Questions Regarding the Prediction of Building Vibrations

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Consider a situation where plans call for construction of a building near an existing road or a rail line and where the building is required to meet definite vibration criteria. How can one predict the traffic-related vibrations in a candidate building design that exists only in the form of drawings? This article discusses some facets of this problem and speculates on approaches to addressing a few of these. It is intended to raise questions, to generate discussion, and to encourage research, rather than to provide answers.

Measurement of Vibrations at the Site

Where to Measure. Some questions about measuring the traffic-induced vibrations at a building's site can be answered relatively easily. Clearly, vibration data should be obtained at a sufficient number of points that cover the footprint of the planned building.

Of course, the measurement transducers need to be well coupled to the soil. Where the surface of the site consists of rock it probably suffices to attach the transducers directly to the rock; where the soil is soft, good coupling can perhaps be best achieved by burying the transducers in fairly shallow holes. In areas where there are rocky outcroppings or curbstones that protrude somewhat from relatively soft soil, it may suffice to attach the transducers to these.

Building Foundations at Grade. If the foundation of a building is to be essentially at the level of the soil surface, one may expect to be able to obtain adequate data by measuring the vibrations at the surface of the ground. However, the vibrations one measures at a point on the bare soil surface will likely differ from those one measures on a slab (representative of the building's slab foundation) located on the soil, because such a slab will in effect integrate the traffic-induced vibrations to some extent over all of the points covered by the slab. Thus, if the building is to be supported on footings at or near the elevation of the soil, it may be best to measure the site vibrations on slabs that replicate the footings. For a building that is to be supported on a slab foundation, it would be useful to measure the vibrations on the actual foundation slab, if that is feasible practically. Otherwise, one might measure on a similar slab of smaller area and scale the results to the larger area. Such scaling, of course, involves some uncertainties. It should take account of the fact that a slab, particularly one of large area, is likely not to act as a rigid body. One might consider the "active" area of an extended slab at a given frequency as that of a square slab with edges that are one-half of the slab's bending wavelength long, taking some account of the effect of the underlying soil on that wavelength.

Building Foundations Below Grade. For buildings whose foundations are to be below the surface of the site one needs to take into account that the vibrations at the level of the foundations may differ from those at the surface. One may consider measuring the vibrations at the bottoms of boreholes of appropriate depths, but practical borehole diameters limit the sizes of the slabs atop which the vibrations could be measured, so that much of the aforementioned integrating effect of a slab would be lost. Measurements made on a representative slab at the bottom of a large enough pit would not suffer from this shortcoming.

In situations where measurement at depth is not feasible, one might consider relying on data obtained from surface measurements and adjusting these data analytically. One may obtain some guidance concerning the difference between the vibrations at a given depth and those at the surface by considering that vibrations travel along the surface of a uniform halfspace predominantly in the form of Rayleigh waves. (At locations within a fraction of a Rayleigh wavelength of the excitation point, other waves may also contribute significantly to the wave field, so that the Rayleigh wave approximation may not be adequate for such locations.) Except at very small depths and low frequencies, the amplitudes of Rayleigh waves decrease considerably with increasing distance from the surface – Figures 1 and 2 show plots of amplitude ratio versus frequency for several depths, calculated for soils with a Poisson's ratio of 0.25.[1] Although the ground is not a uniform halfspace, it might suffice to approximate it as such for the present purposes as long as the soil is relatively uniform down to depths at which the Rayleigh wave amplitude is a small fraction of that at the surface.

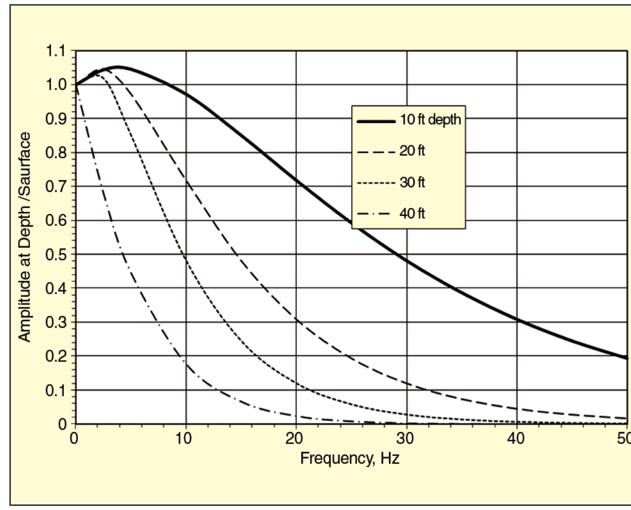

Figure 1. Depth-dependence of vertical amplitude of Rayleigh waves.

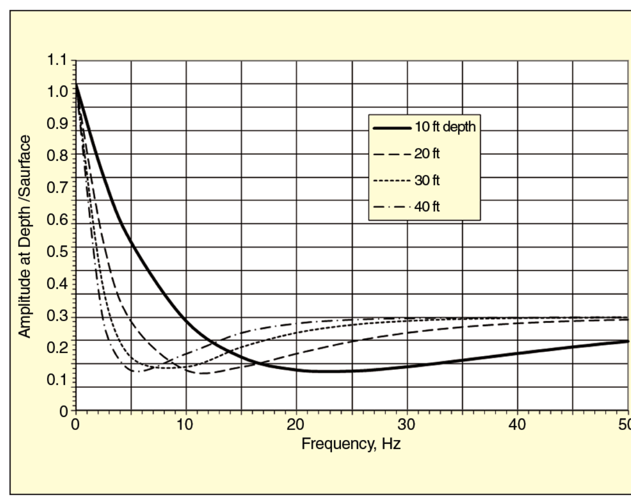

Figure 2. Depth-dependence of horizontal amplitude of Rayleigh waves.

If the halfspace approximation holds and if the integrating effect of slabs can be neglected, then one may use the properties of Rayleigh waves to estimate the vibrations that occur at a given depth from the vibrations measured at the surface. Figures 1 and 2 show, for example, that at 20 Hz the vertical amplitude at a depth of 20 ft is about 30% of that at the surface and that the horizontal amplitude is only about 15% of that at the surface. These may be considered to be reasonable estimates if the soil is relatively uniform down to perhaps 30 ft.

Information on the vibrations expected at a point at depth, as obtained from borehole measurements or the use of Rayleigh wave calculations, probably is most relevant for buildings that are supported on sleeved columns which rest on footings located at depth, because the sleeves prevent soil contact along the lengths of the columns. For cases where the columns are not sleeved is not clear how one should account for the additional vibration transmission along the lengths of the columns.

Another problem arises in the case where the soil over a large area under a building's footprint is to be removed, for construction of a basement, for example. Removing this soil may be anticipated to change the vibrations at the points where at-depth measurements were made. At very low frequencies, at which the Rayleigh waves have wavelengths that are considerably greater than the excavation depth, it is likely that these waves essentially just propagate along the soil surface, even though the surface is depressed over the limited area where a basement is to be located. At these frequencies one may estimate the vibrations at a future excavated area from surface vibration data measured in its vicinity. It is questionable how this estimate can be done defensibly for higher-frequency vibrations.

What to Measure – Data Acquisition and Analysis

Statistical Spectra. Traffic-related vibrations typically are not steady and may depend on such rather unpredictable factors as the types and speeds of passing vehicles, as well as on the traffic situation and the weather. If the "worst case" is to be used as the basis of conservative design, perhaps the best one can do is to sample the traffic-induced vibrations for an extended period and then to analyze the data statistically, e.g., to obtain "peak-hold" or percentile spectra in practically manageable frequency bands. (Peak hold spectra are made up of the greatest magnitudes that are observed in each frequency band during the data acquisition period. Percentile spectra show the magnitudes in each frequency band that are exceeded a given percent of the time.) One-third-octave bands may be most practical, because they typically reveal adequate, but not excessive detail, because many criteria are written in terms of one-third-octave-band spectra, and because such spectra may readily be evaluated by use of available spectrum analyzers.

Because peak-hold spectra can include potentially irrelevant high vibration levels due to unusual events, such as vehicle collisions or thunderstorms, they may overstate the typical traffic-related vibrations. Thus, use of a percentile level to characterize these vibrations is likely to be more appropriate, with the percentile selected on the basis of the building's sensitivity to disturbances. For example, for a building in which occasional disturbances would be very disruptive it might required that the L_1 vibration levels (the vibration levels that are exceeded no oftener than 1% of the time) do not exceed prescribed limits, whereas for a building in which relatively frequent disturbances can be tolerated it may suffice to place limits on just the L_{10} levels (the levels that are exceeded no oftener than 10% of the time).

Steady-State Versus Transient Analysis. Because the vibrations generated by passage of a vehicle clearly are transient and cannot be fully characterized in terms of spectra (which in essence apply to steady-state conditions), one might consider recording the time-varying motions of the measurement points and then determining the model's time-varying response by "playing" these recorded motions into a model of the building under consideration. Because traffic-induced vibrations tend to vary considerably with time, one would need to use long vibration records and then to analyze the results statistically.

Fortunately, the spectrum approach, which is simpler to implement, will suffice in many cases. If a sinusoidal force at frequency f is applied to a structure that is initially at rest, then the motion of each of the structure's modes, starting from rest, will tend toward its steady-state amplitude exponentially. From textbook results for a single-degree-of-freedom system one may find that the time T_n it takes for the response of a not-too-highly damped mode ($\zeta^2 \ll 1$) to reach a fraction n of its steady-state amplitude obeys

$$T_n = -1n(1-n) / 2\pi f \varsigma \tag{1}$$

where ζ denotes the mode's viscous damping ratio (equal to c/c_c with c representing the viscous damping coefficient and c_c representing the critical damping coefficient). This expression holds regardless of the natural frequency of the mode.

From Equation (1) one may determine, for example, that the response of a building's mode with a representative damping ratio of 0.03 to a 5 Hz excitation takes only about 2.4 seconds to build up to 90% of its steady-state value. For higher excitation frequencies the times are proportionately shorter. Since most excitations due to traffic last more than a second or two, the corresponding modal responses would tend to approach their steady-state values closely enough for most practical purposes, so that the use of spectra for determination of the structural responses often may suffice.

When to Measure

A site's vibration propagation characteristics may be expected to depend not only on the local soil's constituents and layering, but also on the soil's moisture content and temperature. The magnitudes of the effects of these parameters are not known, but frozen and/or wet soil likely propagates vibrations relatively well. The depth of the water table also may play a role.

Furthermore, the weather may also have an effect on the generation of vibrations by traffic. Not only may weather conditions influence traffic speeds, but weather also may affect the street surface conditions, even giving rise to ice clumps or potholes.

It is likely that the "worst case" vibrations at a site may be measured at low temperatures and/or after a heavy rain, or in the winter. One again needs to consider the disturbances that can be tolerated in the building and perhaps use a percentile criterion, possibly evaluated over a year or longer.

Vibration of a Building Coupled to Ground

Ground Vibration Change due to Loading by a Building. One may readily visualize that the vibrations observed at a given location under "green field" conditions (that is, on the ground, in absence of a building) will differ from those observed when a building is present at that location, even if the vibrations produced by nearby traffic remain the same.

Let us consider the simplest case, where the motion at a given point on the ground is taken to occur in only one direction – say, the vertical – with the motions in other directions assumed to play no role and with the building's effect confined to only that point and only to that one direction. In this case, the velocity amplitude V_W of a given point on the soil surface with the building in place (which also is the velocity at the building's attachment point to the soil surface) is related to the velocity amplitude V_0 at that same point in absence of a building as

$$\frac{V_W}{V_0} = \frac{1}{1 + Z_B / Z_S} = \frac{1}{1 + K_B / K_S} \tag{2}$$

Here Z_B denotes the driving-point impedance of the building at its attachment point to the soil, and Z_S denotes the driving point impedance of the soil at that point. The Ks represent the corresponding dynamic stiffnesses. (These are related to the impedances via $K = j\omega Z = j2\pi f Z$ in complex or "phasor" notation, where $j = \sqrt{-1}$. The impedances and dynamic stiffnesses are measured in the same direction as the velocities.) If the magnitude of the impedance (or stiffness) of the building is much smaller than that of the soil, then $V_W \approx V_0$; that is, the presence of the building does not change the vibrations significantly. However, if the magnitude of the building's impedance is large compared to that of the soil, then $V_W \ll V_0$. In this case the building's presence results in considerable reduction in the local vibrations – and the vibrations transmitted to the building are much smaller than the vibrations of the soil in absence of the building.

If one models the soil simply as a mass-spring-damper system whose spring is fixed to a rigid base and to whose mass a sinusoidally varying force is applied, one finds its dynamic stiffness to be given by

$$K_S = k_S \left(1 - r_S + j\eta_S\right) \tag{3}$$

where k_S represents the stiffness of the equivalent spring and $r_s = (\omega/\omega_s)^2$, with ω denoting the radian driving frequency and

$\omega_S = \sqrt{k_S / m_S}$ denoting the model's natural frequency in terms of the effective mass m_s. The loss factor η_S represents the system's damping.

If one models a mode of the building as a mass-spring-damper system that is driven by a force applied to the base of its spring, one may determine that its dynamic stiffness obeys

$$K_B = k_B \frac{-r_B\left(1 + j\eta_B\right)}{1 - r_B + j\eta_B} \tag{4}$$

Here the various symbols with subscript B pertain to the building mode and are defined in analogy to those for the soil. This equation, together with Equation (3), permits one to determine V_W/V_0 from Equation (2) for the simple model under discussion.

Response of Building Mode Coupled to Soil

For a building mode modeled as described above, the velocity V_B of the mode (i.e., of the model's mass) is related to the velocity V_W of the soil with the building in place (i.e., of the base of the spring part of the building model) as

$$\frac{V_B}{V_W} = \frac{1}{1 - r_B + j\eta_B} \tag{5}$$

By multiplying this relation by the expression for V_W/V_0 one may find the following expression that relates the velocity of the building mode, coupled to the soil, to the velocity of the soil in absence of the building:

$$\frac{V_B}{V_0} = \frac{\left(1 - r_B + j\eta_B\right)\left(1 - r_S + j\eta_S\right)}{\left(1 - r_B + j\eta_B\right)\left(1 - r_S + j\eta_S\right) - \left(k_B / k_S\right) r_B \left(1 + j\eta_B\right)} \tag{6}$$

This velocity ratio takes on its greatest value at the natural frequencies of the mode and soil combination. These natural frequencies correspond to a vanishing of the real part of the denominator of the previous relation; the related quadratic equation in ω^2 may readily be solved and yields the classical result for the natural frequencies of an undamped two-degree-of-freedom system. The natural frequencies ω_0 obey

$$2\frac{\omega^2}{\omega_B^2} = 1 + \frac{m_B}{m_S} + \frac{\omega_S^2}{\omega_B^2} \pm \sqrt{4\frac{\omega_S^2}{\omega_B^2} + \left(1 + \frac{m_B}{m_S} + \frac{\omega_S^2}{\omega_B^2}\right)^2} \tag{7}$$

The general expression of Equation (6) for V_B/V_0 is too cumbersome to explore analytically, but it is of interest to consider some limiting cases. For the case where the *soil acts like a massless spring*, which corresponds to the situation where the excitation frequency is much smaller than the natural frequency of the soil (visualized as a spring-mass system), one finds that

$$\frac{V_B}{V_0} = \left[1 - m_B\omega^2\left(\frac{1}{k_b\left(1 + j\eta_B\right)} + \frac{1}{k_S\left(1 + j\eta_S\right)} + \right)\right]^{-1} \tag{8}$$

This ratio takes on its greatest value at the resonance frequency ω_{0k} which obeys

$$\omega_{0k}{}^2 = \left[m_B\left(\frac{1}{k_B} + \frac{1}{k_S}\right)\right]^{-1} \tag{9}$$

and corresponds to the building mass supported on the springs of the building mode and the soil in series. At resonance at this frequency the magnitude of the velocity ratio of Equation (8) becomes

$$\left|\frac{V_B}{V_0}\right|_{res} \approx \frac{k_B + k_S}{\eta_B k_S + \eta_S k_B} \tag{10}$$

If the soil is very stiff – that is, if $k_S \gg k_B$ – then this reduces to $1/\eta_B$ which is the same as the result for resonance of the building mode by itself.

For the case where the *soil acts purely as a mass*, corresponding to the excitation frequency being much greater than the natural frequency of the soil, one finds that

$$\frac{V_B}{V_0} = \frac{1 / m_B}{\frac{1}{m_B} + \frac{1}{m_S} - \frac{\omega^2}{k_B\left(1 + j\eta_B\right)}} \tag{11}$$

This ratio takes on its greatest value at the resonance frequency ω_{0m} which obeys

$$\omega_{0m}{}^2 = k_B\left(\frac{1}{m_B} + \frac{1}{m_S}\right) \tag{12}$$

and corresponds to the two masses connected to each other by the building mode's spring. At resonance one finds from Equation (11) that

$$\left|\frac{V_B}{V_0}\right|_{res} \approx \frac{1}{\eta_B\left(1 + \frac{m_B}{m_S}\right)} \tag{13}$$

If the soil is much more massive than the building, so that $m_S \gg m_B$, then this expression again reduces to $1/\eta_B$, the result for resonance of the building mode by itself.

One may observe that here, as well as in the resonant response situations discussed earlier, damping plays a crucial role – as it indeed does in all resonant response cases. Thus, the vibration magnitudes one may compute in these cases suffer from uncertainties in the damping values, which in practice always need to be estimated.

Prediction/Modeling of Building Vibrations

Excitation at Base of Building. How can one predict the vibrations that are transmitted into a building from the ground? A building's motion clearly is much more complex than that of the minimal model discussed in the preceding section. A building obviously does not interact with the ground at just one point – and not just in one direction at each contact point. One would at least need to work with impedances that account for the six displacement (or velocity) components at each point, rather with the single displacement component considered above.

If a building is supported on columns that rest on discrete footings, it may perhaps suffice to consider each footing to act at a single point and to address the motion of that point in terms of six-component impedance matrices. However, in order to determine the motions of the building's floors one needs to account for the totality of the motions transmitted to the floors via all of the columns, so that one needs to consider the different magnitudes and phases of the soil motions that act at the column bases. If the distance between footings is smaller than a quarter wavelength in the soil – that is, at low enough frequencies – the footings likely will be set into motion essentially equally and simultaneously, but this will not be the case at higher frequencies. Note as a point of reference that the Rayleigh wavespeed in dry sand is of the order of 600 ft/sec and that at 10 Hz a quarter wavelength is a mere 15 ft.

Predicting the vibrations of a building on a foundation slab requires consideration of how such a continuous flexible slab couples to the non-uniform motion of the soil. At extremely low frequencies, at which the slab's plan dimensions are considerably smaller than a quarter wavelength in the soil, the slab likely will move approximately uniformly with the soil, but at higher frequencies the relatively complex coupling to the slab to the soil needs to be taken into consideration.

One may obtain some answers by means of finite-element modeling of the building as coupled to the soil. However, the utility of such modeling is confined to low frequencies, even if one has available the computational capabilities for dealing with a large number of elements. One always faces the question of how well a model represents the real situation. The low-frequency results one obtains from a model generally tend to approximate reality better than the results obtained for higher frequencies, because the low-frequency results depend less on how well the model represents the details of the structure. The question is whether the low-frequency range in which the model yields realistic enough results can encompass the phenomena of concern.

Building Motion. Even if one can overcome the previously discussed obstacles to predicting the vibration inputs at the base of a building, one faces difficulties in determining the building's vibrations. Many aspects of an actual building's design are not well defined *a priori*. For example, design drawings only define the gross properties of a building's structure, the actual loads (that is, the weights carried by the structures) generally differ considerably from the specified design loads, and some structural details depend on the workmanship of the construction crew. Thus, one can only arrive at an approximate model of the building, no matter how much computational capacity one may have available, so that one can expect to obtain reasonable approximate predictions only for low frequencies.

In view of the aforementioned uncertainties, one might consider using statistical energy analysis (SEA) modeling, which provides estimates of the average responses of groups of modes of structures that are not well defined. SEA may be of practical use in situations where the building motions of concern are likely to involve a considerable number of modes in a frequency band of interest, provided that the parameters involved in the analysis (the relevant coupling factors and the component loss factors) can be determined adequately. However, the results available from the use of SEA are of limited value in the generally most significant practical cases where the vibration responses are determined by only a small number of discrete modes.

All analytical modeling, and particularly SEA modeling, is better suited to predicting the effects of structural modifications than to predicting absolute vibration response magnitudes. Furthermore, any analytical model needs to be "calibrated" by comparison to corresponding measurement data. This is difficult to do for buildings, unless a new building is similar to an existing one and measurements on the existing building can be used to validate the model.

It also should be noted that neither the magnitudes nor the characteristics of the damping of a building or its components are ever known accurately. Experienced-based estimates of viscous damping coefficients are widely used in computational analyses, although it is clear that the actual damping is not viscous and varies differently with frequency. Inaccurate prediction of damping is particularly troublesome, since the most severe vibrations typically are associated with resonances at which the magnitudes of the motions depend crucially on the damping, as has been mentioned.

Concluding Remarks

The many unknowns and uncertainties discussed above make it virtually impossible for one to develop reliable predictions of the traffic-related vibrations expected in a building, if one relies essentially only on basic principles. Fortunately, in many cases one can arrive at reasonable predictions by making use of data measured in buildings that are similar to the planned one and that are exposed to similar nearby traffic. Scaling of the available data, perhaps by use of some of the principles to which this article alludes, may often be useful. This approach may not be applicable, however, for radically new building designs.

Fortunately, many practical projects only require that prescribed vibration limits not be exceeded in specified parts of the building. In such cases there may be no need for precise prediction of the expected vibrations. Here approximate prediction of the most severe vibrations resulting from a confluence of "worst case" conditions may suffice for determination of the amount of attenuation that would guarantee meeting of the limits. This amount of attenuation would be greater than that absolutely necessary to meet the limits under the actual conditions, but it should generally be obtainable by means of isolation arrangements that involve only a minimal cost penalty.

Reference

1. F. E. Richart, Jr., R. D. Woods, and J. R. Hall, Jr., *Vibrations of Soils and Foundations*, p. 88, Prentice-Hall, Inc. 1970.

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