

An Atomic/Molecular Analogy to Vibration Systems

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Introduction from the world of chemistry. In 1869, Russian scientist Dmitri Mendeleev (1834-1907) arranged all then-known chemical elements into groups according to their properties and predicted several new elements from this categorized information. At that time, he was writing a book for students of chemistry, and one of his goals was to tabulate the chemical elements in a clear and logical progression. He put the name and main properties of each element on a small square of paper and organized these pieces in various table-like combinations. Finally, he created his famous Periodic Table (according to legend, revealed by a dream).

In 1869, the elements (atoms) were considered to be the smallest parts of the world. One step up in size were molecules, each combining several interconnected atoms. Numerous molecules combined to form the continuum of matter. Modern chemistry science now knows of 118 chemical elements.

“Atoms” of the Vibration World. Let’s briefly describe simple vibration systems through the atomic/molecular analogy. Vibration science knows three prime elements: the absolutely rigid mass, the massless spring, and the dashpot, or viscous damper (Figure 1). We may call them elements because each of them represents only one physical property: the mass element simulates inertia, the spring stands for elastic compliance, and the dashpot symbolizes velocity-proportional energy dissipation of the vibration elements. Here, the spring and dashpot are considered to be linear: the elastic and viscous forces are proportional respectively to the displacement and velocity.

In the real world, the dissipation mechanisms may be nonlinear (in particular, dry friction and hysteresis), but the rate of vibration energy attenuation with time is determined by the so-called loss factor that can be evaluated for any mechanism of damping dissipation. It is also noteworthy that Hooke’s Law of Elasticity, named after the 17th-century scientist Robert Hooke who stated it first: *Ut tensio, sic vis* (as the extension, so the force), is just a useful approximation which is relevant only to linear-elastic or “Hookean” materials.

“Molecules” of the Vibration World. Combined together, three such atoms constitute a vibration molecule. There are two ways to connect the spring and the dashpot: in parallel (the Kelvin model, Figure 2a); or in series (the Maxwell model, Figure 2b).

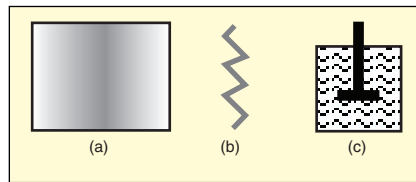


Figure 1. The vibration “atoms.”

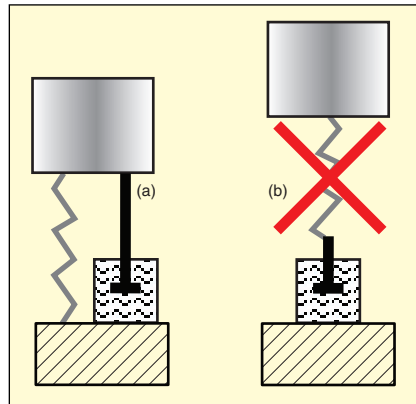


Figure 2. The 1-DOF vibration “molecules.”

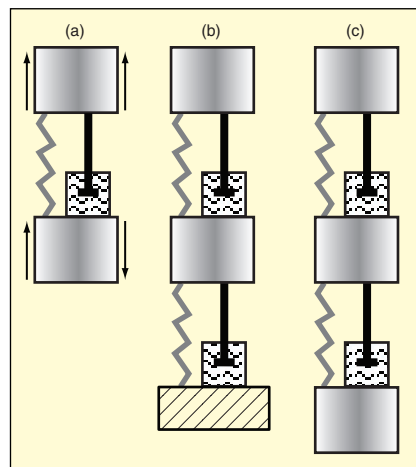


Figure 3. The 2-DOF and 3-DOF vibration “molecules.”

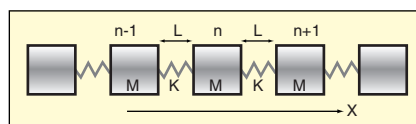


Figure 4. An infinite linear chain of masses and springs becomes the continuum media with $L \rightarrow 0$.

The Kelvin model, connected with one end attached to the mass and with the other end to an immobile base (i.e., Earth), describes single-degree-of-freedom vibration. The Maxwell model simulates creep.

To explain why the Maxwell model can-

not be used for vibration simulation, consider an ideal situation: the viscosity of the dashpot liquid is zero, so the dashpot creates no resistance to the spring movement. In this case, the spring cannot be deformed at all – it just moves like a rigid body with the mass element. More complicated vibration molecules are two-degree-of-freedom mechanical systems. One of them (Figure 3a) consists of two masses connected through the Kelvin body and has only one vibration mode (right-side arrows).

The other is a rigid-body mode of motion (left-side arrows); both masses move synchronously and therefore the spring is not deformed. The other two-degree-of-freedom mechanical system (Figure 3b) consists of two masses and two Kelvin models, with one of the latter being connected to an immobile base. In this case, both motional modes describe vibration. One- and two-degree-of-freedom mechanical systems are classical and are often applied to the simulation of various vibration isolators – Helmholtz resonators, single and double partitions, shock absorbers, etc.

The three-DOF mechanical system (Figure 3c) is also of practical use; in particular, it interprets sound transmission via triple partitions at low frequencies. It has two vibration modes and one rigid-body mode. Engineers know that all models are of limited use in describing the real effects, so the calculated results may be not very accurate. In recent years, there has been a tendency to undermine the importance of simple mechanical models and to rely on computer modeling. However, most experienced specialists continue to find simple models the easiest way to interpret most practical vibration phenomena and to develop technical solutions for vibration control.

From Simple Vibration Systems (molecules) to Elastic Waves in Continuous Media. Consider an infinite linear chain of identical masses connected to each other through similar springs along the X-axis. Each mass element is assumed to have mass M and negligible size. Each spring has length L and stiffness K . The external and damping forces in the chain are ignored.

If w_n is the coordinate of the n th mass, its motion is described as:

$$M \frac{d^2 w_n}{dt^2} + K(w_n - w_{n+1}) + K(w_n - w_{n-1}) = 0$$

This equation can be also take the form:

$$M \frac{d^2 w_n}{dt^2} - KL^2 \left(\frac{w_{n+1} - 2w_n + w_{n-1}}{L^2} \right) = 0 \quad (1)$$

The term in parentheses is the finite-difference approximation of $\partial^2 w / \partial x^2$ so, with the length L approaching the infinitesimal ($L \rightarrow 0$), Equation 1 reduces to the one-dimensional wave equation for continuous media:

$$\frac{\partial^2 w}{\partial t^2} - c^2 \frac{\partial^2 w}{\partial x^2} = 0 \quad (2)$$

where:

$$c = L \sqrt{\frac{K}{M}} \quad (3)$$

is the speed of sound.

For a uniform elastic layer, $K = ES/L$ and $M = \rho SL$; where E and ρ are respectively

Young's modulus and the density of the material and S is the area of the layer. Substituting these relationships into Equation 3, we obtain:

$$c = \sqrt{\frac{E}{\rho}} \quad (4)$$

This simple derivation proves that as in the world of real molecules, the multiple vibration "molecules" approximate the continuum.

Back to Chemistry. The Periodic Table of Elements was not Mendeleev's only achievement. He also was a fruitful researcher in explosives, petroleum, and fuels. His fascination with molecular

weights led him to conclude that to be in perfect molecular balance, vodka should be produced in the ratio of one molecule of ethyl alcohol diluted with two molecules of water, giving a dilution by volume of approximately 38% alcohol to 62% water. As a result of this work, a standard for vodka was introduced in Russia specifying that all vodka be produced at 40% alcohol by volume. Occasionally earthshaking, this interesting fact has nothing to do with the theory of vibration. S.V.

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