

Courting a 15-Year-Old Mistress and a Torsional Vibration Disaster

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The author's early experience with an unauthorized modification to a BMW motorcycle is related to the technology behind torsional vibration absorbers.

S&V contributing editor Mark Rodamaker introduced me to motorcycles when we both worked at the General Motors Proving Grounds. He sold me my first rapid two-wheeler, a Yamaha 250-cc road screamer. We enjoyed many warm-weather adventures together, he upon his new and terrifying spoke-tearing Kawasaki 500 and me upon my quieter and more refined steed. While those Japanese machines were impressive and fun, I craved the precision of one of the German machines I had admired in my youth: a BMW. As a child of the East Coast, I had previously come to nearly worship the offerings of the *Bayerische Motoren Werke*. When I came to own one of their 600-cc motorcycles, I gained a more parochial perspective.

BMW builds beautiful machines; it is easy to become romantically involved with one. I became enamored of 444 pounds of black-skinned Germanic perfection in 1972. The object of my affection was born in 1957, young enough to make my depraved obsession felonious. I knew better, but lust overrode logic. I couldn't ignore the attraction of those horizontally disposed, polished-aluminum cylinder heads. I love the natural and unfettered look of a BMW twin, hanging its working parts in the air-stream to cool for all to see and admire.

Those Teutonic road-cruisers of the '50s were also distinguished by their then-exclusive shaft drive, an unusual front suspension, a wonderfully muffled voice and an enviable reputation for reliability. The frames were hefty and strong. They were normally finished in wonderful glistening black paint with a mile-deep sheen and meticulous white pinstripes, though you could order a machine in red or white.

BMW evolved its sophisticated horizontally opposed, air-cooled, flat-twin, two-cylinder motorcycle engine prior to World War II. Following the war, it applied these (and single-cylinder designs) to increasingly sophisticated motorcycle frames, particularly those mated to sidecars.

Three series of Boxer twins emerged: the 500-cc R50, the 600-cc R60, and 600 cc R69 machines. The R60 engine was the basic offering (28 HP @ 5,600 RPM) optimized for sidecar hauling. The R69 featured 35 HP (at 6,800 RPM) and focused on 'sporting' transport of a single rider.

The culmination of the R69 series was the R69S, which featured 42 HP @ 7,000 RPM. This was accomplished by raising the compression ratio from 8.0:1 to 9.5:1 and using a more aggressive camshaft.

The R69S used highly domed pistons to increase the compression ratio. Its crankshaft was fitted with a torsional vibration damper (a vibration absorber) to deal with the added torsional shaft stress produced by the more powerful engine. This necessitated a 'bump' in the front case cover to clear the damper.

A small number of early R69S models were produced *without* this crankshaft absorber. However, most of the 11,317 units built were equipped with this assembly. (None of the 2,956 preceding R69 models had such a device.) In the same period, the 500-cc, high-compression model R50S was introduced and withdrawn *due to crankshaft failures*.

My machine was a 1957 R69. The chap I bought it from (a GM Proving Grounds driver I will only identify as 'Orville') made some unscrupulous modifications. Most importantly, he rebuilt the engine using the *higher-compression pistons of an R69S* to gain torque and power. However, he did *not* add the vibration damper thought necessary by the BMW factory.



Figure 1. Author and mistress at tale's end.

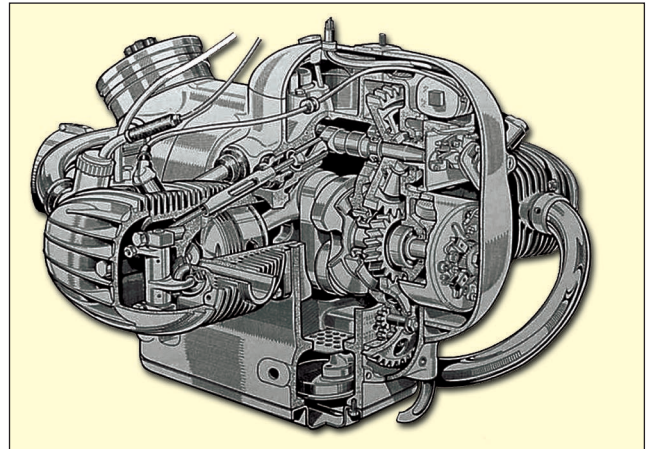


Figure 2. Rendering of the BMW Boxer engine.

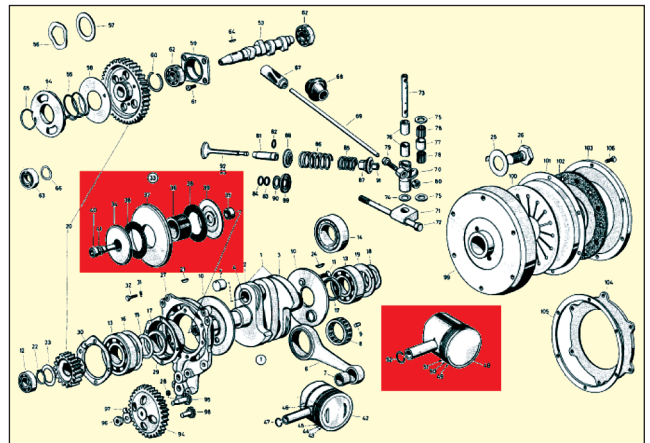


Figure 3. R69 versus R69S (red) crankshaft parts as illustrated in 1966 BMW parts list.

I drove this 15-year-old machine from Michigan to Connecticut. Somewhere in western Pennsylvania, while cruising along at full song, the crankshaft snapped. This was a minor inconvenience as such things go – I was actually able to travel the last 300 miles on the broken shaft!

Though implausible (and ill advised!) this was possible for three

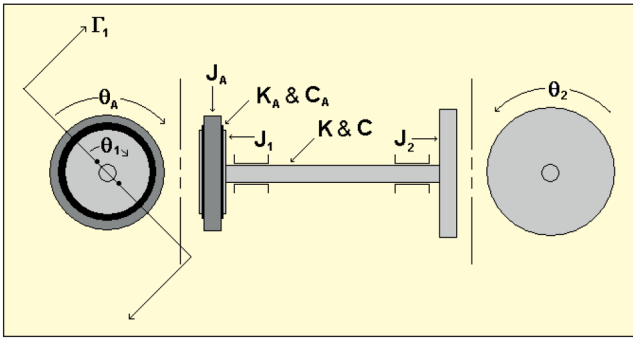


Figure 4. Torsional vibration absorber attached to 2 DOF free-free shaft model.

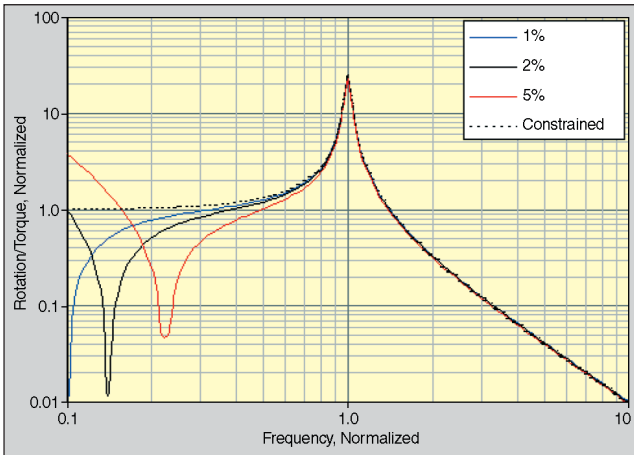


Figure 5. Two-lump 'free-free' shaft model has antiresonance notch determined by mass distribution.

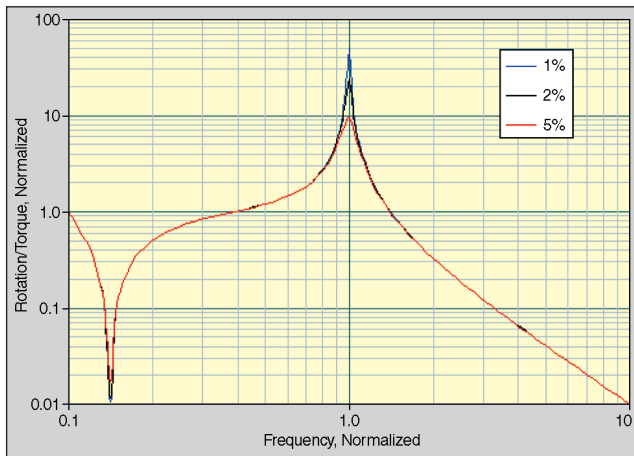


Figure 6. Shaft damping determines peak response.

reasons. First, the break occurred outside the main bearings and inside the small bearing behind the alternator. Second, the ignition system was powered by a separate magneto mounted on the end of the camshaft. So I merely lost the ability to charge the battery and gained some most un-BMW-like foot peg vibration. Third, I was too stupid to realize what had actually happened to the machine. This terrifying comprehension would come days later.

Orville sowed the seeds of disaster with his "shade-tree-mechanic" modifications, then sold me the right to harvest the crop in the bright sunlight of a Pennsylvania afternoon. I was lucky. I only had to pay for his ignorant deed with my money, not with my flesh and bones upon pavement.

So What Is a Torsional Vibration Absorber?

A *vibration absorber* is a tuned system added to a dynamically problematic structure. This appended device is basically a lump of mass that is attached by an elastic support to the parent at a site that vibrates excessively. The natural frequency of the mass

on this suspension is chosen to match the problematic resonance frequency of the structure. When excited by a force at the mounting degree of freedom (DOF) at the previously problematic frequency, the absorber vibrates in resonance while the mounting-point DOF on the parent remains stationary. However, the addition of a tuned absorber produces *two* bounding frequencies at which the mounting DOF will respond resonantly.

The first principles of the vibration absorber are typically introduced (see classic references 1 and 2) by modeling both the parent and the absorber as single DOF spring-mass-damper systems. The parent connects a mass to ground through a parallel spring and damper. The absorber connects a smaller mass to the parent mass in similar fashion. Therefore, the topic is introduced with a *constrained structure* as the parent. This is inappropriate for discussion of a rotating shaft and probably has something to do with the reluctance automotive engineers have to speak of "shaft absorbers." They insist upon calling such devices *vibration dampers*, a misleading term.

In fact, the torsional vibration of a shaft is one of the most profitable targets for the use of a vibration absorber, terminology be damned! However, the basic model of tutelage must be amended. An engine shaft is basically a *free-free* torsional system. Its simplest model contains *two* DOFs, one being the *rigid-body* rotation that makes it useful.

As a result, the minimum model takes the form of two rigid inertial disks connected by a massless elastic shaft (Figure 4). Such a system exhibits two torsional modes. The first is a zero-frequency *rigid-body* mode where the entire shaft rolls. The second is a *twisting* mode of finite frequency, where the disks at either end of the shaft oscillate in phase opposition to one another.

Note that the vibration absorber can be modeled as a ring (of mass inertia, J_A) bonded to one of the disks by an elastic annulus (of torsional stiffness, K_A , and torsional damping, C_A) or as an out-board shaft extension to a third disk. In either case, the equations governing motion resulting from exciting the first DOF (Θ_1) with a sinusoidal torque, Γ_1 , may be written:

$$\begin{bmatrix} [J_1 S^2 + (C + C_A)S + (K + K_A)] & -(CS + K) & -(C_A S + K_A) \\ -(CS + K) & (J_2 S^2 + CS + K) & 0 \\ -(C_A S + K_A) & 0 & (J_A S^2 + C_A S + K_A) \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_A \end{Bmatrix} = \begin{Bmatrix} \Gamma_1 \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

If we define the six identities . . .

$$\begin{aligned} \omega_n &= \sqrt{\frac{K(J_1 + J_2)}{J_1 \cdot J_2}} & 2\xi\omega_n &= \frac{C(J_1 + J_2)}{J_1 \cdot J_2} & \delta &= \frac{J_1}{J_1 + J_2} \\ a &= \frac{\omega_A}{\omega_n} = \sqrt{\frac{K_A}{J_A \omega_n^2}} & \gamma &= \frac{\xi_A}{\xi} = \frac{C_A}{2J_A \omega_n \xi} & \mu &= \frac{J_A}{J_1 + J_2} \end{aligned}$$

. . . we can calculate the complex torque/angle (impedance) frequency response function (FRF) at $S = j\omega = j\beta\omega_n$. Specifically:

$$\begin{aligned} \frac{\Gamma_1}{\Theta_1} &= Z_{\text{real}} + jZ_{\text{imag}} = \frac{\beta^2 K}{1 - \delta} \\ &+ \frac{\left\{ \left[(1 - \beta^2)(\beta^2 - \delta) - (2\xi\beta)(2\delta\xi\beta) \right] + j \left[(2\xi\beta)(\beta^2 - \delta) + (2\delta\xi\beta)(1 - \beta^2) \right] \right\}}{(\beta^2 - \delta)^2 + (2\delta\xi\beta)^2} \\ &+ \frac{\mu\beta^2 K}{\delta(1 - \delta)} \left\{ \frac{\left[\alpha^2(\beta^2 - \alpha^2) - (2\alpha\gamma\xi\beta)^2 \right] + j \left[\alpha^2(2\alpha\gamma\xi\beta) + (\beta^2 - \alpha^2)(2\alpha\gamma\xi\beta) \right]}{(\beta^2 - \alpha^2)^2 + (2\alpha\gamma\xi\beta)^2} \right\} \end{aligned} \quad (2)$$

Note that the first term in Eq. 2 describes the two-disk shaft without an absorber. The second term reflects the actions of the appended tuned-absorber or 'damper' structure. The complex angle/torque (mobility) FRF may simply be evaluated as:

$$\frac{\Theta_1}{\Gamma_1} = A_{\text{real}} + jA_{\text{imag}} = \frac{Z_{\text{real}} - jZ_{\text{imag}}}{Z_{\text{real}}^2 + Z_{\text{imag}}^2} \quad (3)$$

Figure 5 presents the magnitude of the complex FRF of Eq. 3. That is, it reflects the response when $\mu = 0$. Note that the two-disk model exhibits a peak motional response at its *resonance* frequency.

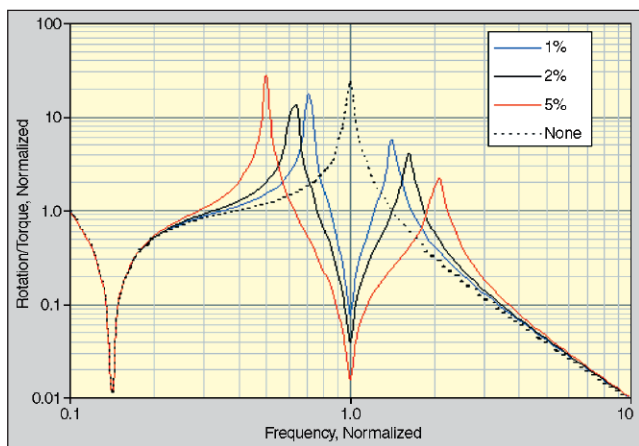


Figure 7. Absorber creates antiresonance at tuned frequency.

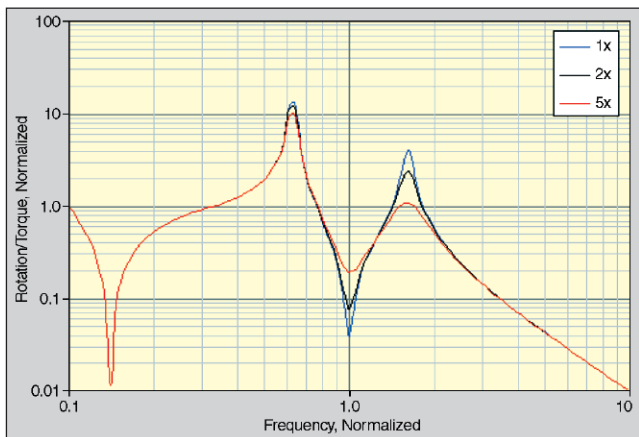


Figure 8. Absorber damping reduces the two 'split' peaks, but not equally.

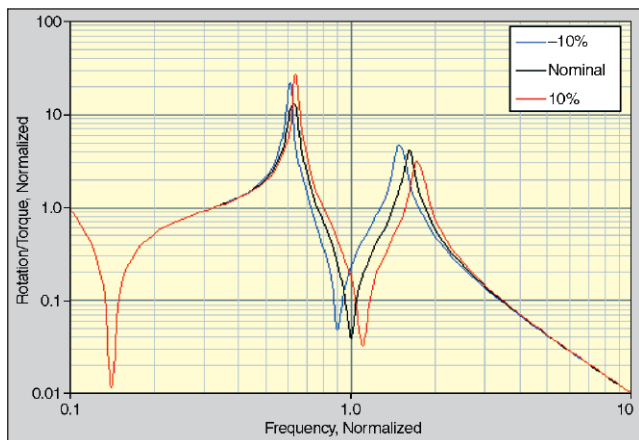


Figure 9. Mistuning the absorber increases peak.

The horizontal axes of all frequency response plots presented here are normalized to this frequency. The response also exhibits a 'notch' (minimum) value at a lower frequency. At this frequency, the "driven end" stands still, and the remainder of the shaft vibrates to generate a reaction force that holds the input DOF immobile. This frequency is termed an *antiresonance*.

The antiresonance frequency is determined by the resonance frequency and the mass distribution. If the rotational *moment of inertia* of the driving DOF divided by the total moment of inertia of the shaft results in a ratio δ the antiresonance frequency is approximately $\sqrt{\delta}$ times the resonance frequency. Therefore, as the drive-site becomes a more massive fraction of the whole, the antiresonance notch frequency *increases*. Figure 5 shows results for the drive DOF being 1, 2 and 5% of the total shaft rotational inertia. The dotted line also shows the result for the *constrained* system described in references 1 and 2.

It is difficult to add significant *damping* or energy absorption

to a freely rotating torsional system. The bulk of the damping that controls the resonant peak must come from the *shaft material*. Therefore, torsional *damping factors* of (at most) a few *percent of critical damping* are expected, and fractional percentage damping is not at all unusual. Figure 6 illustrates damping factors of 1, 2 and 5% of critical applied to our tutorial model.

The vertical axis of all of these plots has been normalized to the *static response* of the shaft to a rotational moment. Therefore, the peak plot value is a multiple of the 'static' ($1/K$) rotation that would result from applying a fixed twisting moment to the shaft. This amplification is termed a *quality factor Q*. For a system exhibiting 100% damping, the quality factor is $Q=1/2\zeta$. Note that the shaft damping has virtually no effect on the behavior near the antiresonance.

Appending a torsional vibration absorber tuned to the shaft resonance creates an antiresonance at the previous resonance frequency. Two new resonances appear, one above and one below the original (tuned) frequency, as shown in Figure 7. The bandwidth between these peaks is determined by the inertia of the appended structure. Increasing the mass inertia of the absorber increases the frequency span between them. Figure 7 illustrates application of absorbers with rotational inertia of 1, 2 and 5% of the shaft's total.

The damping factor of the appended system influences the height of the two resonance peaks and the depth of the new antiresonance notch. Figure 8 illustrates applying an absorber with 1, 2 and 5 times the (untreated) shaft's damping factor. Note the asymmetrical effect of increased damping; the larger peak at lower frequency is much less affected than the higher frequency peak.

The frequency "tuning" of a shaft absorber is very critical. Figure 9 shows the result of mistuning the absorber frequency by $\pm 10\%$. This has little significant impact on the "notch depth," but changes the peak values of both surrounding peaks. Note that the value of the lower frequency (and maximum amplitude) peak increases in magnitude with either an *increase* or *decrease* in the tuned frequency.

Conclusions

The "big picture" concept here is that the *damping* added by the appended structure does a whole lot less to correct a structure's dynamics than does *moving a resonance frequency* from coincidence with a major forcing function (such as peak engine torque). Automotive types may continue to speak of "dampers" out of ignorance; dynamacists should continue to correct their lexicon.

The ability to shift a resonance frequency away from a significant forcing function is the driving motivation for incorporating a vibration absorber, be it linear or torsional. That it can further reduce the peak response amplitude by introducing system damping is a minor but appreciated benefit. Hermann Fram's 1915 invention remains a dynamics blessing, allowing a small mass to make a big change. Equations 1 through 3 are easily implemented in Excel®, MatLab® or other software for personal study of the first principles. Since these are largely forgotten, it may prove time well spent.

Epilog

I made it to my appointed Memorial Day rendezvous in one piece. I had enjoyed frightening introductions to solid concepts in handling dynamics, including the powerful bow waves of fast trucks on Route 80 and the destabilizing effect of rain-grooves cut in the road on narrow (3.5×18 in) tires. I had the sobering experience of opening my wounded engine and extracting a *piece* of the crankshaft attached to the alternator rotor. My young mistress may have enticed and performed again, but never for me. She stayed astern and was sold into possibly sordid servitude after I returned to Michigan by far less exciting air transport.

References

1. Den Hartog, J. P., *Mechanical Vibrations*, McGraw-Hill, New York, NY, 1934, 1940, 1947, 1956.
2. Thomson, W. T., *Mechanical Vibrations*, Prentice-Hall, Englewood Cliffs, NJ, 1948, 1953.

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