Effects of Filtering Shock Data

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The plateau of the pseudo velocity shock spectrum (PVSS), when depicted on a four-coordinate plot (4CP – frequency, displacement, velocity, acceleration), shows the frequency range of shock severity. Peak modal stress is proportional to PV.[1-4] Therefore, filtering effects can be quantified according to changes in the plateau. Maximum acceleration usually defines the high-frequency extent of the plateau, and low-pass filtering reduces the peak acceleration levels of the shock data. Low-pass filtering of the data hides the high-frequency content of the shock. This is demonstrated in both the time history record as well as the PVSS analysis. Both Butterworth and Bessel filters are compared to see if the linear-phase attribute of the Bessel filter causes any changes in the PVSS.

I am trying to influence a major change in shock analysis technology to move from emphasis on the acceleration shock spectrum to the pseudo velocity shock spectrum shown as a four-coordinate plot (PVSS on 4CP). This change has many advantages. Most importantly, it specifically shows the damage capacity of the shock. It provides a way to quantify the effects of filtering on shock data. This article presents a brief examination of some of the effects.

I have not done an exhaustive study of the mechanical shock filtering literature. Two documents seem to summarize results from many authors: Piersol's 1992 *Sound & Vibration* article,[5] and the *IEST Recommended Practice Handbook*.[6] Some filtering recommendations for pyroshock (high-frequency shock) from both documents are:

- Low-pass filtering should not be used with cutoff frequencies of less than 20 kHz without a thorough analysis indicating why. The low pass cutoff should always be 1.5 times the highest frequency for later data analysis.[5,6]
- For high-pass filters used to remove electrical offsets or drift in the transducer instrumentation, the cutoff frequency should be less than 20 Hz or 0.1% of the lowest frequency of subsequent data analysis computations. Cutoff frequencies higher than this might remove artifacts such as a temporary zero shift that would invalidate the analysis.

My experience is that a 75-100 Hz high pass filter will easily hide an invalid zero shift, and make the data look gorgeous (Smallwood shows examples of this[7]). A general rule for low pass filters, often repeated, is that the cutoff frequency must be at least 1.5 times the highest analysis frequency. That is, the shock spectrum of a filtered shock is only accurate to two-thirds of the filter cutoff frequency. The testing I'm reporting here is that an accurate shock spectrum may only extend to half of the cutoff frequency.

Matlab® hit the market during 1988, and it makes manipulating and plotting digital data quite easy. All of the calculations and plots for this document were made with Matlab Release 12.[8] In 1988, it became easy for people to filter digital data on their PCs. Before then, it took a serious programmer, electronic technician, or one skilled in electronics to evaluate filtering effects of shock data. I have books on writing filter routines in C and FORTRAN that are dated 1991 and 1993, so people were still writing C programs during this period. They refer to "designing" filters, which seems nuts; I use filters and assume Bessel and Butterworth did the designing years ago. Filtering was trusted to instrument makers who were trying to sell a product.

Now, because we know the PVSS-4CP plateau level shows the severity of shock, we can look at filtering effects from a much more sensible point of view. We can evaluate filtering in terms of what it does to the plateau. Papers on filtering dated before 1987 weren't done with Matlab and were very difficult to produce. In early to mid 1970s, I was evaluating no-name, low-pass digital filtering of shock data. The filters were FORTRAN programs, programmed by Dan Carlton[9] from WES (Waterways Experiment Station); I remember him instructing me to run them forward and backward to remove any phase shift, so I did. I didn't want any phase shift; would you? That's terrible. (Matlab's "filtfilt" function does the forward-backward filtering.) However, I tested the results by calculating the PVSS with the files filtered both ways and could not detect any difference.

Now with Matlab, it has become very easy to filter data, high pass and low pass, with Butterworth, Bessel, Chebyshev, Elliptical, of any order you desire, forward or back. It's time for more testing of these filters on shock data. I'll start that ball rolling with this article, and I invite your comments, corrections, or better ideas.

This work will be easier to understand for those who are now convinced that PVSS on a 4CP is the only way you can evaluate severe shock. We believe that stress is proportional to modal velocity and high modal velocities are around 100 ips (2.5 m/s). Modal velocities at the elastic limit of materials range from 100 ips for mild steel up to 1000 ips (25 m/s) for ultra strong materials. The modal velocity a shock can deliver to equipment is well measured by the 5% damped PVSS on a 4CP.

So we are going to look at filtering shock data according to what it does to the 5% damped PVSS on a 4CP of the shock data. Low-pass filtering is a high-frequency information erasing operation;

On May 18, 2008, NATO Standardized Agreement (STANAG) 4549: *Testing of Surface Ship Equipment on Shock Testing Machines* was adopted. In Annex A, a system of describing shock test severity in terms of the (metric) displacement, velocity and acceleration asymptotes of the pseudo-velocity shock spectrum is introduced. The NATO standardized level notation is defined to take the form: **NS LEVEL (m, m/s, m/s²)**. Therefore, a required environment may be completely defined in a compact notation of unambiguous intent.

July 19, 2009 saw the acceptance of ANSI/ASA S2.62-2009: *Shock Test Requirements for Equipment in a Rugged Shock Environment*. This specification defines 10 severity levels in terms of (metric) velocity change at impact, Level 1 being a mild shock of 1 m/s velocity change and Level 10 a severely punishing event of 10 m/s. Annex D defines the computation of the PVSS and links its central plateau to the velocity change frequency limited by drop height and peak acceleration. Note that this document refers to the PVSS as the Pseudo Velocity Shock Response Spectrum (PVSRS).

Two recent testing specifications recognize the PVSS as a useful and accepted tool. Neither of these specifications makes any reference to the passé acceleration shock spectrum measurement. Two commercial instrument manufacturers (Spectral Dynamics and Vibration Research) have bundled PVSS measurement into specialized versions of commercial products. Hopefully, their marketing studies will lead them to introduce new products that more fully capitalize upon PVSS technology. In the meantime, my Matlab® m-scripts for PV shock spectrum analysis and plotting are available for free download at www.SandV.com along with the PDF file of this article. If you don't own Matlab, you might want to experiment with one of its freeware clones, such as Octave available at www.gnu.org/software/octave. My missionary task will be nearing completion when everyone can afford a dedicated PVSS analyzer for use in drop, hammer, shaker shock, or explosive testing. I will know I have been fully heard when you can control a shaker using PVSS feedback to conduct a shock test of specified severity.
Howard A. Gaberson

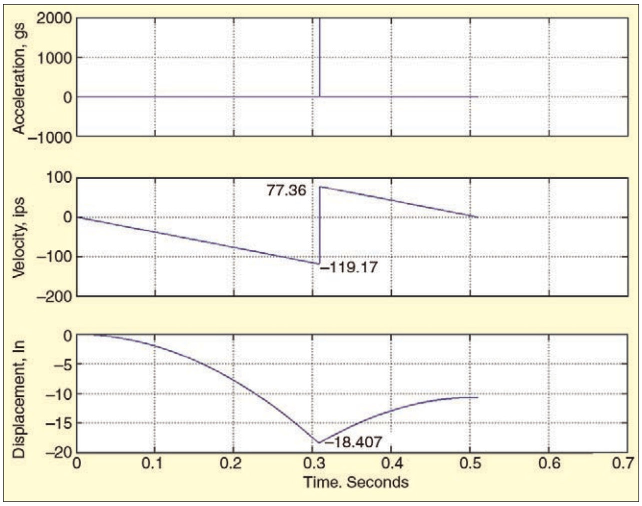

Figure 1. Acceleration, velocity and displacement of a severe high-frequency, half-sine shock.

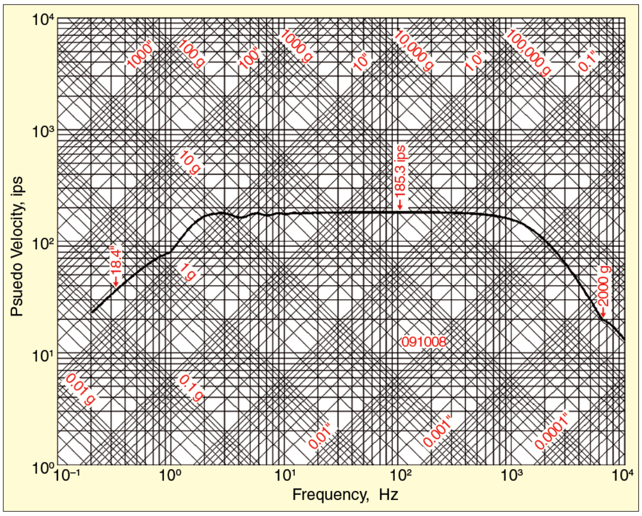

Figure 2. The 5% damped pseudo velocity shock spectrum (PVSS) shown on a four-coordinate plot (4CP) for the high-frequency shock of Figure 1 (long plateau from 2 Hz to about 800 Hz).

and high-pass filtering is a low-frequency erasing operation. I'll demonstrate and explain the information erased by low pass filtering. It cuts the peak acceleration which erases the high-frequency portion of the plateau. Remember that the plateau shows the frequencies where the shock is most damaging.

Simple Drop Table Shock Testing

I'll start with a simple equation-specified half-sine drop table shock. I'll use a high frequency half sine shock of 2000 g and 0.0004 second duration, including the 18.4-in. drop and a rebound with a coefficient of restitution of 0.65, producing a velocity change at impact of 197 ips. Figure 1 shows its time history and integrals.

Figure 2 shows its 5% damped PVSS. The plateau comes out to be 185.3 ips and is limited by high- and low-frequency asymptotes of 2000 g and 18.4 in., respectively. (Stopping the analysis at 10,000 Hz barely lets us see that it hits the 2000 g asymptote.)

I call this a high-frequency shock because it has a high PV content near 200 ips (5 m/s) close to 1000 Hz. Since PV indicates stress, this shock is severe for equipment with modal frequencies from 2 to 1000 Hz. Figure 3 shows the effect of low-pass filtering of this shock data with two-pole Butterworth filters having cut off frequencies of 1000, 500, and 250 Hz.

Figure 4 shows the effect of low-pass filtering on the 5% damped PVSS of our high-frequency, half-sine simple shock. The flat portion or plateau of this shock is at a PV of 180.54 ips. Recall that this is proportional to stress. I have drawn a thin blue line at 90% of this value or 162.40 ips. Where this line intersects the PVSSs one might consider the high-and low-frequency limits of the shock;

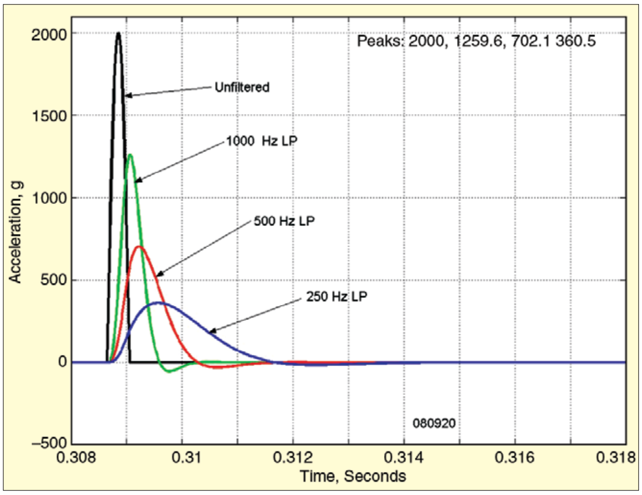

Figure 3. Comparison of filtered time histories to unfiltered shock. Notice the drastic effect on peak acceleration. Black is unfiltered, green is 1000 Hz low passed, red is 500, and blue is 250 Hz low passed. The maxima of the filtered half sines are 1000: 1259.6, 500: 702.1, 250: 360.5.

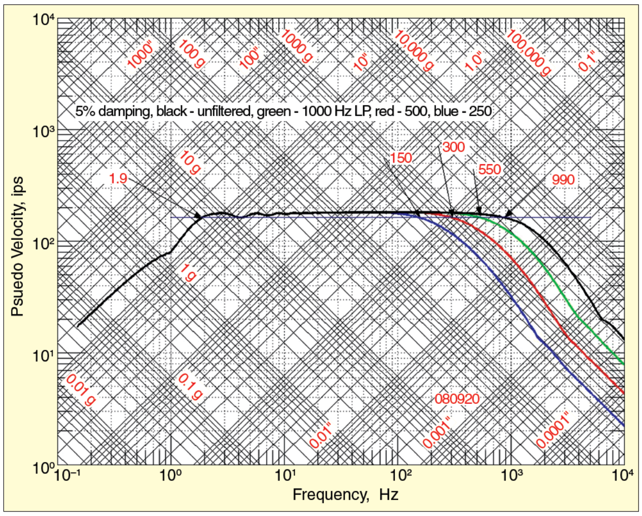

Figure 4. PVSS of the unfiltered and Butterworth two-pole, low-pass filtered high frequency simple drop table shocks. Notice thin blue line at 90% of plateau, 162.40 ips. This is where the stress has dropped to 90%. Filtering obscures part of the severe plateau.

it's where the stress has dropped to 90% of its peak value. So the thin line intersection with the four PVSS plots shows the high frequency limits of the shock. I estimate these intersections to be at 900, 550, 300, 150, and 1.9 Hz.

The unfiltered shock has high PV content from 1.9 Hz to 900 Hz; 1000 Hz low pass cuts the plateau upper frequency to 550 Hz; 500 Hz LP cuts it to 350 Hz, and 250 Hz LP cuts it to 150 Hz. The unfiltered data is the shock felt by the equipment. The filtered shock is what we might show in a report acknowledging that the data had been filtered, but probably misleading the reader about the extent of this effect. One would certainly expect a 1000-Hz, low-pass filter to leave the plateau untouched until after 1000 Hz. It is shown here that a 1000-Hz, low-pass filter hides high-frequency data beyond 550 Hz. The conclusion is that low-pass filtering hides the high-frequency damage potential of the shock.

Figure 5 shows six-pole Butterworth filtering of the shock. Six poles means the cutoff is sharper. While a two-pole filter rolls off at 12 dB/octave, a six-pole filter attenuates at 36 dB/octave. (When searching for filter cutoff rates, I found many references stating that Butterworth filters provide 6-dB/octave attenuation per pole.[10]) The sharp cutoff causes a ringing that can be seen as undulations in Figure 5. Notice also that the peak g levels are also reduced greatly.

Figure 6 shows the PVSS of these six-pole filtered shocks. I defined a simple shock PVSS characteristic I call the droop zone.[11] The droop zone is where peak acceleration exceeds the terminal

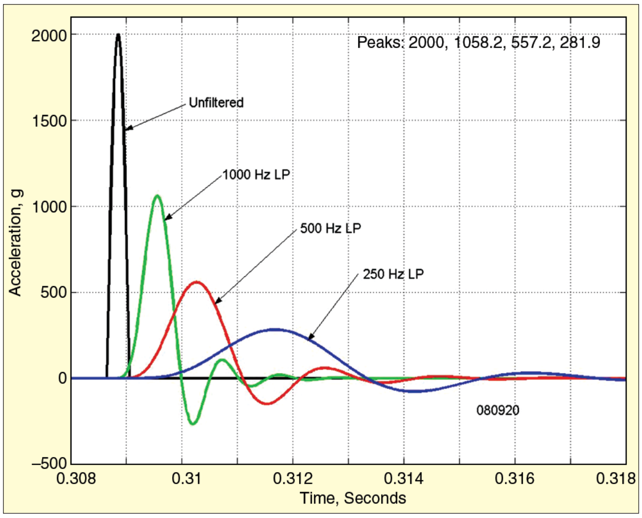

Figure 5. Comparison of the six-pole Butterworth filtered time histories to the unfiltered shock. Notice the drastic effect on the peak acceleration. Black is unfiltered, green is 1000 Hz low passed, red is 500, and blue is 250 Hz low passed. Notice also the ringing or decaying waviness of the 6-pole filter. Maxima of filtered half sines are 1000: 1058.2, 500: 557.2, 250: 281.9.

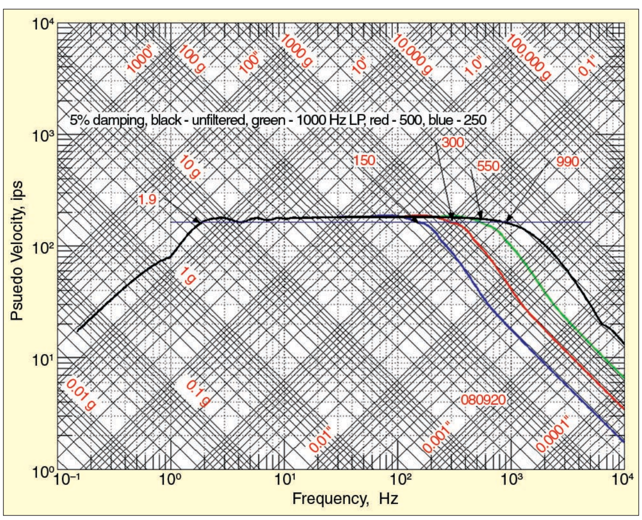

Figure 6. Effect of using a six-pole Butterworth is essentially the same as using a two pole.

acceleration asymptote. In comparing two- and six-pole Butterworth filtering effects, the droop-zone duration is reduced. Look at the blue PVSSs of Figures 4 and 6. On Figure 4, it ends at 1700 Hz, while on Figure 6, it ends at 550 Hz. Notice that the blue curve in Figure 6 has an acceleration asymptote of 300 g, and the droop zone, which is between 150 and 600 Hz, rises to about 450 g.

Bessel filters are reported to be best for low-pass filtering because of their linear phase characteristics. I'll examine the two- and six-pole Bessels to see how they compare with the two- and six-pole Butterworth filters. I won't use the forward-backward filtering, because it introduces a precursor.

Figures 7 and 8 show the time histories and the PVSSs for the two-pole Bessel filtered half sine. The time history shows essentially no overshoot, or ringing waviness, and the droop zones in the PVSSs are very smooth. Figures 9 and 10 show the same thing for the six-pole Bessel filterings. Smoother droop zone; still about the same hiding of the high-frequency plateau.

In Figure 10, I've dropped the plateau cut off line to 20% just to see the change in the plateau limiting frequencies. The upshot is that a 1000 Hz LP cuts things off at 450 Hz; the 500 Hz LP cuts off at 250 Hz, and the 250 LP at 120 Hz.

Table 1 gives a good summary of the drastic effects of filtering. We thought that a 250-Hz low pass would not distort meaningful content below 250 Hz, and that's simply not true. Butterworth filters hide the plateau at 50-60% of cutoff; Bessel filters hide the plateau at 40-50% of cutoff. Table 2 lists the peak accelerations

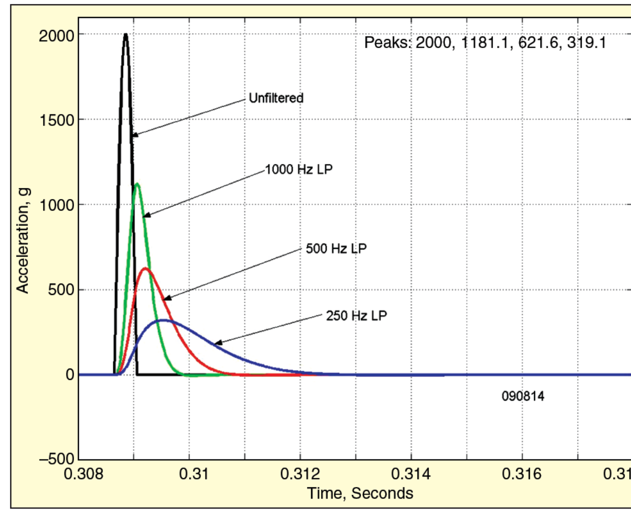

Figure 7. Comparison of filtered time histories to the unfiltered shock. Notice the drastic effect on the peak acceleration. Black is unfiltered, green is 1000 Hz low passed, red is 500, and blue is 250 Hz low passed. Maxima of the filtered half sines are 1000: 1181.1, 500: 621.6, 250: 319.1.

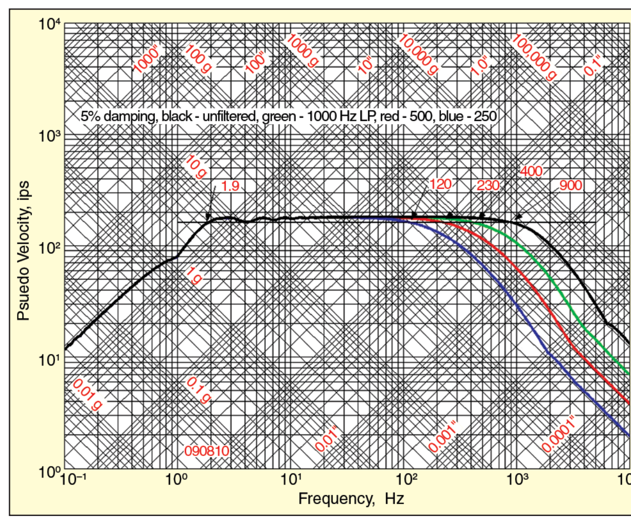

Figure 8. PVSS of two-pole, Bessel-filtered, high-frequency half sine shock. Smoother droop zone.

for the different filters.

Multicycle Explosive Shock

Now let's examine the effects of filtering on a nonsimple explosive shock motion. The results are similar. Figure 11 shows Navy Mil-S-901 heavyweight shock test acceleration and its integrals. The test is done by mounting the equipment in a barge and setting off an underwater explosion nearby to simulate ship shock motions. It is a good example for this analysis. To filter this shock I used a

Table 1. Estimated intersection values of PVSS with depressed plateau by 10%.

Frequency Intersect	Low	Unfiltered	1000 Hz	500 Hz	250 Hz
Butter 2 pole, 10 %	1.9	850	520, 52%	300, 60%	150, 60%
Butter 6 pole, 10 %	1.9	850	510, 51%	290, 58%	140, 56%
Bessel 2 pole, 10 %	1.9	850	410, 41%	230, 46%	110, 44%
Bessel 6 pole, 20%	1.6	1300	430, 43%	230, 46%	120, 48%

Table 2. Maximum values of filtered half sines.

	Unfiltered	1000 Hz	500 Hz	250 Hz
Butter 2 pole	2000	1259.6	702.1	360.5
Butter 6 pole	2000	1058.2	557.2	281.9
Bessel 2 Pole	2000	1181.1	621.6	319.1
Bessel 6 Pole	2000	786.1	408.6	205.9

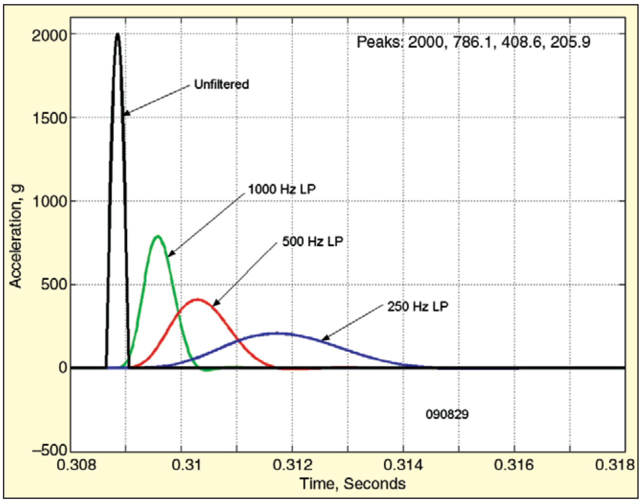

Figure 9. Comparison of six-pole, Bessel-filtered time histories to unfiltered shock. Notice the drastic effect on the peak acceleration. Black is unfiltered, green is 1000 Hz low passed, red is 500, and blue is 250 Hz low passed. Smoother droop zone; still about the same high frequency plateau hiding. Maxima of the filtered half sines are 2000: 786.1, 500: 408.6, 250: 205.9. Shocks are delayed more, flattened, and look symmetrical. No steep rise and gradual tail-off. Very slight ringing.

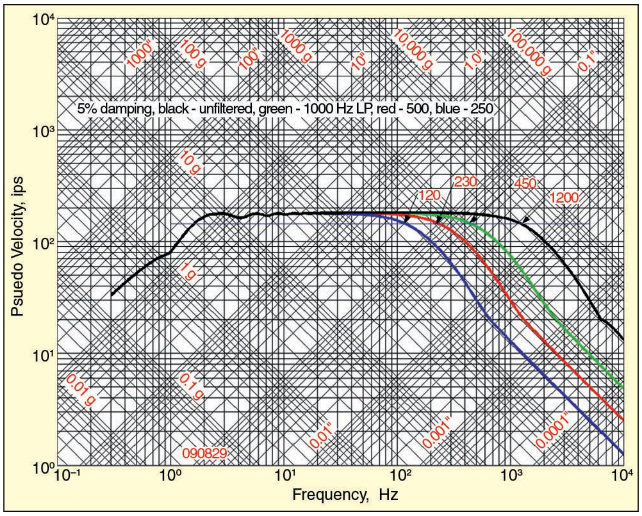

Figure 10. PVSS of six-pole Bessel low pass filtered 0.4 ms, 2000 g half sine with cutoffs at 1000, 500, and 250 Hz.

two-pole Bessel filter, because its linear phase will not affect the time history. Figure 11 shows the time history and integrals of the shock test.

In Figure 12, I tried to show the filter effect on peak acceleration, but it is not as clear as I would like. The graphs are auto-ranged so you have to look at the scale on the ordinates. What I did in Figure 13 is to repeat this, but only for the high intensity first 10 msec and not auto-range to show the dramatic effect on the acceleration. It is interesting to see how the peak accelerations are reduced and yet the PVSSs of Figure 14 are unaffected at low frequencies and gracefully reduced at high frequencies.

Notice in Figure 14a how the high-frequency plateau is successively reduced by the filtering, while the low frequency is unaffected. The filtering was done with a Bessel two-pole filter, which has a linear phase and is not supposed to affect the time history or the PVSS.

Figure 14b is an expanded view of the affected high-frequency plateau portion, so we can estimate the frequencies at which the filterings cause a 10% reduction of the plateau. The 250 Hz low pass (blue) reduces the PV by 10% at about 95 Hz; the 500 Hz low pass (red) causes a 10% reduction at about 190 Hz, and the 1000 Hz low pass (green) at about 300 Hz. The upshot is that 500 Hz low pass filtering does not mean you are only cutting content above 500 Hz at all. It's much worse. The content appears in the spectrum but at deceptively low levels.

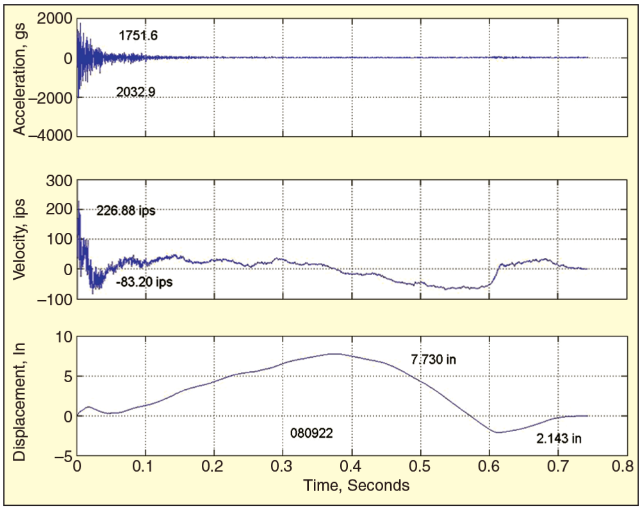

Figure 11. Example of Navy heavyweight shock test – multicycle real data shock for test of low-pass filtering effects. These are often filtered.

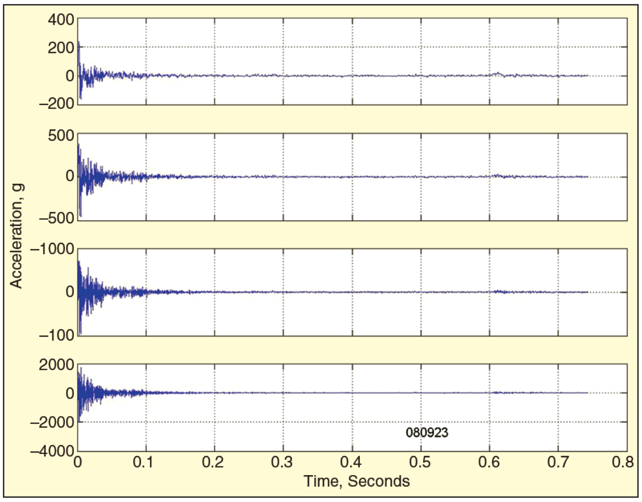

Figure 12 Filtered acceleration time histories. Unfiltered on bottom, 1000 next one up, 500 second from top, and 250 on top.

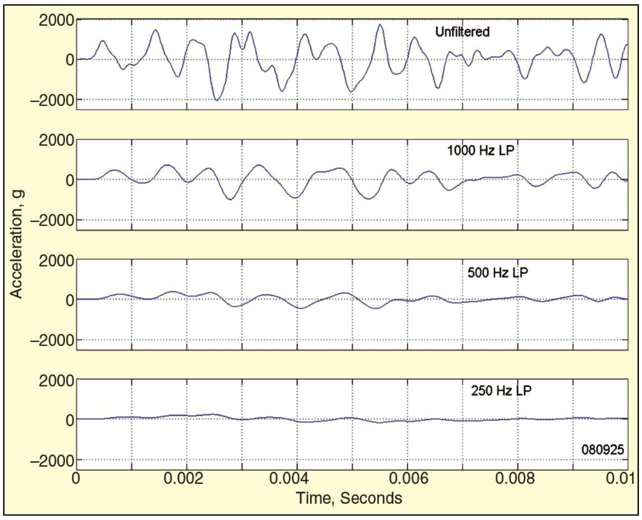

Figure 13. Peak acceleration is surprisingly reduced by low-pass filtering.

I want to emphasize this:

- A 1000-Hz, two-pole Bessel low pass filter causes a 10% plateau depression at 300 Hz, 30% of 1000 Hz.
- A 500-Hz, two-pole Bessel low pass filter causes a 10% plateau depression at 190 Hz, 38% of 500.
- A 250-Hz, two-pole Bessel low pass filter causes a 10% plateau depression at 95 Hz, 38% of 250.

So rather than the PVSS being affected at 67% of the cutoff

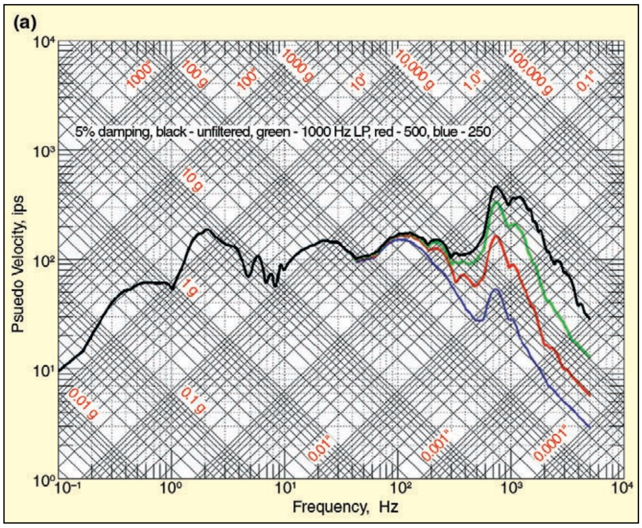

Figure 14a. Estimating the frequency at which a filter has reduced the PV by 10%, the 250 Hz low pass reduces PV by 10% at about 90 Hz; the 500 Hz low pass at about 200 Hz, and the 1000 Hz low pass at about 300 Hz.

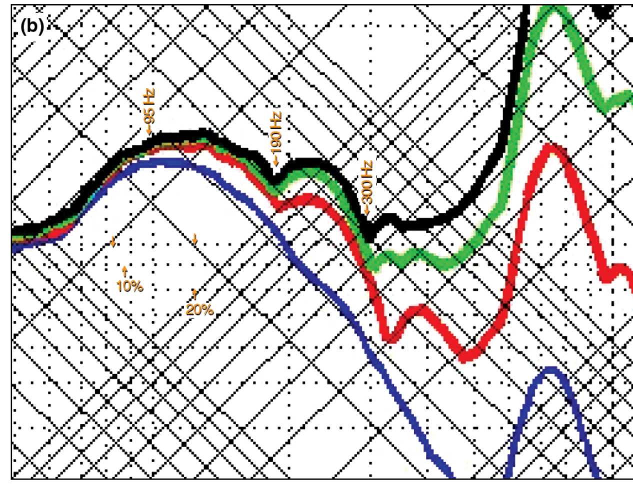

Figure 14b. Expanded view of high-frequency plateau region as it is affected by three different low-pass filters. Notice the four vertical arrows that show height or distance representing 10% and 20%. Green is a 1000 Hz low pass, red: 500, and blue 250 Hz.

frequency it is affected by 10% at about 35% of the cutoff filter frequency.[5,6]

Conclusions

The conclusion has to be that low pass filtering of shock data has a more drastic effect on the shock severity analysis of the pseudo velocity shock spectrum than one is led to believe by the filter cutoff frequency. The PVSS plateau is reduced by 10% at frequencies of about one-third to 60% of the filter cutoff frequency. Standard guidance[5,6] has been that the SRS is good to two-thirds of the cutoff frequency. Similarly, anti-aliasing filters are going to hide true plateau levels at about one-third to 60% of their cutoff frequencies. An anti-aliasing filter with a cutoff frequency of 20 kHz is likely to reduce the calculated level of the plateau at frequencies above 6.7-12 kHz.

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Why Do Things Break When We Drop Them?

George Fox Lang, Associate Editor

A pseudo velocity shock spectrum (PVSS) clearly documents the damaging severity of a particular shock. Note that it is the height of the spectrum's pseudo velocity plateau that indicates (in velocity units) how likely the measured shock is to break a structure, *not* the peak acceleration observed during the shock. However, the peak acceleration does limit the frequency span of the velocity plateau, exhibiting itself as the frequency-descending, high-frequency PVSS asymptote. The lower limit of the plateau's width is determined by a rising-with-frequency displacement line asymptotic to the test's drop-height. In essence, the plateau velocity level indicates damage potential while the drop-height and peak acceleration determine the frequency span over which that potential is realizable.

Remember that any shock spectrum actually presents the simulated response of a hypothetical structure to the test pulse. It is not a reversible transformation of the shock pulse, nor is it an analysis of a particular structure. At each frequency evaluated, the SS amplitude is equivalent to the peak response of a base acceleration-excited spring-mass-damper system. Each of these single degree-of-freedom systems is independent of all the others. Each has a different resonance frequency equal to its frequency position in the spectrum. All of these SDOF systems have the same percent critical damping and all are subjected to the same base acceleration input. In long employed acceleration-response spectra, the output was the maximum (unsigned) value of the mass's absolute acceleration. In modern PVSS measurements, the response is the pseudo-velocity, (unsigned) peak relative displacement across the spring and damper, multiplied by the SDOF's natural frequency in radians per second. The PVSS yields (essentially) the same plateau value as does a shock spectrum using the relative velocity between the excited base and the mass as the response. Unlike relative velocity, the pseudo velocity yields a shock spectrum with clear displacement and acceleration asymptotes bounding the velocity plateau.

Choosing the correct response variable for the SS provides a nearly constant damage indicator over a pulse shape determined bandwidth. Should a structure subjected to the studied pulse have a resonance (or resonances) within this bandwidth, the pseudo-velocity level indicates the probability of inducing a failure. A PV spectrum level of 1 m/s (39.37 ips) is not apt to damage an industrial structure; a level of 10 m/s (393.7 ips) almost certainly will. For example, five copies of a draft fan failed when subjected to very different pulse shapes, all with PVSS plateau levels of 180-200 ips (4.6-5.0 m/s).[12] A sixth identical fan was subjected to a different very severe pulse, but test facility limitations prevented the PVSS level from reaching 180 ips. This fan survived.

Why is it that velocity rather than acceleration tracks shock damage potential? Experience with static machine design and faith in $F = ma$ certainly suggest acceleration should be the damage metric. But stop for a moment and consider a drop test. The test object (of mass, M) is raised a distance, h, above an immovable target. In this pretest state, its potential energy is Mgh, where g is the Earth's gravitational acceleration. Then the object is released. It free-falls for $\sqrt{2h/g}$ seconds until it reaches the target, striking with velocity, $V_1 = \sqrt{2gh}$. At this point its potential energy has fallen to zero while the kinetic energy has risen to $½MV_1^2 = Mgh$. The test object stays in contact with the target for a short time duration, say D. Then it may rebound upward with velocity, $V_2 = -\varepsilon V_1 = -\varepsilon\sqrt{2gh}$, where ε is the coefficient of restitution. During its contact with the target, the test object experiences a velocity change of:

$$\Delta V = V_1 - V_2 = (1+\varepsilon)V_1 = (1+\varepsilon)\sqrt{2gh}$$

The energy dissipated in the collision is thus:

$$E_d = ½M(V_1^2 - V_2^2) = ½M(\Delta V)^2(1-\varepsilon/1+\varepsilon)$$

Therefore, the ferocity of this single impact is defined in terms of the velocity change at impact, ΔV, not a specific measure of the acceleration. ΔV is nothing more than the area under the acceleration time-history curve during the D seconds of target contact. For example, if the acceleration during target-contact were of perfect half-sine shape with a peak acceleration value of A, that area would be $\Delta V = 2AD/\pi$. If the acceleration pulse shape during contact differed from a half sine, the resulting peak acceleration (for the same ΔV) could be quite different from A. Clearly then, it is the velocity change at impact, not the peak acceleration during the impact that defines an impact's damage potential. Note (for a single impact shock) that the plateau value of a PVSS is a very close approximation of the velocity change at impact.

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