Techniques for Implementing Near-Field Acoustical Holography

Sean F. Wu, Wayne State University, Detroit, Michigan

Near-field acoustic holography (NAH) has fundamentally changed noise diagnostics in that it enables one to get all acoustic quantities such as the acoustic pressure, particle velocity, acoustic intensity, sound power, normal surface velocity and structural wave number information simply by taking the acoustic pressure measurements in the near field of a target source surface. The insight into the acoustic characteristics of a sound source that one can get from NAH cannot be matched by any conventional methods. This article describes commonly used ways to implement NAH for reconstructing acoustic quantities in 3D space. The original implementation of NAH is through a Fourier transform that is suitable for a surface containing a level of constant coordinates in a source-free region. To extend NAH to arbitrary geometry, the Helmholtz integral theory is employed and implemented through the boundary-element method (BEM). An alternative is the Helmholtz equation least-square method (HELS) that offers an approximate rather than an exact solution to the acoustic field generated by an arbitrary source. Other methods are developed to visualize an acoustic field radiated from a source in motion or that from a source subject to an impulsive excitation. The efforts of able researchers have made NAH an ever more powerful tool to gain an insight into the characteristics of sound generation and propagation in 3D space.

Noise and vibration issues have always been one of the major concerns to automotive, aircraft, appliance and machinery manufacturers. Identifying the root causes of undesirable noise and vibration and understanding their interrelationships are the critical first steps toward solving these problems and enhancing product performance. Traditionally, noise diagnosis is carried out using a microphone to measure the sound pressure level (SPL) and spectrum of a target structure to determine the noise level and frequency content. Another common approach is to scan a sound intensity probe over a structure to show the "hot spot" from which the acoustic energy is flowing into the surrounding fluid medium.

The main advantage of these traditional methods is that they provide direct measurements of specific acoustic quantities such as SPL and sound intensity. Their limitations are that:

- They provide specific acoustic quantities at the measurement location only.
- The measured values are discrete and uncorrelated; therefore, it is not possible to get a global view of a sound field
- It is difficult to pinpoint the location of a noise source, which is especially true when there are other sources or reflecting surfaces nearby
- It is not possible to identify the structural waves that are traveling along the surface or to visualize the out-of-plane vibration pattern, which may have a direct impact on the resultant structure-borne sound.

These limitations can be circumvented by using near-field acoustical holography (NAH). The major advantage of NAH is that it enables one to reconstruct all acoustic quantities such as the acoustic pressure, particle velocity and acoustic intensity not only at a measurement location, but in 3D space and on a source surface by measuring the acoustic pressure in the near field of the target source surface. Moreover, it allows for visualization of the structural waves traveling along the surface of a structure, yielding an invaluable insight into the interrelationship between sound and vibration.

Based on a paper presented at NOISE-CON 08, Institute of Noise Control Engineering of the USA, Dearborn, MI, July 2008.

Fourier-Transform-Based NAH

In the original NAH technique,[1-3] the acoustic pressure was obtained by taking an inverse Fourier transform of the angular spectrum of the acoustic pressure measured on a hologram plane multiplied by a propagator, where the angular spectrum is a spatial Fourier transform of the measured acoustic pressure into the wave number domain and the propagator represents a phase shift from the hologram plane to any parallel plane in a source-free region. Once the acoustic pressure is reconstructed, the particle velocity can be obtained by Euler's equation and acoustic intensity be specified by multiplying the acoustic pressure and particle velocity. Therefore, all the acoustic quantities are determined once the acoustic pressure on a hologram plane is measured. This is the essence and power of NAH.

The Fourier-transform-based NAH is suitable for a surface containing a level of constant coordinates such as an infinite plane, infinite circular cylinder, and a sphere. Table I displays their formulations, where $H_n^{(1)}(kr)$ and $h_n^{(1)}(kr)$ are the cylindrical and the spherical Hankel functions of order n of the first kind, respectively, $Y_n^m(\theta,\varphi)$ stands for the spherical harmonics, $k = \omega/c$ is the acoustic wave number, and (r,θ,ϕ) imply the spherical coordinates of a field point.

Theoretically, a hologram plane must be infinite so as to facilitate the Fourier transform. If the acoustic pressures on this hologram plane could be measured continuously and exactly, the spatial resolution of a reconstructed acoustic image would be infinitely high. In practice, however, such a scenario is nonexistent, because the measurement space is limited, and input data contain errors or are insufficient. As a result, the reconstructed acoustic images

Table 1. Fourier-transform-based NAH for planar, cylindrical, and spherical geometry.

Planar NAH

$$p(x,y,z_S;\omega) = \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}\left\{\mathcal{F}_x\mathcal{F}_y\left[p(x,y,z_h;\omega)\right]G_p\left(k_x,k_y,z_S-z_h\right)\right\}$$

$$v_n(x,y,z_S;\omega) = \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}\left\{\mathcal{F}_x\mathcal{F}_y\left[p(x,y,z_h;\omega)\right]G_v\left(k_x,k_y,z_S-z_h\right)\right\}$$

$$\text{where } G_p\left(k_x,k_y,z_S-z_h\right) = e^{ik_z(z_h-z_s)},\ G_v\left(k_x,k_y,z_S-z_h\right) = \frac{k_z}{\rho_0 ck}e^{ik_z(z_h-z_s)}$$

$$\mathcal{F}_x\mathcal{F}_y[\psi] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi e^{-ik_x x}e^{-ik_y y}dxdy,\ \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}[\Psi] = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi e^{ik_x x}e^{ik_y y}dk_x dk_y$$

$$k_z = \sqrt{k^2-k_x^2-k_y^2}$$

Cylindrical NAH

$$p(r_S,\phi,z;\omega) = \mathcal{F}_\phi^{-1}\mathcal{F}_z^{-1}\left\{\mathcal{F}_\phi\mathcal{F}_z\left[p(r_h,\phi,z;\omega)\right]G_p\left(k_\phi,k_z,r_S-r_h\right)\right\}$$

$$v_n(r_S,\phi,z;\omega) = \mathcal{F}_\phi^{-1}\mathcal{F}_z^{-1}\left\{\mathcal{F}_\phi\mathcal{F}_z\left[p(r_h,\phi,z;\omega)\right]G_v\left(k_\phi,k_z,r_S-r_h\right)\right\}$$

$$\text{where } G_p\left(k_\phi,k_z,r_S-r_h\right) = \frac{H_n^{(1)}(k_r r_s)}{H_n^{(1)}(k_r r_h)},\ G_v\left(k_\phi,k_z,r_S-r_h\right) = \frac{k_r}{i\rho_0 ck}\frac{H_n'^{(1)}(k_r r_s)}{H_n^{(1)}(k_r r_h)}$$

$$\mathcal{F}_\phi\mathcal{F}_z[\psi] = \frac{1}{(2\pi)}\int_0^{2\pi}\int_{-\infty}^{\infty}\psi e^{-in\phi}e^{-ik_z z}d\psi dz,\ \mathcal{F}_\phi^{-1}\mathcal{F}_z^{-1}[\Psi] = \frac{1}{(2\pi)}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi e^{in\phi}e^{ik_z z}dk_z$$

$$k_r = \sqrt{k^2-k_z^2}$$

Spherical NAH

$$p(r_S,\theta,\phi;\omega) = \mathcal{F}_\theta^{-1}\mathcal{F}_\phi^{-1}\left\{\mathcal{F}_\theta\mathcal{F}_\phi\left[p(r_h,\theta,\phi;\omega)\right]G_p\left(k_\theta,k_\phi,r_S-r_h\right)\right\}$$

$$v_n(r_S,\theta,\phi;\omega) = \mathcal{F}_\theta^{-1}\mathcal{F}_\phi^{-1}\left\{\mathcal{F}_\theta\mathcal{F}_\phi\left[p(r_h,\theta,\phi;\omega)\right]G_v\left(k_\theta,k_\phi,r_S-r_h\right)\right\}$$

$$\text{where } G_p\left(k_\theta,k_\phi,r_S-r_h\right) = \frac{H_n^{(1)}(kr_s)}{H_n^{(1)}(kr_h)},\ G_v\left(k_\theta,k_\phi,r_S-r_h\right) = \frac{h_n'^{(1)}(kr_s)}{i\rho_0 c h_n^{(1)}(kr_h)}$$

$$\mathcal{F}_\theta\mathcal{F}_\phi[\psi] = \int_0^{2\pi\pi}\int_0\psi Y_n^m(\theta,\phi)^*\sin\theta d\theta d\phi,\ \mathcal{F}_\theta^{-1}\mathcal{F}_\phi^{-1}[\Psi] = \sum_{n=-\infty}^{\infty}\sum_{m=-n}^{n}\Psi Y_n^m(\theta,\phi)$$

may be distorted.

A finite measurement aperture can cause wrap-around errors in convolving the measured acoustic pressures with respect to a propagator. In other words, it introduces some artificial wave numbers that are nonexistent. The amount of wrap-around errors cannot be determined exactly, but the effect of the aperture can be greatly reduced by making an aperture four times as large as the source size.[3,4] Recently, patch NAH has been developed,[5-8] which uses analytic continuation of the patch pressure and singular value decomposition to eliminate the need to scan large measurement surfaces. This patch NAH enables one to significantly reduce the number of measurement points normally required by NAH and tackle a large-scale structure.[9]

Planar NAH has been employed to reconstruct the structure-borne intensity as well as the normal acoustic intensity, a technique known as SIMAP (structural intensity from measurements of the acoustic pressure).[10] Williams *et al.*[10] showed that an acoustic intensity map can fail to find the root cause of structural vibration. To identify the real driver of structural vibration, one needs to rely on a structural intensity map. Further, one needs a supersonic intensity map to identify the locations and strengths of the actual acoustic sources of a structure.[11] This supersonic intensity is responsible for sound radiation to the far field, which can be obtained by integrating the acoustic intensity over wave numbers bounded by the radiation circle.

In practice, few structures have a level of constant coordinates. Therefore, the Fourier-transform-based NAH is limited in its applications. For example, when planar or cylindrical NAH is used to reconstruct an acoustic field from a nonplanar or noncylindrical source, one can back propagate the acoustic field to a surface conformal to the hologram surface tangential to the source surface. This is because extrapolation of an acoustic field from one surface to another is valid in a source-free region, beyond which it is no longer source free, and back propagation becomes invalid.

Boundary-Element Method

The concept of NAH can be generalized by using the Green's function that satisfies either the homogeneous Dirichlet or Neumann condition on a hologram surface.[4,12-15] Such a Green's function is available for simple geometry such as an infinite plane but extremely difficult to derive for arbitrary geometry. One alternative is to solve the Fredholm integral equation of the first kind by approximating an acoustic quantity as a linear combination of a set of functions with the associated coefficients determined by an implicit least-squares method.[16] Another approach is to employ the Helmholtz integral theory.

The first attempt to reconstruct the acoustic quantities in an interior region bounded by an arbitrarily shaped body was shown by Gardner and Bernhard,[17] who used the Helmholtz integral theory to describe the interaction of acoustic sources on the surface and field points in an interior region. Numerical solutions are obtained by the boundary-element method (BEM). Because this is an inverse problem, this approach is known as IBEM-based NAH.

The formal derivations of the IBEM-based NAH were given by Veronesi and Maynard:[18]

$$\mathbf{p}(\mathbf{x}_m;\omega) = \mathbf{T}_p(\mathbf{x}_m|\mathbf{x}_S;\omega)\mathbf{p}(\mathbf{x}_S;\omega) \quad \text{and} \quad \mathbf{p}(\mathbf{x}_m;\omega) = \mathbf{T}_v(\mathbf{x}_m|\mathbf{x}_S;\omega)\mathbf{v}_n(\mathbf{x}_S;\omega) \tag{1}$$

where $\mathbf{p}(\mathbf{x}_m;\omega)$ and $\mathbf{p}(\mathbf{x}_S;\omega)$ represent the acoustic pressures measured on a hologram surface at $\mathbf{x}_m$ and the acoustic pressures on the source surface at $\mathbf{x}_S$, respectively; $\mathbf{v}_n(\mathbf{x}_s;\omega)$ is the normal surface velocity at $\mathbf{x}_s$; and $\mathbf{T}_p(\mathbf{x}_m|\mathbf{x}_S;\omega)$ and $\mathbf{T}_v(\mathbf{x}_m|\mathbf{x}_S;\omega)$ stand for the transfer matrices relating measured pressures $\mathbf{p}(\mathbf{x}_m;\omega)$ to surface acoustic quantities $\mathbf{p}(\mathbf{x}_S;\omega)$ and $\mathbf{v}_n(\mathbf{x}_S;\omega)$, respectively:

$$\mathbf{T}_p(\mathbf{x}_m|\mathbf{x}_S;\omega) = (4\pi)^{-1}\left\{\mathbf{D}(\mathbf{x};\omega) + \mathbf{M}(\mathbf{x};\omega)\mathbf{M}_S^{-1}(\mathbf{x}_S;\omega)\left[2\pi\mathbf{I} - \mathbf{D}_S(\mathbf{x}_S;\omega)\right]\right\} \tag{2a}$$

$$\mathbf{T}_v(\mathbf{x}_m|\mathbf{x}_S;\omega) = (4\pi)^{-1}\left\{\mathbf{D}(\mathbf{x};\omega)\left[2\pi\mathbf{I} - \mathbf{D}_S(\mathbf{x}_S;\omega)\right]^{-1}\mathbf{M}_S(\mathbf{x}_S;\omega) + \mathbf{M}(\mathbf{x};\omega)\right\} \tag{2b}$$

where $\mathbf{M}_S(\mathbf{x}_S;\omega)$ and $\mathbf{D}_S(\mathbf{x}_S;\omega)$ represent the effects of monopole and dipole on surface points, respectively; while $\mathbf{M}(\mathbf{x};\omega)$ and $\mathbf{D}(\mathbf{x};\omega)$ represent those of monopole and dipole on field points, respectively. The acoustic pressure $\mathbf{p}(\mathbf{x}_S;\omega)$ and normal velocity $\mathbf{v}_n(\mathbf{x}_S;\omega)$ in Eq. 2 are solved by using singular value decomposition (SVD).[18]

An example of using SVD for an axisymmetric object was illustrated by Borgiotti *et al.*,[19] where the measurements were taken on a surface conformal to the source geometry as close as feasible. Since Eq. 1 is an ill-posed problem, it is regularized by truncating matrices to include only the singular values that are larger than a certain tolerance.[20]

Over the next 10 years, many research papers emerged using SVD and the Helmholtz integral formulations to reconstruct acoustic radiation from an arbitrary structure.[21-28] SVD is a powerful technique for solving a general matrix equation.[29] It enables one to express any $(M \times N)$ complex matrix as a diagonal matrix in proper bases together with the domain and range spaces. SVD yields the least-squares solution for an overdetermined system of equations $(M > N)$.[30]

The most valuable information that SVD offers is the singular value of an ill-conditioned or singular matrix. This is because when an ill-conditioned matrix is inverted, the small singular values become huge. As a result, any small errors in the input data may be significantly amplified and the reconstructed images completely distorted. Therefore, by eliminating the small singular values of an ill-conditioned matrix, one can stabilize the matrix equation and get a well-conditioned solution. This truncated SVD (TSVD) works well for an ill-posed problem for which many elementary methods fail. However, numerical computations involving SVD for a large and complex matrix can be time consuming, especially at high frequencies.

The main advantages of IBEM based NAH include:

- It allows for reconstruction of the acoustic quantities on an arbitrarily shaped structure.
- There are no restrictions on the locations of measurement points on a hologram surface as long as they conform to the contour of the target surface in a near field.
- There are no restrictions on the locations of reconstruction points.
- It is applicable in both exterior and interior regions.

However, since the acoustic quantities on an arbitrary surface are obtained using a spatial discretization, one must have a minimum number of nodes per wavelength to avoid distortions in reconstruction. Each node contains two unknown variables: the surface acoustic pressure and the normal surface velocity. For a complex structure such as a vehicle, the number of discrete nodes required to display surface acoustic quantities accurately may be huge. Accordingly, the number of measurement points required may be excessive, making the reconstruction process extremely time consuming.

Another shortcoming of IBEM-based NAH is that it fails to yield a unique solution when an excitation frequency is near one of the eigenfrequencies of the corresponding interior boundary value problem. While the nonuniqueness can be overcome by combining the exterior and interior integral formulations, known as a CHIEF method[31] or by combining the single- and double-layer potentials,[32,33] numerical computations can become even more complicated and time consuming. Moreover, the presence of $1/R^2$ in the integrands of IBEM can make the numerical computations unstable when the field point is close to a source surface; that is, $R \approx 0$, which is very undesirable in NAH. Kang and Ih[34] developed nonsingular integral formulations that circumvent this difficulty but at the expense of greatly raising the complexities of the integral formulation.

Helmholtz Equation, Least-Squares

An alternative to Fourier transform and IBEM based NAH is the Helmholtz equation, least-squares (HELS) method.[35,36] Unlike the first two methods, HELS does not seek an exact solution to the acoustic field generated by an arbitrary surface, but rather an approximation for an acoustic field using an expansion of the admissible basis functions with errors minimized by least squares. This approach simplifies the problem, yet still enables one to tackle complex situations with relatively few measurements.

One can choose different coordinate systems for HELS expansion. For example, spherical coordinates yield an approximate solution in spherical wave functions, and cylindrical coordinates offer an approximate solution in cylindrical wave functions. The coefficients in the expansion are determined by matching an assumed-form solution to measured acoustic pressure, and the errors in this process are minimized by least squares.

Mathematically, HELS can be written in a matrix form as:

$$\mathbf{p}(\mathbf{x};\omega) = \mathbf{G}_p(\mathbf{x}|\mathbf{x}_m;\omega)\mathbf{p}(\mathbf{x}_m;\omega) \quad \text{and} \quad \mathbf{v}_n(\mathbf{x};\omega) = \mathbf{G}_v(\mathbf{x}|\mathbf{x}_m;\omega)\mathbf{p}(\mathbf{x}_m;\omega) \tag{3}$$

where $\mathbf{p}(\mathbf{x};\omega)$ and $\mathbf{v}_n(\mathbf{x};\omega)$ represent, respectively, the column vectors of the acoustic pressure and normal component of particle velocity at any desired location $\mathbf{x}$, which can be in the field or on a source surface; $\mathbf{p}(\mathbf{x}_m;\omega)$ is the column vector that contains the acoustic pressure measured on a hologram surface; $\mathbf{G}_p(\mathbf{x}\,|\,\mathbf{x}_m;\omega)$ and $\mathbf{G}_V(\mathbf{x}\,|\,\mathbf{x}_m;\omega)$ stand for the transfer matrices that correlate $\mathbf{p}(\mathbf{x}_m;\omega)$ to $\mathbf{p}(\mathbf{x};\omega)$ or to $\mathbf{v}_n(\mathbf{x};\omega)$.

$$\mathbf{G}_p(\mathbf{x}|\mathbf{x}_m;\omega) = \Psi(\mathbf{x};\omega)\Psi(\mathbf{x}_m;\omega)^{\dagger} \quad \text{and} \quad \mathbf{G}_v(\mathbf{x}|\mathbf{x}_m;\omega) = \frac{1}{i\omega\rho_0}\frac{\partial\Psi(\mathbf{x};\omega)}{\partial n}\Psi(\mathbf{x}_m;\omega)^{\dagger} \tag{4}$$

where $\Psi(\mathbf{x}_m;\omega)^{\dagger}$ is known as a pseudo inversion.

$$\Psi(\mathbf{x}_m;\omega)^{\dagger} = \left[\Psi(\mathbf{x}_m;\omega)^H \Psi(\mathbf{x}_m;\omega)\right]^{-1}\Psi(\mathbf{x}_m;\omega) \tag{5}$$

where the superscript H indicates a conjugate transposition, and the elements of matrix Ψ are the particular solution to the Helmholtz equation, which is expressible in spherical coordinates as:

$$\Psi_{nl}(r,\theta,\phi;\omega) = h_n^{(1)}(kr)Y_n^l(\theta,\phi) \tag{6}$$

where $h_n^{(1)}(kr)$ and $Y_n^l(\theta,\varphi)$ are the spherical Hankel functions of the first kind and the spherical harmonics, respectively; the indices j, n and l are related via $j = n^2 + n + l + 1$, with n starting from 0 to N and l from $-n$ to $+n$.

Equation 3 offers an approximate solution to an acoustic field based on the information collected on a hologram surface at $\mathbf{x}_m$. HELS always attempts to yield an optimal result for any given set of input data using the least squares and optimization. So the more accurate the input data, the better the approximation. Needless to say, conformal arrays of microphones should be used and be placed at very close distances to a target source surface.

The main advantages of HELS-based NAH are its simplicity in formulation, efficiency in computation and flexibility in application. Since HELS solves the Helmholtz equation directly, it is immune to the nonuniqueness difficulty inherent in IBEM-based NAH. The main limitation of HELS is that there is no single set of coordinate systems that can provide a good approximation for all surface geometries. For example, the spherical coordinate system is good for blunt and convex surfaces but not for a highly elongated one; the cylindrical coordinate system is ideal for slender bodies but not for a flat or blunt surface.

Since Ψ_{nl} represents the outgoing spherical wave, the solutions provided by Eq. 10 converge very fast when measurement and reconstruction points are outside a minimum sphere that circumscribes an arbitrary surface under consideration. In fact, when the measured acoustic pressures are exact, the reconstructed acoustic pressure converges to the true value as the number of expansion terms $J \to \infty$.[37]

On the surface, HELS seems similar to a Rayleigh series in terms of the spherical Hankel functions and spherical harmonics, with their coefficients determined by orthogonality properties of spherical harmonics. The interrelationships between HELS and Rayleigh series were revealed by Semenova and Wu.[38] In particular, they considered an infinite cylinder with an arbitrary cross section and discovered that outside the minimum circle that circumscribed the singularities of the cylinder, the Rayleigh series gave a result identical to HELS. This is because the high-order terms in the Rayleigh series are negligibly small, so the difference between the Rayleigh series and HELS solution (a truncated expansion) is minuscule.

Inside a minimum circle, however, a Rayleigh series diverges beyond the region bounded by singularities,[38] which confirms Millar's theory on the validity of the Rayleigh hypothesis.[39-44] On the other hand, HELS yields reasonable results on an arbitrary surface not subject to the same restriction as a Rayleigh series. It is interesting to note that even if the Rayleigh series is truncated at the same order as that of HELS, the errors the Rayleigh series inside the minimum circle are still much larger than those of HELS.[38]

A major difference between HELS approximations and the Rayleigh series is that the former expresses the acoustic pressure in terms of an optimal number of expansion functions and uses least squares to determine expansion coefficients; while the latter describes the acoustic pressure as an infinite series and employs orthogonality properties of the spherical harmonics to calculate the expansion coefficients.

A rigorous mathematical justification of HELS in reconstructing acoustic quantities on an arbitrary surface was provided by Isakov and Wu,[45] who proved that any radiating solution to the Helmholtz equation outside a bounded Lipschitz domain with a connected complement could be approximated by a family of special solutions. Using this approximation and conditional stability estimates in the Cauchy problem for an elliptic equation, they demonstrated that the solution was bounded outside an arbitrarily shaped surface and converged to the exact solution, provided that it converged to the exact solution on the measurement surface.

Like IBEM, HELS is executed through a conformal array of microphones at a very close distance to a vibrating object. Unlike IBEM, HELS allows for patch reconstruction. For example, it enables one to reconstruct the acoustic quantities on a portion of a source surface, which can be very convenient in practice. Specifically, one can take measurements over an area that is one row and one column of microphones larger than the surface area of interest. Because HELS employs the same criterion to set microphone spacing as that of Fourier-transform based NAH but covers a smaller area, HELS needs fewer measurement points than Fourier-transform based NAH does.

HELS has been used to reconstruct the acoustic fields produced by an arbitrary structure in both exterior[46,47] and interior regions.[48,49] In particular, it allows for piecewise reconstruction, which can be very handy in engineering applications.

Other Developments of NAH

One can supplement input data by measuring both the field acoustic pressure and normal component of particle velocity. The concept of using mixed acoustic pressure and normal surface velocity was examined numerically by Kang and Ih on a rectangular plate.[50] In this study, the field acoustic pressures were measured at M points, while the normal surface velocities were specified on n ($< M$) nodes to form an overdetermined system of equations. The surface acoustic pressures and normal velocities were obtained by using the least-squares method and SVD.

Another effort to improve the accuracy and efficiency in reconstruction was made by Wu and Zhao[51] in developing a combined HELS and IBEM, known as CHELS, for arbitrary surfaces. To account for the effects of sound reflection from surrounding surfaces, a hybrid NAH method is developed that expresses the acoustic pressure in terms of both outgoing and incoming spherical waves simultaneously.[52] This expansion can be combined with BEM to enhance the efficiency of reconstructing vibro-acoustic responses on the surface of an arbitrary structure. Examples of this hybrid NAH are demonstrated in reconstruction of acoustic radiation from an engine block[53] and from an elongated circular cylinder with a diameter-to-length aspect ratio of 1:10.[54]

Recently, much effort has been made to empower NAH to analyze relative contributions from individual sources that are mutually incoherent.[55,56] Under this condition, one can place the reference microphones at apparent sources and calculate their individual contributions by taking cross correlations. Noise rejection methods can be used to discern contributions from individual sources as well.[57,58] In this case, the effect of a target source on the overall output is treated as a true signal and those from other sources as noises. By calculating the coherence functions among all sensors and comparing them with that of the sensor close to a target source, one can recognize the contribution from this target source. The shortcoming of this approach is that it requires some *a priori* knowledge of the locations of individual sources before applying NAH.

To analyze contributions from individual sources without any prior information on source locations, Nam and Kim proposed employing spatial information of a hologram to infer a source position.[59] They introduced an optimal decomposition plane to reduce the spatial overlap in reconstruction. This method seems to work well for planar surfaces. For an arbitrary structure, however, more effective methods need to be developed.

There are many other developments and applications of NAH to tackle cases that involve transient excitations,[60-63] source convectional motion,[64-68] equivalent source distributions,[69-73] partial field decomposition,[74-79] etc. For more detailed discussions of other NAH developments, readers are referred to Reference 80.

Conclusions

NAH has changed significantly from a simple concept of visualizing acoustic radiation to a popular diagnostic tool over the past three decades. In particular, NAH has been recognized by the U.S. Naval Research Laboratory as one of the 75 most innovative technologies over the past 75 years.[81] Based on the pace of evolution, it will be hard to predict what NAH may become in the next three decades. Indeed, with able researchers working diligently in this field, nothing is impossible.

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The author can be reached at: sean_wu@wayne.edu.