Review of Recent Developments in Multiple-Reference Impact Testing

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Impact testing was one of the first practical applications of the fast-Fourier transform (FFT) technique in the late 1960s. Prior to developing the FFT, measuring the frequency response function (FRF) was limited to sinusoidal testing procedures. The sine testing methods were slow, and required elaborate fixtures for excitation (electro-mechanical or hydraulic exciters). Impact testing provided an order of magnitude faster test time and minimal fixtures. As a result, it became a very good field-testing and troubleshooting method, as well as a pretesting method for controlled laboratory testing. This article is a general review of the evolution of impact testing from its development in the 1960s and '70s to the present time, with a more extensive review of recent developments in testing procedures and parameter estimation for multiple-reference impact testing (MRIT).

In the mid 1960s, the mathematical properties of the Fourier transform were well known, but its applications were limited. It was the development of the FFT that made the numerical computations of the Fourier transform practical. The FFT was a revolutionary breakthrough that led to many developments in digital signal processing; these were applied in many disciplines, including acoustics, controls, and structural dynamics.

In 1966 and '67, a project to develop a transient testing procedure for measuring frequency response functions was initiated for a master's thesis. In this initial effort, an impact hammer was used to excite a machine tool structure, with measurement of the transient input and response on an FM tape recorder. Tape loops of the transient responses were played back into a transfer function analyzer (TFA). The input and response signals were processed by using the tracking filters in the TFA to filter and ratio the response to the input signal, thereby estimating the frequency response between the input signal and the response signal. This method proved to be impractical due to signal-to-noise problems. However, it triggered the investigation of other methodologies including Fourier analysis.

A prototype real-time analyzer was made available by Spectral Dynamics Research Corporation in the late '60s, and this single-channel spectrum analyzer was used to estimate the response spectrum from an impact to a machine tool base. This spectrum measurement had good agreement with the response spectrum estimated from the measured FRF using sinusoidal testing. Based on this result, a serious effort was initiated to develop a measurement process that would use the newly developed FFT algorithm to estimate an FRF from the FFTs of the digitized input and responses signals. In 1968 and '69, the large applied dynamics computer in the Department of Electrical Engineering at the University of Cincinnati was used to develop a software program that used the analog part of the hybrid computer to digitize the force and the response (accelerometer) signal measured by testing a machine tool base. The IBM 1130 computer (digital part of a hybrid computer) was used to compute FRFs and coherence functions. These measurements were compared to the FRF function measured with a Spectral Dynamics transfer analyzer, and the comparisons were good.

The hybrid computer filled a complete room. As a result, only small test objects could be taken into the computer room to be tested in real time. For large test articles, the measurement data had to be recorded on an AM or FM tape recorder; these recorded data were processed with the hybrid computer. In general, this was the same situation in other organizations where data were recorded and processed in their computer centers.

In the late '60s and early '70s, small minicomputer systems manufactured by Hewlett-Packard and Time Data Corporation (DEC computer) became available; they were portable and used Fourier analysis. These systems could be located next to the test article, making laboratory and field testing practical. These systems were an important step in the evolution of measurements from the analog to digital arena.

Transient Testing Developments

The advent of the portable Fourier analyzer system totally revolutionized the experimental measurement arena in the early '70s. The revolution was the ability to measure power spectrum and frequency response functions using a wide variety of different signal types (sinusoidal, random and transient). Note that a portable two-channel system in the early '70s was the size of a large TV or small refrigerator instead of a room full of equipment.

This article concentrates on the evolution of testing methods utilizing transient signal types. We will briefly itemize important developments of the 1970s and 1980s and will mainly emphasize the developments in the late 1980s thru the 2000s.

1970s – Test Procedures and Digital Signal Processing

The University of Cincinnati Structural Dynamics Research Laboratory (UC/SDRL), was loaned a prototype HP 5450 Fourier

Acronyms	
3D	Three dimensional
ADC	Analog digital converter
ARMA	Autoregressive moving average
CMIF	Complex-mode indicator function
EFRF	Enhanced frequency response function
EMIF	Enhanced-mode indicator function
ESSV	Extended-state space vector
DFRF	Directional frequency response function
DOF	Degrees of freedom
DOT	Department of transportation
DSIT	Digital system interface transmitter
DSP	Digital signal processing
DSS	Digital sensor system
ERA	Eigenvalue realization algorithm
GVT	Ground vibration test
FEM	Finite-element model
FRF	Frequency response function
FFT	Fast-Fourier transform
HP	Hewlett Packard
IRF	Impulse response function
LSCE	Least-squares complex exponential
MAC	Modal assurance criteria
MDOF	Multiple degrees of-freedom
MIMO	Multiple-input multiple-output
MRIT	Multiple-reference impact testing
NASA	National Aeronautics and Space Administration
PTD	Polyreference time domain
SDA	System dynamic analysis
SDOF	Single degree of freedom
SDRL	Structural Dynamics Research Laboratory
SDRC	Structural Dynamics Research Corporation
SST	Spatial sine testing
SVD	Singular value decomposition
UC	University of Cincinnati
UMPA	Unified-matrix polynomial algorithm
UIF	Unit impulse function

analyzer system in 1970. The HP 5450 was based on the HP 2114 8K minicomputer system. This system was used initially to evaluate the potential of using Fourier analysis with transient testing. A graduate student performed an impact test on a small milling machine that had been previously evaluated for its chatter characteristics. The modal characteristics and important directional frequency response functions (DFRF) had been measured using standard analog techniques of the 1960s, and these characteristics were measured with impact testing. The results were very encouraging.

This testing was discussed in a technical note from Hewlett-Packard[1] and a paper[2] presented at a conference at the University of Birmingham in England, including a live demonstration of impact testing as part of the conference. After the conference, a seminar and a demonstration of impact testing was presented at the University of Leuven in Belgium. This was the start of a very high-energy effort to develop an understanding of the digital signal process and the measurement aspects of Fourier analysis. This effort spanned the decade of the '70s.

Based on these results, the HP system was updated to a HP 5451A system, and a more extensive set of tests was performed by the undergraduate advanced vibration class (class of 1971 and '72). Groups of 2 to 3 students were given a project to use impact or step relaxation for testing of a variety of test objects. There were approximately 10 groups per class. Fourier analysis was in its infancy, and the student projects actually contributed significantly to the transient testing state of the art. Some of the contributions of the student projects to transient testing:

Impact Testing
- An impact hammer with a built-in load cell was developed (Hammer could easily be roved.)
- A load cell with an impact surface was mounted on the structure. (The load cell measured the transient input force and the transient force due the mass added by the load cell. Thus, the mass additive effect was taken into consideration. This testing condition can be used when the accelerometer is roved.)
- A ratio calibration process was developed for calibrating instrumented hammers.
- Tips and mass of impact hammers can be changed to control frequency content of the input.
- Moving systems can be impacted. (A student group wrote a technical paper describing the use of impact testing to measure the FRF of a rotating spindle with a hydrodynamic bearing where the stiffness of the bearing depended on the rotation of the spindle.[3]

Unit Step Function (step relaxation testing)
- A large machine tool isolation foundation was tested with a step relaxation method and a response ratio method for measuring the mode shapes. The force was not measured, and an accelerometer mounted at the input point was used to trigger data acquisition and to serve as the reference sensor.
- A small shear model of a high-rise building was tested using a load cell to measure the input force. The FRFs were computed using step input. The input force and response channels were AC-coupled to reduce the influence of the DC component.

The major activity in the 1970s was the development of signal processing techniques to make good measurements with different excitation methods. The secondary effort was the development of parameter estimation methods that could be used to extract characteristic functions (modal parameters and impedance functions) that could be used to characterize the systems being tested. Some of the important developments related to transient testing in the 1970s are summarized here:

Special Transient Testing Windows. In the early '70s, the most important signal processing development for impact testing was the development of force and exponential windows. This development was the fallout from work being done by Ron Potter, who was the person at HP primarily responsible for the development of the HP 5450 series Fourier analyzer system. He was the real guru of Fourier analysis in the late '60s and '70s. As part of one of his activities, he was developing a parameter estimation algorithm for extracting modal parameters. He was trying to get starting values

for the modal parameters, using the shift theorem of the Fourier transform to reduce the apparent damping in his FRF measurements. The shift theorem states that multiplying a time function by an exponential function will shift the damping or frequency axis in the transformed domain. In other words, the apparent damping of a system can be changed in a predictable way simply by multiplying the unit impulse response measurement of a system by a damped exponential. It became apparent that this could be used as a window to eliminate "leakage errors" and to improve the signal to noise ratio of impact measurements. As result, an exponential window could be applied to both the input force and the output response signal. An additional force window could be applied to the force signal to eliminate noise on the force channel after the impact. These windows have been documented in a number of references.[4,5]

Animated Display. Impact testing was often used for troubleshooting vibration problems in the field. The ability to visualize mode shapes quickly and conveniently is especially important in the field. Initially, the mode shapes were plotted by hand, which was time consuming. Several plotting routines were developed in which the mode shapes could be plotted in the field. Some of these plotting programs would generate 3D images that could be viewed with special training (crossed-eye 3D images). In general, this was not completely satisfactory, since many people had difficultly viewing these images; as a result, this technique was quickly abandoned as a functional method. Later, there was the development of an animated display for the HP 5451B system, in which the mode shapes could be viewed directly on the display. This was revolutionary.

Completion Algorithms. In testing, there were often only a few points defined on a given component and measurement at a given point in only a few directions. These conditions led to confusing displays. These findings led to the development of methods where the measured DOFs could be used to estimate missing DOFs. One method was to estimate rigid-body properties for a rigid component or for a section of the component that behaved in a rigid fashion. The rigid body properties were estimated from the measured data, and the resulting rigid body characteristics could be used to interpolate and predict the response for missing DOFs. A second method used a slave DOF (a point whose motion is the same as a measured point to estimate missing DOFs. Completion algorithms were particularly important for impact testing, since it is difficult or impossible to make measurements at certain points and in certain directions.

Reducing Periodic Noise. One of the negative aspects of the exponential impact window is that periodic noise components are smeared by the window over a frequency range centered around the frequency of the periodic noise. In an initial effort in the 1970s, the DC component and frequency components that were periodic in the window were filtered by taking the FFT of both input and response channels and setting the Fourier coefficient of the noise components to the mean values of the adjacent Fourier coefficients. The data were then transformed back into the time domain. The force/exponential window could then be applied to the force and response signals.

Impact Hammers. A modally tuned hammer was developed in which the influence of the modes of the hammer was controlled to reduce artifacts in the FRF measurements due to the hammer dynamics. A wide range of impactors was developed, ranging from very small hammers to large masses, whose weight could exceed hundreds of pounds and could be used as a pendulum to impact large test objects.

Parameter estimation for transient testing methods, particularly troubleshooting applications, were normally restricted to simple SDOF methods, with quadrature being the most popular. However, MDOF algorithms were developed during the '70s, including: complex exponential algorithm (CEA)[6] Ibrahim time domain algorithm(ITD)[7] and the least-squares complex exponential algorithm (LSCE)[8]

1980s – Multiple-Channel Data Acquisition
The major innovations in transient testing methodology were

accomplished in the 1960s and '70s. In the 1980s, developments in data acquisition and parameter estimation led to the significant advancements in multiple-reference impact testing (MRIT) of the '90s.

Data Acquisition Developments. It became clear in the '70s that to improve modal parameter estimation, it would be necessary to develop multiple-reference parameter estimation algorithms and affordable multiple-channel data acquisition systems. One of the major problems with parameter estimation was the consistency of the measurement database. Measurements taken at difference times from different reference points were inconsistent. As a result, estimated modal parameters were inconsistent even when the fit to individual measurements appeared to be excellent. To address this problem, a large multiple-channel affordable data acquisition system would be required. In the late '70s, the groundwork for the application of a multiple-channel acquisition system was developed with the formulation of the multiple-input/multiple-output (MIMO) FRF technique. Initially, a four-channel system with two inputs and two outputs was used, followed by testing with an eight-channel system with two inputs and six roving response channels. Several vehicles were tested in this manner with encouraging results.

The dream was a system with two to four inputs and hundreds of responses that could be measure simultaneously and multiple reference parameter estimation algorithms that could extract modal data from this set of measurements.

Parameter Estimation Developments. In the early 1980s, with the breakthrough development of the polyreference time domain algorithm (PTD)[9] the parameter estimation part of the dream came true. This was followed by the development of the eigenvalue realization algorithm (ERA)[10] a few years later. The PTD algorithm was a multiple-reference version of the least-square complex exponential algorithm, and the ERA algorithm was effectively a multiple-reference version of the ITD method. The PTD method could run effectively in a small minicomputer system; however, in the early '80s, the ERA required a larger mainframe computer. As a result, two different groups of users were employing the two methods: NASA was the primary user of ERA; industrial users (machine tool, auto companies, etc) used the PTD method. By the mid '80s, a more general unifying approach to parameter estimation was being developed. The unified matrix polynomial approach (UMPA) concept was being developed in the late '80s and early '90s. Using the UMPA concept, all important parameter algorithms could be rederived from a common starting point.

Important mathematical techniques like singular value decomposition (SVD) became a significant part of these developments. In fact, a parameter estimation procedure based on SVD, the complex mode indication function (CMIF), was developed in the late '80s and perfected in the early '90s. It became a standard parameter estimation tool used with multiple-reference impact testing (MRIT) in the '90s

The second part of the dream, the ability to measure hundreds of channels simultaneously, took a little bit longer to develop. In 1981, Boeing Aircraft Company conducted the first large-scale modal test of the Boeing 767, where up to 128 channels of responses could be measured simultaneously. The raw time data was recorded to a large disk file and was post-processed into FRF measurements. This was the ideal case; by recording the raw data, it was available for post-processing after the test object was released. Different signal processing could be used to enhance certain aspects of the analysis, such as zooming into a certain frequency band to enhance an important target mode.

Unfortunately, this type of data acquisition was too expensive in the early '80s for most users. The development of the Struccel system[11] in the mid '80s significantly reduced the cost of the multiple-channel sensor systems. The widespread demand for CD players and further developments in digital music led to mass production of delta sigma ADCs that were available and very inexpensive by the early '90s. This made it possible to design a very inexpensive multiple-channel data acquisition system. The possibility of conducting a test with hundreds of channels became practical in the 90s for groups outside the aerospace and auto industries.

Figure 1. Seymour Street Bridge.

1990s to Present – Transient Testing

Advances in state-of-the-art data acquisition and modal testing can be clearly illustrated by comparing impact testing between the 1970s and 2000s. In the '70s, a two-channel Fourier analyzer system cost approximately $75,000-$100,000, and senior undergraduate engineering students used this system to measure the modal parameters of a guitar. In 2002, a 13-year-old middle school student competing in a science fair was given a small instrumented hammer and an inexpensive accelerometer. He programmed the soundboard in his $600 PC to collect data from a guitar using impact testing with a pseudo-random sequence of impacts. Given access to MATLAB, minor help from his father and a MATLAB animation program, he duplicated the effort of the students of the '70s and was a major winner in his science fair. This comparison demonstrates the clear advancements in the cost of data acquisition and the availability of powerful computational tools. An ordinary PC of the 2000s is much more powerful than the most powerful main frame of the 1970s; data acquisition costs have been reduced by orders of magnitude; and new parameter estimation techniques with powerful new mathematical tools and concepts were available by the early 2000s.

Transient Parameter Estimation Procedures. Obviously, any parameter estimation procedure developed to extract modal parameters from measured FRFs can be used with FRFs measured with transient testing. With the development of rather inexpensive and portable data acquisition systems in the early '90s, multiple-reference impact testing (MRIT) became practical. In MRIT, a large number of excitation points are used in the testing. This significantly increases the amount of redundant spatial information available in the parameter estimation process, which improves the possibility of uncoupling closely coupled modes. The measurement of this enhanced multiple-reference database led to the development of specialized multiple-reference parameter estimation algorithms for transient testing in the early '90s.

In the late '80s, an inexpensive multiple channel sine testing system was developed to take advantage of the additional spatial information. This testing method was identified as the spatial sine testing method (SST), where a large number of electro-mechanical exciters were distributed on a structure, and at a given frequency, a number of forcing patterns were used to excite the structure. This process was repeated at a number of frequencies, and the forcing vectors and resulting response vectors were processed with a first-order UMPA model to estimate the modal parameters. Details of this method can be found in Reference 12.

In the early '90s, a major infrastructure test program was initiated by the Ohio Department of Transportation. This program was initiated to develop a procedure for testing medium-sized bridges, typical of those found on interstate highways, for accidental damage due to seismic events, flood, or accidental impact from large vehicles. The standard inspection method includes visual inspection and measurement of static deflection due to large loads.

The initial effort used changes in the bridge's modal parameters as a measure of its health. This study had limited success. It was discovered that significant measured changes in discrete modal parameters were not sensitive enough to reliably predict the health
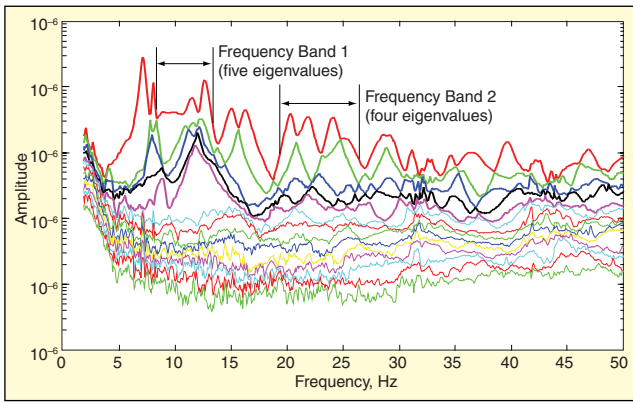
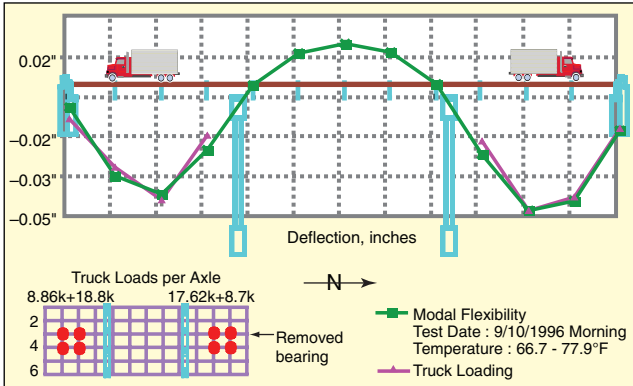*Figure 2. CMIF plot, Seymour Street Bridge.*

Figure 3. Typical comparison between static deformation due to truck loads and predicted deformation from modal model.

Figure 4. EMIF reconstruction of Band 1, Figure 1, for Seymour Street Bridge.

of a bridge. In fact, the deformation of the bridge to static loads was a better indicator of significant bridge damage. The static deformation is a measure of the simultaneous contribution of a number of bridge modes. Unfortunately, it was easy to load the bridge but was difficult to measure its static deformation. Today, with some of the newer laser systems available, it may become more practical.

The proposed solution was to experimentally measure a modal model of the bridge and predict its flexibility to a variety of diagnostic loading conditions. This required the ability to not only get estimates of the bridges eigenvalues and eigenvectors but also to generate a scaled modal model.

A bridge was located that was scheduled to be demolished. The bridge (Figure 1) was in reasonably good shape but was being demolished because the road it serviced was being retired. The first phase of the project was to determine the best testing procedure. The Civil Engineering Department of the University of Cincinnati was responsible for the project, and they had access to both large hydraulic, electromechanical and impact exciter systems. An initial study indicated that a MRIT test was the best testing method based on the requirement for a fast testing cycle. The bridge testing process used is described here later; see References 13 and 14.

A MRIT testing procedure was used to generate multiple reference FRF data sets, and the commercial parameter estimation of the early '90s was used to process these data sets with little success. The criterion for success was to predict the static deflection of the bridge to statically applied truck loads. A special load frame was built to measure the static deflection of the bridge using an array of potentiometers connected between bridge and load frame. The initial expectation was that the static deformation would only require getting good estimates of the lowest 10 or so modes of the bridge and was assumed to be possible.

Unfortunately even for the lower modes, the problem of trying to sort out good estimates of the modal parameters from computational parameters was determined to be impractical using the MDOF parameter estimation techniques available at the time. It was not possible to generate a modal model that gave results that correlated well to the measured static deformations. Fortunately, a technique developed in the mid to late '80s as a mode indicator
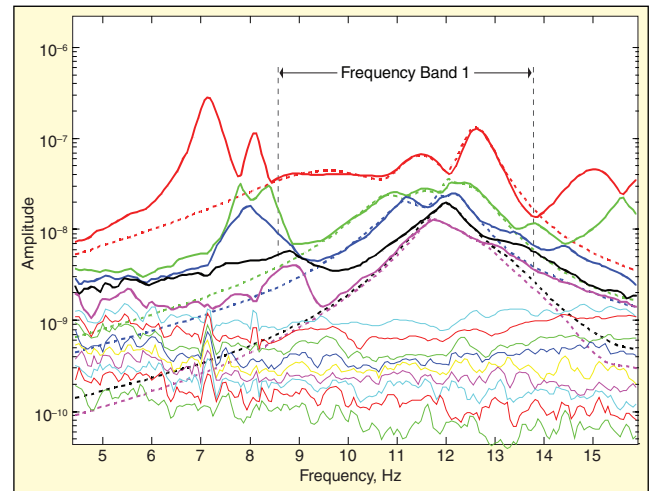
function (CMIF) was modified to give estimates of the eigenvectors; then using the estimates of eigenvectors as modal filters to estimate an enhanced frequency response function (EFRF). The eigenvalues and the modal scale factor could be extracted from the EFRF.

The resulting eigenvalues, eigenvectors and modal scale factors defined a modal model of the system. The modal model generated with this method successfully predicted the static deflection of the bridge-to-truck loads. A complete description of this method is documented in Reference 15. Figure 2 shows a typical CMIF plot for the bridge, and Figure 3 shows a comparison between predicted static deflection of the bridge due to truck loads and measured deflection. Note that the static deflection was not measured completely across the bridge using potentiometers because of the difficulty in constructing a load frame the length of bridge with traffic flow under the bridge.

For cases in which the modal density makes it difficult to use the CMIF method to extract the modal parameters, a method developed in the late '80s as part of the spatial sine testing development was modified for MRIT testing. This enhanced-mode indicator function (EMIF) method used a first-order frequency domain UMPA parameter estimation model. The mathematical formulation of the model is shown in the following equation:

$$\left[(j\omega)[A_1]+[A_0]\right]\{\bar{x}(\omega)\}_k = \left[\sum_{m=10}^{M}(j\omega)^m[B_m]\right]\{\bar{F}(\omega)\}_k \quad (1)$$

where:

$[A_1]$ = response coefficient matrix
$[B_j]$ = input coefficient matrix
$\{X(\omega)\}$ = response vector
$\{F(\omega)\}$ = input vector
M = order-of-input polynominal
K = Index for k^{th} equation

The number of eigenvalues estimated by the first-order UMPA[16] was fixed based on the number of eigenvalues observed to be active in the narrow frequency band of interest using the CMIF plot of the quadrature FRF responses. The FRF matrix is a 3D matrix of (input times the response times the frequency) for the structure being tested. The EMIF eigenvalues are an average value of the estimates of the eigenvalues where the A_0 and A_1 were used to normalize the UMPA model in a least-squares sense. This is similar to the normalization used in the (H_1 or H_2) FRF estimation process. For a particularly difficult region of the CMIF plot (see Figure 4), the CMIF plot for the fit region of the measured FRF matrix and the synthesized FRFs that were estimated using the EMIF algorithm is shown for comparison. The correlation is very good.

In the 2000s, better formulations of the overspecified UMPA models and automated methods for sorting of the computational and real eigenvalues makes it possible for novices to use commercial software to obtain acceptable modal parameters from MRIT testing of complex structures like bridge testing of the early '90s. The general UMPA model formulation is given below for the time

and frequency domains.[17]

UMPA Models – General Formulation for k^{th} equation

$$\sum_{i=0}^{n}[A_i]\{x(t_{k+1})\} = \sum_{j=0}^{m}[B_i]\{f(t_{k+j})\} \quad \text{Time Domain} \quad (2)$$

$$\sum_{i=0}^{n}(s_k)^i[A_i]\{X(s_k)\} = \sum_{j=0}^{m}(s_k)^i[B_j]\{F(s_k)\} \quad \begin{array}{c}\text{Generalized}\\\text{Frequency}\\\text{Domain}\end{array} \quad (3)$$

where:

t = time
s = scaled frequency (if $s=j\omega$, ω is the unscaled frequency)
$[A]$ = response coefficient matrix where responses can be virtual
$[B]$ = input coefficient matrix
$\{x\}$ = response vector – time response
$\{f\}$ = input vector – time response
$\{X\}$ = response vector – generalized responses can include states
$\{F\}$ = input vector – generalized response
n = order-of-input polynomial
m = order-of-response polynomial
= number

In the 2000s, there were several significant advancements in implementing the parameter estimation processes that made it easier for the user. One of the biggest problems for users involved selecting a realistic set of eigenvalues from an overdetermined set of estimates. In general, the UMPA model is drastically overspecified to help reduce the influence of noise. The process increases the order of the UMPA model that likewise increases the estimated number of eigenvalues. The real eigenvalues of the system is a subset of the estimated eigenvalues.

The solution process involves:

1. Selecting a particular UMPA model.
   a. Time Domain – equations are very well conditioned.
   b. Generalize frequencies domain.
      i. Unscaled frequency – very poorly conditioned.
      ii. There has been much research on how to scale the frequency to improve the conditioning of the solution.
         1. Normalized frequency – frequency is scaled by maximum frequency so frequency values are between plus and minus one.
         2. Orthogonal polynomials – effectively reduce the order of the polynomial.
         3. Reduce the order of the polynomial by state-space expansion or by measuring more response points.
         4. Complex z mapping – effectively maps the data to the unit circle or uses the equivalent of the inverse Fourier transform, which effectively maps the frequency domain data to the scaling of the time domain. As a result, the conditioning is similar to the time domain.
   c. High-order model.
   d. Low-order model.
2. Solving a set of linear equations for the coefficients of a UMPA model.
   a. There are many more equations than unknowns so that a pseudo inverse solution is required.
      i. Selection of objective function (function for minimizing the error).
         1. Normalizing equations so that $[A_n]$ equals the identity matrix. In this case the damping is overestimated; in other words, the estimated eigenvalues appear to be more heavily damped. In general, this makes it more difficult to sort out the computational modes.
         2. Normalizing equations so that $[A_0]$ equals the identity matrix. The damping of the eigenvalues is underestimated. This makes it easier to sort out the computational modes. In fact, many computational modes show negative damping, which is unrealistic, making it easy to reject these modes. As a result, this is often the objective function used. One of the negative aspects is that very lightly damped system modes will often have negative damping.

         3. A total least-squares objective function can be used.
      ii. Least-squares solutions.
         1. Normal equation solution.
         2. Total least squares – eigenvalue solution.
         3. etc.
      iii. Using transformations.
         1. SVD.
         2. LU, etc.
3. Solving for the eigenvalues/eigenvectors of the UMPA model.
4. Filtering the estimated eigenvalues into system eigenvalues and computational eigenvalues due to the overspecification of the model. This was an area that had the biggest impact in the 2000s in terms of usability. This is also the step where there is a little black magic or art involved in the process. Every vender of commercial parameter estimation software has made an effort to make this step more intuitive or possibly more autonomic. Note that Steps 1-3 in the solution process and the characteristics of the FRF database have a big influence on the filtering process. We will not detail the many methods for filtering the data here but will concentrate on methods that work well for MRIT testing.

With MRIT testing, a large number of excitation points are typically used during the test. For a typical MRIT test, 10 to 30 impacting locations are often used. For the previously described bridge, 15 excitation points were used, and up to 128 response points were measured. As a result, there is significantly more spatial information available from a MRIT test. This spatial information can be used to help filter out computational information. The methods used in the early '90s for the bridge test were the CMIF or EMIF method. The CMIF method is a purely spatial domain method where the eigenvectors are estimated from the SVD of the FRF matrix. The EMIF uses a reduced first-order frequency domain UMPA model that can solve for a fixed number of eigenvalues in the frequency range of interest.

In the early 2000s, an alternate method using a more conventional UMPA model approach was used with the MRIT database. In general, a first-order UMPA model in either the time or frequency domain can be used. In the time domain, the UMPA model is the equivalent of the ERA algorithm. The first-order ERA algorithm is generated by a state expansion of the second-order equations of motion. For the bridge example with 128 response points, the UMPA model would have a solution with 256 eigenvalues, and the eigenvector would be a state space vector with length of 256. The state space vector is the system's eigenvector augmented by the system's eigenvector multiplied by its eigenvalues. Note that the UMPA can be further expanded by adding additional state space expansions where each expansion would expand the solution for this example by 128 eigenvalues. This state space vector is referred to as the extended-state space vector (ESSV)[18]. Using the ESSV will generate a model with hundreds of computational modes. To filter the computational modes from the systems modes, the correlation between the state space vectors estimated by using the A_0 and A_1 solutions for the same model can be used. The system modes should be highly correlated and computational modes poorly correlated. The correlation value can be used as the filter cutoff. Note that the correlation computation is equivalent to the modal assurance criteria (MAC) often used to compare modal vectors.

We have used this procedure successfully with MRIT testing for troubleshooting applications since the early 2000s. Specialized MATLAB programs and the X-MODAL program developed by UC/SDRL have been used in the data processing for MRIT testing.

To demonstrate the application of using an ESSV as a spatial filter, the following MRIT data set will be used to extract the eigenvectors and eigenvalues with a specialized MATLAB program. Note that this same type of analysis can be done using X-MODAL. The data set to be analyzed was taken from a multiple reference data set taken in the late 1970s on a circular plate and has been used as an example often and in many modal courses over the past 30 or more years. The data were taken with a two-channel analyzer by mounting a transducer at a point on the structure and impacting at 36 points, moving the transducer to 6 other points, and repeating the process. By using reciprocity, this generated a FRF 3D matrix (seven inputs by seven responses by 512 spectral lines). The CMIF
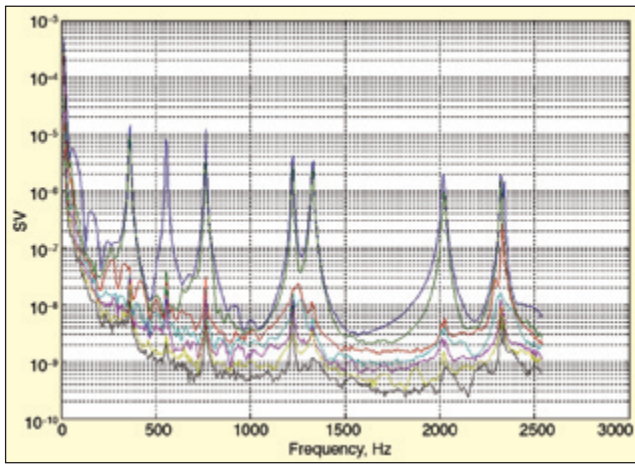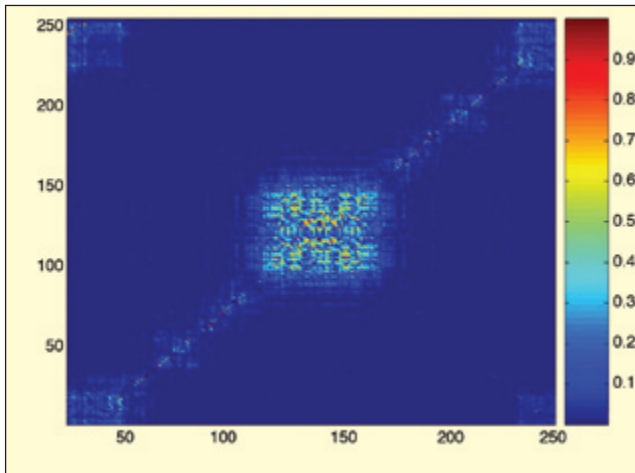
Figure 5. CMIF plot, C-plate.

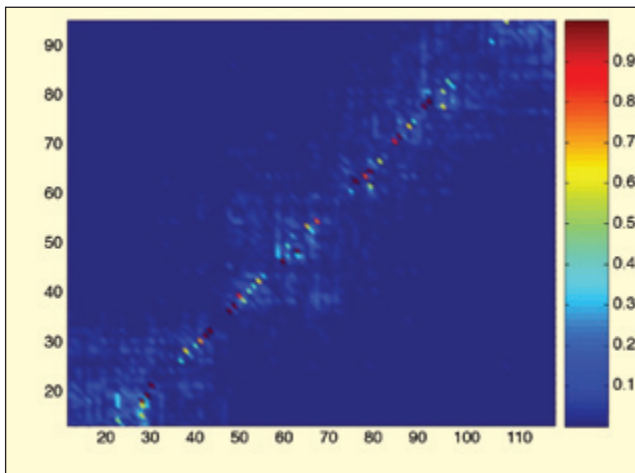Figure 6. P-color plot of MAC between A_0 and A_1 eigenvector estimates.

Figure 7. Zoom of Figure 6 showing filter poles that appear as small, dark, brown spots.
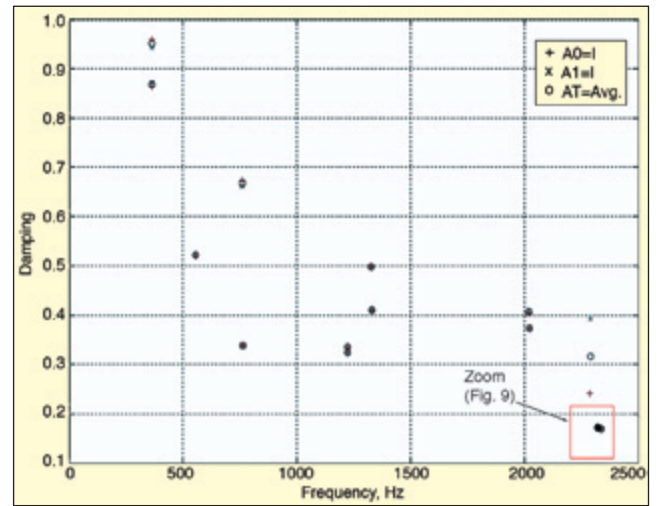

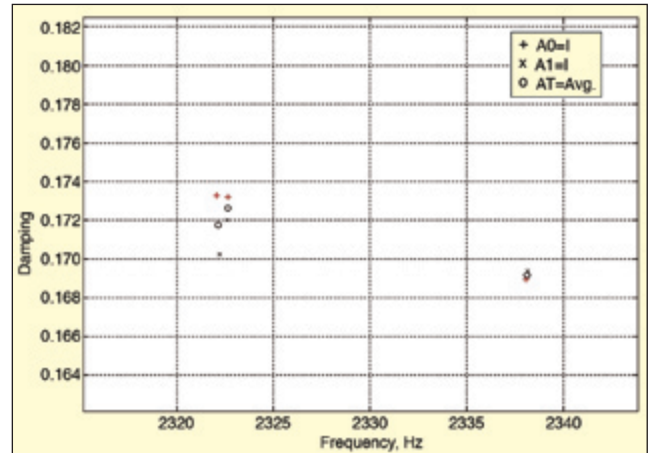Figure 8. C-plate filtered pole plot; Figure 9 shows zoomed area.

Figure 9. Zoom of region in Figure 8 showing three closely coupled modes, a repeated mode around 2322 Hz, and a single mode around 2338Hz. Legend shows the pole location for the A_0, A_1 and average estimates for each pole.

plot for this data set is shown in Figure 5. This data set is a historical data set but the C-plate structure is a standard test article. C-plate-type structures have been tested over the past 30 years with every new version of data acquisition and sensors, and they have been analyzed with nearly every type of parameter estimation algorithm. This original data set has been used as a test case for evaluating many software packages. It has repeated modes, is lightly damped and the data has a number of inconsistencies. This data set is typical of a MRIT dataset. Note that in a more modern test, all the response data would be taken simultaneously, which would generate a more consistent data set.

The data were analyzed with a standard UMPA model emulating the ERA time domain algorithm with an extended-state space

vector with seven state space shifts or extensions. This extended model will generate 7×36, or 252, eigenvalues, while the CMIF plot indicates that there are 28 eigenvalues. The eigenvalues are filtered by comparing the MAC value between the A_0 and A_1 solutions estimated eigenvectors. The filter cutoff was set to accept any MAC value greater than 0.95. Note that for this data set, all the selected data have MAC values greater than 0.99. In Figure 6, a MATLAB P-color map (252×252) of the MAC values is shown. In Figure 7, a zoomed region is shown so that highly correlated eigenvalues are visible. The small dark brown spots are highly correlated solutions. The correlated poles (eigenvalues) are plotted in Figure 8 and a zoom of the region around the three highest frequencies modes is shown in Figure 9. For each pole, the estimated pole from the A_0 and A_1 solutions is plotted and the average value of the two solutions is plotted. The average value is the value used as the estimate for the pole. From experience, using this method with analytical data sets, the average values are generally a better estimate than either the A_0 or A_1 solutions.

There are two parameters used in the process:
- The number of state space extensions: In ERA, a state space extension is often described as adding a virtual measurement set to the data set, where the virtual measurement set is simply a time shift of the previous measurement set.
- The MAC value for the filter cut-off: To select the cutoff, a stability type plot can be generated, where the filtered cutoff is plotted versus pole frequency. As the MAC cutoff is reduced, more modes will become visible. The CMIF plot can be used as a guide to how many modes are active in a given frequency range. In the end, however, it requires an engineering judgment to make a good decision as to what is a reasonable cutoff. In

Figure 10. Ground vibration testing, a new concept in 2003.

other words, like most things associated with modal parameter estimation, experience is important.

For narrow frequency ranges, the generalized frequency equivalent of the ERA algorithm can be used with either unscaled or scaled frequency in the same manner as in the example given above.

In the future, a more complete examination of this type of spatial filtering will be developed with several more detailed case histories.

1990s to Present – Transient Testing Procedures

Two historical transient testing procedures were developed in the 1970s. In one testing procedure, the transient inputs were input at one or more points, and roving response measurements were taken to estimate the FRFs of interest. In the second method, reference response sensors were mounted at the input points, and a roving transient force (normally an impact but occasionally a step relaxation) was used to excite the points of interest. Reciprocity was used to determine the FRFs of interest. In both cases, transient testing makes it is easy to measure data from a large number of reference points, since there is minimal fixturing and data acquisition involved. As a result, transient testing has become a powerful field-testing and troubleshooting tool. Historically impact testing has been the transient method of choice probably more than 90% of the time.

In the 1990s and 2000s, a large MIMO modal test using exciters involves the use of two to four simultaneous exciters and hundreds of response sensors (normally accelerometers). A number of exciter configurations may be used in the overall modal testing program. A single data set consists of the data taken from one of the MIMO configurations.

A large MRIT modal test involves mounting potentially tens or hundreds of sensors and impacting at a large number of fixed input points. A different impacting device can be used at the various input points. One data set consists of a single input and all of the response points. For MRIT testing, it is not necessary to use MIMO signal processing.

Since the MIMO testing procedure is a more controlled test, it has been the method of choice for large laboratory modal testing. In the pretest phase, however, impact or transient testing has often been used to get an initial set of data to:
• Determine or check exciter and sensors locations.
• Obtain an initial estimate of frequencies, damping and modal density.
• Identify potential local modes that can present a problem during the testing.
• Identify rattles, clearances, or other local noise sources or nonlinearities.

In 2003, a new ground vibration testing (GVT) technique, was conducted at Boeing Aircraft (Figure 10) to evaluate a number of concepts:
• Using a reduced suite of sensors with the existing modeling technology for modal model verification.
• Transient testing methods as a primary modal testing method.

- MRIT impact testing.
- Step relaxation testing – transient force was not measured; in most cases, a known static force was applied and then released by suddenly cutting the static restraining line.
- Evaluation of amplitude-dependant modal parameters (non-linear ID).
• New digital sensor system (DSS) drastically reduced cabling requirements.
• Testing aircraft on it's landing gear with reduced tire pressure acting as a soft support.
• Noise floor of current generation of sensors.

The results of this program were encouraging related to the use of a reduced sensor set and the possibility of using transient testing procedures. Results are summarized in Reference 19. Impact testing has been used historically not only as a pretest procedure during a GVT but also for documenting aircraft components such as:
• Control surfaces that have been tested using impact testing by several aircraft vendors.
• Stores configurations that have been tested both by the Air Force and by vendors.

1990s to Present – Sensors and Data Acquisition

Data acquisition improved significantly in the late 1980s and early '90s by incorporating inexpensive 24-bit sigma-delta ADCs in data acquisition. Fairly portable multiple-channel acquisition systems and powerful notebook computers that are well suited to MRIT testing also became available. Eight- to 32-channel systems can conveniently be transported and used in the field. These systems make troubleshooting or testing of infrastructure in the field practical. Depending on the application, either a roving input or response could be used.

In the year 2000, a prototype data acquisition system, the digital sensor system (DSS), was developed. In it, a number of sensors could be mounted along a single wire. This system was demonstrated in the early 2000s at IMAC, JMAC, ISMA and several other conferences and organizations internationally. It significantly reduced the cabling problems for conducting a high-channel-count modal test. It was small and portable and worked well for MRIT applications. The digital components in the prototype system would not fit into a small modal sensor, so it required a small patch panel (digital system interface transmitter, or DSIT) to be mounted along the cable to interface with the sensor. A prototype seismic digital sensor that could be mounted on the cable was developed. However, to build smaller sensors, it would require developing an application specific integrated circuit (ASIC). To develop the ASIC and/or to further commercialize the existing system was judged to be too expensive by developers. As a result, DSS system development was stopped in the mid 2000s.

A steady improvement in the cost, sensitivity and size of sensors has improved from the '70s, with low cost, high sensitivity and small size being desirable. The fantasy of every test engineer has been wireless sensors. Wireless systems with a limited number of channels are commercially available. However, for large-channel-count applications with hundreds of sensors, the technology is still not practical. A wireless hammer channel would certainly be desirable for roving-hammer applications, and several hammers of this type have been developed but have not been widely accepted by the marketplace.

Latest Transient Testing Methodologies. The impact of the current state-of-art in sensors, data acquisition, parameter estimation and testing techniques on several prominent historical applications of transient testing are examined in this section.

Troubleshooting and Field Testing. One of the first applications that used transient testing techniques was troubleshooting vibration and acoustics problems (forced and self-excited) in the field. The testing techniques have not changed significantly from those used in the '70s. The difference is in the number of channels of data acquisition and the improvements in sensors. There are more channels of acquisition, and the ADCs have a 24-bit dynamic range. Historically, auto-ranging the data acquisition was important for obtaining good measurements. This was a very time consuming process, particularly when a number of acquisition channels were

Figure 11. Troubleshooting test to solve self-excitation problem, which was due to thermal baring of rubber roll in paper calendar stack.

Figure 12. Completed mode shape at measured points.

Figure 13. Completed mode shape at interpolated points.

used. The 24-bit ADC greatly reduces the magnitude of this problem. The newer generations of IPC sensors also have an increased dynamic range, which improves the process.

Data acquisition software has been modified so that if one of the response channels is overloaded during an impact, the data from the channel is rejected, and the gain of the overloaded channel is reduced for the next impact. Channels that are not overloaded are averaged. As a result, the number averages taken per channel is different, but good data are not rejected for the channels that are not overloaded. If the input is overloaded then all the channels are rejected, and the input channel gain is reduced for the next impact. Some software packages have the capability to process data in real time but can also save all the impact data for post-processing. If necessary, the saved raw data can be reprocessed at a later time for data enhancement if there are data analysis questions.

For testing a large piece of infrastructure like the bridges described previously, hundreds of sensors can be mounted on the structure, and the structure can be impacted at 10 to 30 input points. For the bridge testing where testing time is important, the sensors are premounted on the cabling and can be unwound and located on the bridge in a very short time. The bridge is impacted at 10 to 20 points to take a complete data set, the cables rewound, and the testing is moved to the next bridge.

For large troubleshooting projects where it is difficult to find good impacting points, a small array of triaxial accelerometers are roved over the structure. The triaxial sensors are located on different regions of the structure so that it is easy to quickly relocate them for the next measurement cycle.

Triaxial sensors are used to get a good three-dimensional animated display of the mode shapes during the testing process. Visualization is important for troubleshooting, since it is a very interactive process where new points are often selected on the fly.

A portable data acquisition system with 8 to 32 channels is typically used for these applications. A 32-channel system can be configured with 10 roving triaxial accelerometers and one fixed reference accelerometer located at an important point on the structure. For each measurement configuration, five to ten common input points are impacted. Standard MRIT signal processing for each measurement cycle is used.[12] The modal information is also
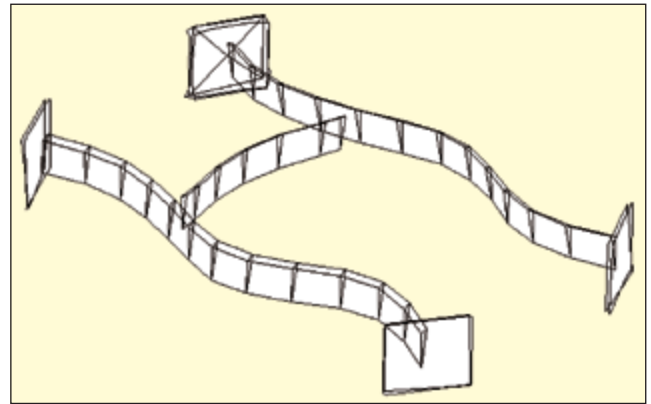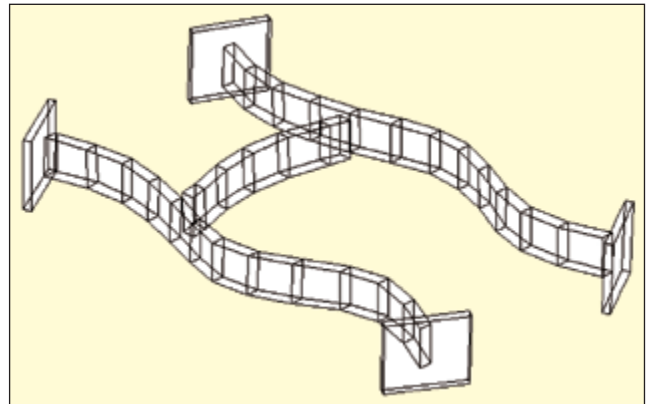
processed on the fly using the CMIF parameter estimation process, where mode shapes are quickly animated to be used as feedback for selecting new response locations on the next pass. The raw impact data should be saved for post-processing after the test is completed to enhance understanding of the problem.

For troubleshooting, an iteration process is often performed where a fix is proposed and quickly implemented, and a second test is performed. Figure 11 illustrates a typical troubleshooting project in an industrial testing environment where a roving hammer is often difficult and dangerous to use.

Laboratory Testing – Verifying or Generating System Models. In the past 10 to 15 years, an increasing number of MRIT tests have been performed in the laboratory on system components (frames, bodies in white, engine blocks, etc.) with some success. As was the case in the troubleshooting discussion, impact testing techniques and signal processing have not changed significantly since the '70s, only the number of simultaneously measured channels. The vast amount of spatial information that is available using the MRIT testing procedures significantly improves the ability of the modal parameter estimation procedures to extract a good modal model of these components. In laboratory testing, most of the components have been evaluated by roving the input, mounting a number of reference accelerometers, and using reciprocity. Since with impact testing, it is not possible to impact tangential to the surface of the test article, completion algorithms are often used to determine missing DOFs or to interpolate and estimate the modal data at unmeasured points.

An example of this is taken from the system dynamics analysis (SDA) course sequence at the University of Cincinnati. A modal test of an H-frame structure is performed by the students in the course using MRIT and a completion algorithm to generate a modal model of the H-frame structure with six DOFs at important connection points. A typical mode is shown, with the mode shape completed at the measurement points in Figure 12. In Figure 13, the same measured mode is shown where the data are interpolated to points common to some of the nodes in the FEM model using the completion algorithm. In this figure, the modal coefficients at 90% of the points are interpolated from a small subset of measured DOF.

Six-DOF information is estimated by the completion algorithms at each of the displayed points on the end masses.

The students use the experimentally estimated modal model to predict modifications made to the H-frame. They also use the modal model to verify a finite-element model (FEM) of the H-frame, which is built by a subset of students in the SDA class who are also taking the FEM course.

Conclusions

Impact testing was one of the first applications to use Fourier analysis in the area of structural dynamics and has a long history as a method well suited for field testing and troubleshooting. Recently it has been used more frequently for developing and validating modal models. It has advantages both in the laboratory and in the field due to simplified test setup and ease of measuring a dataset with significant amounts of spatial information. The additional spatial information has benefits in developing or validating a modal model. It may not replace a well-controlled MIMO laboratory test, but it may certainly augment the capabilities and speed up testing in the laboratory. It is clear that transient testing has many historical applications, and the advancement in sensor, data acquisition and data processing is expanding its horizons.

References

1. "Dynamic Testing of Mechanical Systems Using Impulse Testing Techniques," Application Note 140-3, Hewlett-Packard Corporation, 1972.
2. Morse, I. E., Shapton, W. R., Brown, D. L., Kuljanic, E. "Application of Pulse Testing for Determining Dynamic Characteristics of Machine Tools," 13th International Machine Tool Design and Research Conference, University of Birmingham, England, 1972.
3. Allemang, R. J., Powell, C., Graef, T., "Dynamic Characteristics of Rotating and Non-Rotating Machine Tool Spindles," ASME Paper No. 73-DET-29.
4. Brown D. L, "Grinding Dynamics", Ph.D. Thesis, Department of Engineering, University of Cincinnati, 1976.
5. Brown, D. L., Halvorsen, W. G., "Impulse Techniques for Structural Frequency Response Testing," *Sound & Vibration*, pp. 8-21, Nov. 1977.
6. Spitznogle, F. R., "Representation and Analysis of Sonar Signals, Volume 1: Improvements in the Complex Exponential Signal Analysis Computational Algorithm," Texas Instruments, Inc. Report No. U1-829401-5, Office of Naval Research Contract Number N00014-69-C-0315, 1971, 37 pp.
7. Ibrahim, S. R., Mikulcik, E. C., "A Method for the Direct Identification of Vibration Parameters from the Free Response," *Shock and Vibration Bulletin*, Vol. 47, Part 4, pp. 183-198, 1977.
8. Allemang, R. J., Zimmerman, R. D., Mergeay, M., Brown, D. L., "Parameter Estimation Techniques for Modal Analysis," SAE Paper Number 790221, *SAE Transactions*, Volume 88, pp. 828-846, 1979.
9. Vold, H., Rocklin, T., "The Numerical Implementation of a Multi-Input Modal Estimation Algorithm for Mini-Computers," *Proceedings, International Modal Analysis Conference*, pp. 542-548, 1982.
10. Juang, J. N., Pappa, R. S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 4, pp. 620-627, 1985.
11. Poland J. B., "An Evaluation of a Low Cost Accelerometer Array System – Advantages and Disadvantages," *Master of Science Thesis*, University of Cincinnati, Dept of Mechanical and Industrial Engineering, 1986.
12. Fladung, W. F., "Multiple-Reference Impact Testing," *Master's Thesis*, University of Cincinnati, 1994.
13. Catbas, F. N., Lenett, M., Brown, D. L., Doebling, C. R., Farrar, C. R., Tuner, A., "Modal Analysis of Multi-reference Impact Test Data for Steel Stringer Bridges," *Proceedings IMAC*, 1997.
14. Lenett, M., Catbas, F. N., Hunt, V., Aktan, A. E., Helmicke, A., Brown, D. L., "Issues in Multi-Reference Impact Testing of Steel Stringer Bridges," *Proceedings, IMAC Conference*, 1997.
15. Catbas, F. N., "Investigation of Global Condition Assessment and Structural Damage Identification of Bridges with Dynamic Testing and Modal Analysis," *Ph.D. Dissertation*, University of Cincinnati, Civil and Env. Engineering Department, 1997.
16. Fladung, W. F., "A Generalized Residuals Model for the Unified Matrix Polynomial Approach to Frequency Domain Modal Parameter Estimation," *Ph.D. Dissertation*, University of Cincinnati, 146 pp., 2001.
17. Brown D. L., "Review of Spatial Domain Parameter Estimation procedures and Testing Methods," *Proceedings, International Modal Analysis Conference*, 23 pp., 2009.
18. Brown, D. L., Phillips, A. W., Allemang, R. J., "A First-Order, Extended-State Vector Expansion Approach to Experimental Modal Parameter Estimation," *Proceedings, International Modal Analysis Conference*, 11 pp., 2005.
19. Pickrel, C. R., Foss, G. C, Phillips, S; Allemang, R. J., Brown, D. L., "New Concepts in Aircraft Ground Vibration Testing," *Proceedings, International Modal Analysis Conference*, 6 pp., 2004.

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