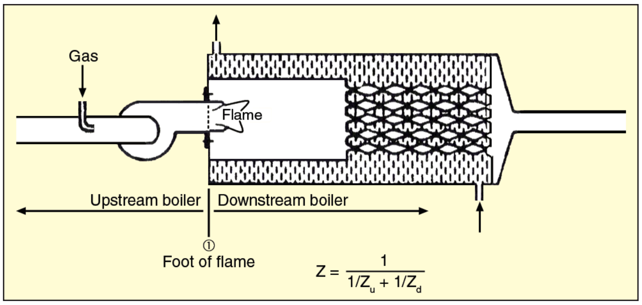

Figure 3. Boiler schematic.

Today every engineer has a computer on his or her desk. So optimizing the effect of a hole does not need elaborate tests. Programs are readily available for solving the matrix equations[10] that describe the acoustic properties of an assembly of simple elements that are a reasonable representation of a boiler with various lengths of flue pipes and various configurations of the air and mixture supply system, including holes. Simple models do have to use simplifying assumptions. Therefore, the validity of those assumptions has to be verified experimentally. ASHRAE TC 6.10 has proposed a research project to do just that.[11] Recently, the authors have jointly produced a simple model that we plan to contribute for use in the proposed project.

The Toolbox

The task of solving combustion oscillation problems has something in common with the task of a fire department: In both cases one has to be prepared to respond quickly and with an assortment of tools for handling the different aspects of the task. Tools useful for solving oscillation problems in heating boilers are:

1. A two-channel FFT analyzer capable of displaying not only spectra but also waveforms and transfer functions. Since oscillations in some boilers occur at a very low frequency, the low-frequency cutoff of the instrumentation needs to be considered.
2. An assortment of small, rugged microphones and pressure transducers, some with probe tubes for exploring the pressure field inside the boiler.
3. An acoustic pulse generator for exciting resonances within the boiler.
4. A simple one-dimensional model for calculating the transfer functions Z and H of the boiler and the mixture supply system shown in Figure 2.
5. An acoustic impedance head for measuring the transfer functions Z and H under cold no-flow conditions to validate the model.
6. A Langmuir probe for exploring the spatial extent of the flame oscillations may also be useful.

In this article, there is not enough space to discuss all of these tools. Therefore, we will concentrate on Item 4. Details on the other items may be obtained by contacting me at peterbaade@aol.com.

Basic Features of the Model

The boiler shown in Figure 3 is called a forced-draft fire tube boiler in ASHRAE terminology. The heat exchanger can be vertical or horizontal, and the blower may be above, below, or to the side of the heat exchanger. There are always some elbows in the flue pipe and in the air supply system. These details are irrelevant for modeling. The important feature is that the gases flow in a closed path from inlet to outlet and that this path consists of a series of segments that have different lengths and cross sections.

The model must allow calculation of the acoustic impedance Z that describes the properties of the top part of the feedback loop shown in Figure 2 and also calculation of the transfer function H that is involved in the bottom part. To calculate Z, it is necessary to calculate separately the impedance of the downstream path Z_d and that of the upstream path Z_u each starting at the flame, and then combining the two as shown by the equation in Figure 3.

The first segment of the upstream path is the acoustic resistance of the burner ports. According to Doebelin,[12] this acoustic resistance is related to the pressure drop across the burner. This resistance is a very important source of damping and a key factor in the stability of all gas- and oil-fired boilers. For a given burner, the pressure drop depends on the flow rate, which varies with the load. Therefore, the damping is greater at full load than at partial load.

Next comes a string of segments, starting with the volume inside the burner can and progressing to the end of the air vent pipe. Each of these segments is represented by a transmission matrix. The matrix equations for all of these segments are identical. The only differences are in the values used for the different temperatures, lengths, and cross sections of the elements.

This similarity means that one needs to type the equation for the transmission matrix only once and then use a text editor to copy and paste it into the script that MATLAB® calls an 'm-file.' At the end of this string of transmission matrixes is a different one that represents sound radiation from the end of the air vent pipe, another significant source of damping. For a closed system such as that shown in Figure 3, the transfer function H is simply the negative reciprocal of Z_u.

The modeling procedure for the downstream path is almost the same, again starting at the flame. The first segment is the combustion chamber surrounding the flame. It connects to the tube bundle, which can be modeled as a single tube of the same total open area. This can be broken down lengthwise into about a dozen segments having consecutively lower temperatures. This is followed by a collecting chamber and then by the flue pipe, the length of which varies from one installation to the other.

The type of boiler shown schematically in Figure 3 typically has relatively rigid walls. Rigid walls make a boiler quite easy to model, because one does not need to deal with vibrations of flexible walls. Procedures for boilers with more flexible walls and different heat exchange surfaces are to be developed in the proposed ASHRAE Project.[11]

Models for Estimating Flame Transfer Functions

The transfer functions H and Z that can be calculated with our model apply to those parts of the boiler that are under control of the boiler manufacturer. Plots of the magnitude and phase of these functions alone may provide valuable insights for solving a combustion oscillation problem, but to get the full benefit of the model, one also needs information on the transfer function of the flame.

The flame transfer functions shown in Figure 4 were measured with a burner deck consisting of a thin perforated sheet with 2-mm holes, on 4-mm centers. This hole pattern is an approximation of the patterns commonly used in burners for high-efficiency, low-NO_X boilers.

The gain is seen to be pretty flat for frequencies from 0 to 300 Hz and then dropping off rapidly. The plot of the phase is close to a straight line, starting at the origin. Note that the phase is essentially independent of the flow rate, which is quite different from the behavior of flames driven by modulation of the mixture composition.

It may be surprising that the phase angle is independent of the flow rate, but there is a simple explanation: Increasing the flow rate will stretch the flame, but this increase in flame height is proportional to the increase in mixture velocity. So the time delay does not change.

Figure 4 shows a phase lag of about 210° at 400 Hz. The time required for a cycle of oscillation at 400 Hz is 2.5 milliseconds. Since 360° amounts to a full cycle, 210° amounts to a 0.58 cycle. So the time delay at this frequency is about 1.5 milliseconds. This value applies to all frequencies below 400 Hz as well, because the phase plot in Figure 4 is essentially a straight line through the origin. This value of 1.5 milliseconds will be used in the analysis below.Reference 6 shows that the time delay and, therefore, the phase lag are correspondingly larger for holes 3 mm in diameter and that the magnitude drop-off occurs at lower frequencies.

At the present time, burner manufacturers, with one laudable exception,[5] are not equipped to provide flame transfer function data. So one has to rely on estimates based on the literature. Kornilov[14] provides an excellent survey of what has been published so far. He has found that for flames driven by modulation of the mixture velocity, all the published flame transfer function data,

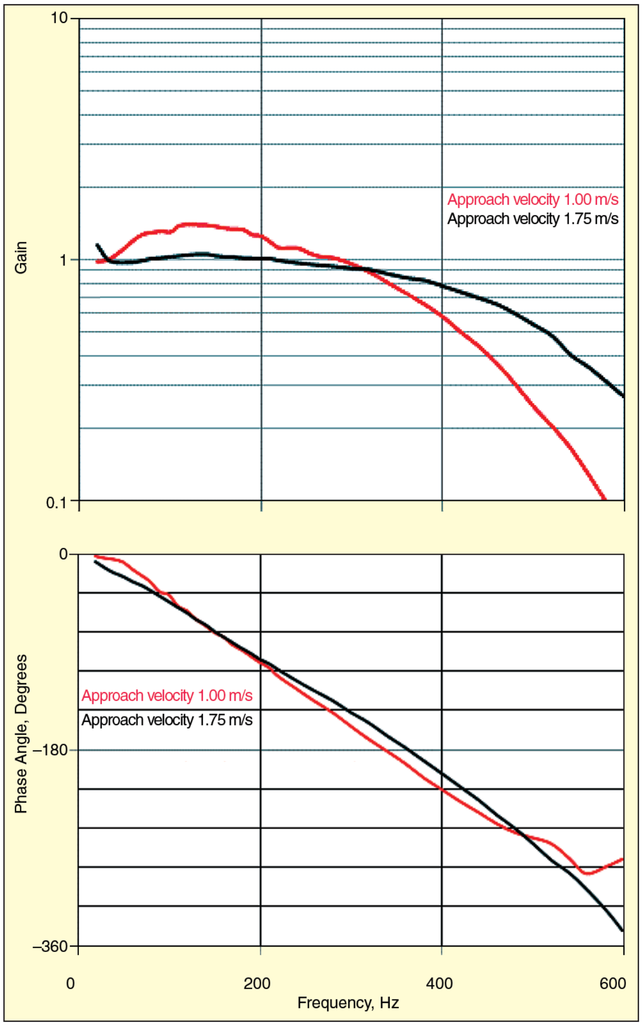

Figure 4. Measured transfer functions for two flow rates of a lean flame driven by modulations of the mixture flow rate.[13]

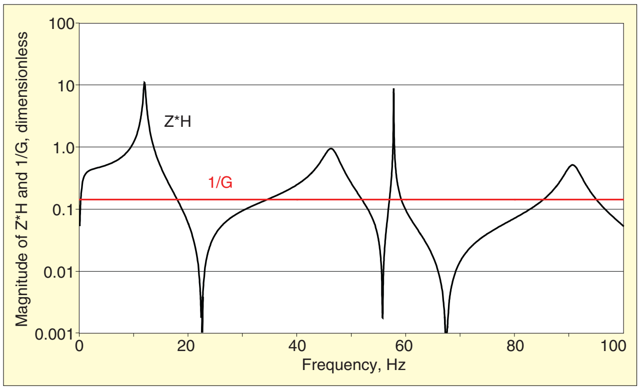

*Figure 5. Estimated magnitude of the product of Z*H and of 1/G.*

as well as those he measured in his own extensive research, show the same trends as the curves in Figure 4.

He has derived an empirical model for such flames, which should be useful solving an oscillation problem if measured data are not available. When using his equations in conjunction with our model, note that the definition of our transfer function G differs from his definition of gain. Therefore, the gain has to be multiplied by a volume expansion factor:

$$|G| = gain * [(\rho_u / \rho_b) - 1] \tag{2}$$

where:
ρ_u = density of the unburned mixture
ρ_b = density of combustion products

Since the density is inversely proportional to the absolute

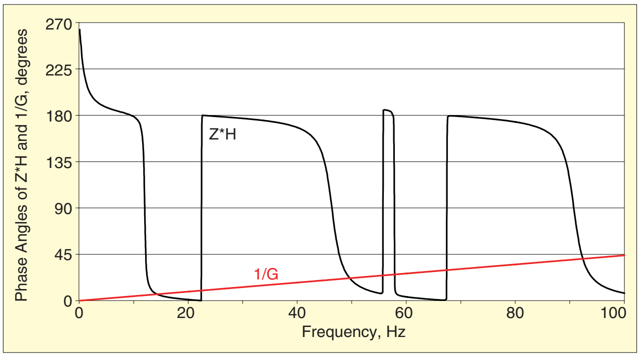

*Figure 6. Estimated phase angle of the product Z*H and of 1/G.*

temperature, the value of the factor in the brackets of Eq. 5 can be estimated to be somewhere between 6 and 8 for most flames. A value of 7.0 will be close enough for the application of the model discussed below.

Note that Kornilov's model does not apply to flames driven by modulations of the mixture composition. The difference between these two driving mechanisms was the key clue in the following case history. Some cases involving flames driven by modulations of the mixture composition are discussed in Reference 9.

Application of Boiler Model to a Puzzling Problem.

Figure 5 shows the estimated magnitude of the product of the transfer function Z and H for a boiler that had an oscillation problem at about 12 Hz, and Figure 6 shows the corresponding phase angle.

To use these plots, one must also consider the transfer function *G* of the flame. The most convenient way[15] to do that is by adding a curve for the magnitude of 1/*G* in Figure 5 and a curve for the phase angle of 1/*G* in Figure 6. This method is based on the fact that the stability criterion stated in connection with Figure 2 can be cast into the following form:

*Oscillations will occur only if the product |Z| x |H| > |1/G| at any frequency at which the phase angle of Z*H is equal to the phase angle of 1/G.*

Note that the phase angle of *Z*H* is the sum of the phase angles of *Z* and *H* and that the phase angle of 1/*G* is equal to the negative of the phase angle of *G*. The curves for 1/*G* in these figures are estimates based on Figure 4.

At the time when this oscillation problem arose, our model did not exist yet. Therefore, the solution had to be found by a combination of "back-of-the-envelope" type of calculations described by Seebold[9] and some diagnostic tests. The calculations led to a plausible hypothesis that was verified by the tests and ultimately led to the solution. This is a process that requires quite a bit of experience in acoustics. Since boiler test engineers typically do not have much training in acoustics, we feel that the use of a computer-based model like ours is essential for them do these calculations and solve oscillation problems in a rational manner.

Figure 5 shows that the magnitude criterion is met at all the peaks of *Z*H* in the frequency range shown. Thus there are multiple frequencies at which oscillations *could* occur, *if* the phase criterion stated above is also met. A logarithmic scale is used for the magnitude of the product *Z*H* to provide adequate detail not only for the highest peak but also for the entire plot. Logarithmic scales are commonly used in acoustics. Each decade in this figure corresponds to 20 decibels.

The black curve in Figure 6 shows the sum of the phase angles of

Appendix A – Development of the Model

In developing our model, we started with the premise that, when one has to solve a problem that has arisen during prototype testing of a new boiler, one needs a model that can be implemented quickly, using only basic dimensional data. This led us to pick a one-dimensional approach that is limited to modeling only plane wave modes. Such models are called "low-order models" because they disregard all higher-order modes. Since the fundamental frequency of boiler oscillations is typically low enough that the half wavelength is much larger that the largest transverse dimension of the boiler, this is not a serious limitation.

Within the class of low-order models one can chose between using impedance, scattering or transmission matrixes. We have opted for the latter, because there is considerable evidence in the literature that transmission matrixes have been found useful for modeling oscillations in gas turbines. Furthermore, we have chosen to use volume velocity as the flow variable.

This results in the following matrix equation for a typical transmission element between stations *i* and *i*+1:

$$\begin{Bmatrix} p \\ q \end{Bmatrix}_i = \begin{bmatrix} \cos kL & j\dfrac{\rho c}{S}\sin kL \\ -j\dfrac{S}{\rho c}\sin kL & \cos kL \end{bmatrix} \begin{Bmatrix} p \\ q \end{Bmatrix}_{i+1} \qquad (3)$$

where:
p = pressure, Pa
q = acoustic volume velocity, m^3/s
L = length of segment, m
S = cross section of segment, m^2
ρ = density of gas in the segment, kg/m^3
c = speed of sound of gas in the segment, m/s
k = wave number, $k = 2\pi f/c$, 1/m
f = frequency, Hz

There are undoubtedly good reasons why some investigators have made different choices such as the use of Riemann invariants, but we leave it to the proposed ASHRAE project to evaluate the relative merits of those other options. This also applies to our choice of MATLAB® for the computing platform that we picked simply because both of us had some experience with that platform. For the same reason we decided to handle all the plotting with Excel. One desirable feature of MATLAB is that the input can be a script called an m-file. Since this is a text file, it can easily be edited to adapt it to any particular boiler.

A schematic of our model is shown in Figure A1. We have not broken the heat exchanger down into several segments, because there are many boilers for which this is not useful. These boilers use finned coils surrounding the flame in a manner that does not lend itself to treatment as a string of segments.

For such boilers, it seems to be best to lump the combustion chamber and the heat exchanger into a single element, the properties of which have to be established by fitting the model to a set of suitable test data. This is one of the things that the proposed ASHRAE project is expected to clarify. Another open question is how to account for wall vibration in a boiler that has thin, flexible walls.

Figure A1 shows that our model does provide for including a hole in the air vent pipe just upstream of the point where the gas is introduced. The calculations discussed here were run with that hole closed, because the case involved did not include a hole.

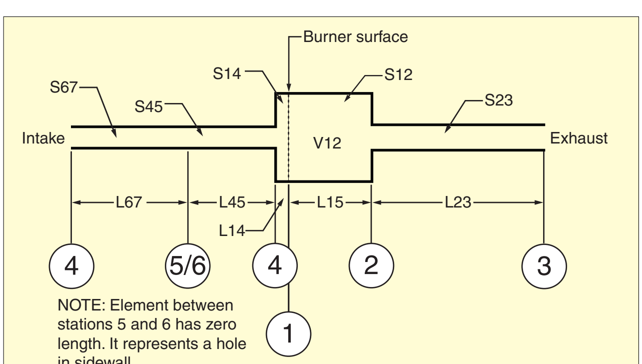

Figure A1. Model schematic.

Appendix B – Further Discussion of Figure 5

Acoustic impedance is the mathematical relation between pressure oscillations and the volume oscillations that cause them. Figure B1 shows a plot of the calculated magnitude of the acoustic impedance in the flame area of the boiler model used here. The volume oscillations involved are oscillations of the excess volume of combustion products produced by the oscillating flame in the boiler.

The peaks in Figure B1 occur at almost the same frequency as those in Figure 5. Thus, it is useful to find out what causes them.

The equation in Figure 3 is:

$$Z = \frac{1}{1/Z_u + 1/Z_d} \quad (4)$$

where:

Z_u = impedance of upstream part of system
Z_d = impedance of downstream part

In Figure B2, the peak in the curve for $1/Z_u$ at about 45 Hz represents the first half-wave resonance in the upstream path; i.e. the frequency at which the wavelength is twice the total length of the supply duct. Since we have approximated the supply system as a duct of uniform cross section for the present purposes, there are additional peaks at all multiples of that frequency. Peaks in the curve for $1/Z_d$ represent half-wavelength resonances in the flue pipe. These peaks are much sharper than the peaks of $1/Z_u$ because the model of the downstream system does not yet consider the damping due to radiation from the surfaces of the boiler and due to the pressure drop across the heat exchanger coils.

Of particular interest in Figure B2 is the intersection of the two curves at about 12 Hz. At that frequency, the two magnitudes are equal, but their phases are almost 180° apart. This means that the two terms in the denominator of Eq. 4 nearly cancel each other out, resulting in the peak in the impedance magnitude at about 12 Hz in Figure B2.

Similar near cancellations occur at the intersections of all the positive sloping parts of the $1/Z_d$ curve with negative sloping parts of the $1/Z_u$ curve. At all these frequencies, Figure B2 has a peak. All these peaks are indications that there is a potential for oscillations at those frequencies if the magnitude and phase criteria are met.

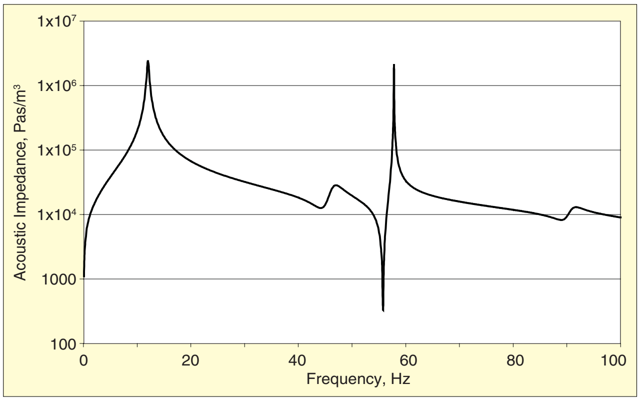

Figure B1. Estimate of magnitude of acoustic impedance at flame location.

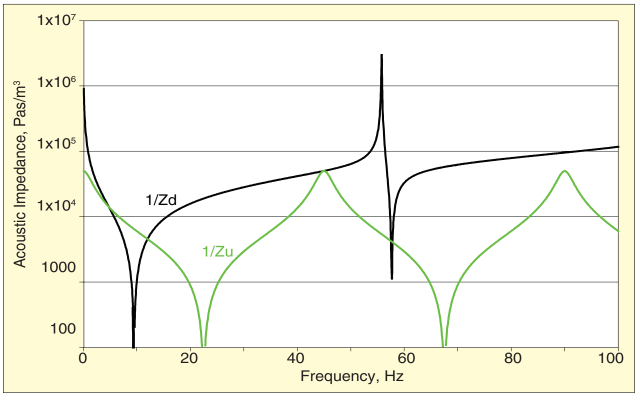

Figure B2. Plots of magnitudes of reciprocals of impedances of upstream and downstream paths in boiler model.

Z and *H*, while the red curve is the negative of the phase angle of *G*. The phase criterion is fulfilled at the intersections of these curves, which occur at about 14, 22, 49, 56, 58, 68, and 92 Hz. Since Figure5 shows that the magnitude criterion is not met at 22, 56, and 68 Hz, the intersections at these frequencies can be disregarded.

Of the remaining four intersections, the one at about 14 Hz is the only one of interest for this particular case, because it is so close to 12 Hz, the frequency at which the oscillations actually did occur. Closer examination of Figure 5 shows that the peak magnitude of $Z*H$ at 12 Hz is about 10. At the intersection frequency of 14 Hz, however, the magnitude is much smaller, only about 0.8. This is not much above the magnitude of $1/G$, which means that the feedback loop should be only marginally unstable. In such borderline cases, the sound pressure spectrum typically has only a few harmonics. But in this case, the spectrum had lots of harmonics, which is an indication that the phase lag at the oscillation frequency must have been close to 90°.

These considerations raised the suspicion that the flame may not have been driven by modulations of the velocity of the mixture flow at all but rather by modulations of the composition. Modulations of the composition always originate at the point at which the gas is injected into the air stream. From there, they are convected at the velocity of the mixture flow. The time delay involved in reaching the flame is much longer than the delay derived from Figure 4, which means that the phase lag should be much greater.

A model of the flame transfer function for composition-driven flames is given in Reference 8. A rough estimate of the time delay using that model did show that the phase delay must have been very close to 90°. This supported the hypothesis that the oscillations in this boiler were driven by modulations of the mixture composition, originating at the point where the gas is injected.

To make sure that this was the case, a simple diagnostic test was devised: the venturi where the gas is injected was moved 6 inches further upstream. This did eliminate the oscillations at 12 Hz, which could not possibly have happened if the flame had been driven by modulations of the velocity of the mixture flow.

While a relocation of the venturi was not suitable for production, this simple test did identify the true nature of the problem. Ultimately, a few more diagnostic tests pinpointed the gas valve as the culprit, and the solution was to switch to a more robust valve.

Conclusions

This case history shows that it is helpful to use a combination of simple modeling and diagnostic testing when confronting a combustion oscillation problem. Perfectionists might argue that this approach is merely a form of educated guessing. There may be some justification for that view, but we submit that *educated* guessing is a lot better than the random guessing inherent to the trial-and-error approach commonly used.

Our model has shown considerable promise for diagnosing oscillation problems in cases where it has been applied so far. We are aware that there are parts of the model that should be improved. In particular, estimates of damping due to the pressure drop across the heat exchanger and due to acoustic radiation from the surfaces of the boiler ought to be included. It is very likely that this will show that the magnitude criterion of Figure 5 was really not satisfied at the frequency at which the curves of Figure 6 intersected.

We have copyrighted the input script for our model but have no intention of commercializing it. We will gladly share it with anybody who wants to work on it. Requests should be addressed to peterbaade@aol.com or tomarchio@gmail.com. In return, we only expect that we will be informed of any errors that a user may find and we will welcome suggestions for improvements. We would also appreciate help in locating suitable test boilers for the proposed ASHRAE Research Project. These should be boilers with a maximum output of less than 500,000 Btu/hr that either have an oscillation problem or can be made to oscillate with some minor change.

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