

# A New Method for Separating Discrete Components from a Signal

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**A new procedure is proposed that uses the real cepstrum to localize and edit the log amplitude of the original signal, removing unwanted discrete frequency components, and then combines the edited amplitude with the original phase spectrum to return to the time domain. This cepstral editing procedure (CEP) is used to remove discrete frequency components from signals measured on two machines with a faulty bearing, and then perform envelope analysis on the residual signal to diagnose the bearing fault.**

Signal processing used for condition monitoring purposes is usually concerned with separating various signal components from each other to identify changes in any one of them. This has to be done blind, since measured responses are a sum of components from a multitude of sources, and include deterministic (discrete frequency at constant speed), stationary random, and cyclostationary random components. The latter are typically produced by modulation of random signals by discrete frequencies and are often produced by rotating and reciprocating machines.

A fundamental division is into discrete frequency and random components (both stationary and cyclostationary) for which a number of techniques have been developed over the years. This will normally separate gear from bearing signals, for example, since the former are deterministic and phase-locked to shaft speeds, while the latter can be treated as cyclostationary. As pointed out in Reference 1, the signals generated by local faults in rolling-element bearings are actually “pseudo-cyclostationary,” since the repetition frequency is affected by random slip and is not known exactly in advance, but the signals can still be treated as cyclostationary for envelope analysis. Signals produced by bearings with extended spalls are truly cyclostationary if the discrete carriers (such as gear-mesh harmonics) are modulated at a fixed cyclic frequency (shaft speed for an inner race fault). The roughness of the spall surface introduces randomness in the modulation and allows separation from deterministic gear faults.

Where the deterministic components are of primary interest, such as in gear diagnostics, the best separation method is undoubtedly time-synchronous averaging (TSA),<sup>2,3</sup> where a separate signal is obtained for each fundamental period over which the averaging is performed (typically a different result obtained for each individual gear in a complex gearbox). It can be used to obtain the residual signal after removal of all deterministic components<sup>4</sup> but is quite arduous, since the signal has to be separately order tracked for each independent shaft speed (e.g. in a gas turbine engine) and in any case resampled to an integer number of samples per period for each periodicity even after order tracking (removal of small speed fluctuations by resampling at uniform increments in rotation angle rather than time). After removing all discrete components, the signal has to be converted back to a time axis by reverse mapping. However, this procedure does result in the least disruption of the residual signal, though this is rarely necessary. It is limited to the removal of harmonics only and will not, for example, remove modulation sidebands unless these are also harmonics of one of the fundamental frequencies. The process also only works over the full frequency band, including zero frequency, and cannot be used for partial bands (zoom).

Where the primary interest is in removing the discrete frequency components to obtain the random residual, often dominated by bearing signals in certain frequency bands, other methods have been developed based on the different correlation length of discrete frequency and random signals. The first is SANC (self-adaptive noise cancellation),<sup>5</sup> which is a modification of ANC (adaptive

noise cancellation). The latter procedure has two input signals, a primary signal containing a mixture of two components to be separated and a reference signal containing only one. This does not have to be identical to the corresponding component in the primary signal, just coherent with it. An adaptive filter finds the linear transfer function between the two versions and subtracts the corresponding component from the primary signal, leaving the other in the residual. It has been used to separate gear and bearing signals in situations where the primary signal was measured on a faulty bearing in a gearbox and the reference signal on another remote bearing with no fault signal present.<sup>6,7</sup> In SANC, the reference signal is a delayed version of the primary signal, with a delay just longer than the correlation length of the random component so that only the deterministic part is recognized by the adaptive filter.

In Reference 8, it was shown how the same effect could be achieved much more efficiently by the so-called “discrete/random separation” (DRS) technique using efficient FFT processing in the frequency domain. The transfer function between the primary signal and its delayed version (representing only the deterministic part) is calculated in the same way as the  $H_1$  frequency response function (FRF). This has a value near 1 at discrete frequencies and near zero at other frequencies. It is used to filter the whole signal by “fast convolution” in the frequency domain and once again allows the deterministic part to be separated out and the random residual signal to be obtained by subtraction.

For both the SANC and DRS procedures, all discrete frequency components are removed as long as they have a correlation length longer than the delay time. The correlation length of bearing signals is generally short – in the vicinity of the high frequency resonances that are normally demodulated for diagnostic envelope analysis. For example, it is normal for the random slip in bearings to give about 1% variation in the characteristic fault frequencies. It is common for the resonance frequencies excited by the bearing faults to be two orders of magnitude higher than the repetition frequency, in which case the harmonic orders above about 50 will be smeared together. This is the same as saying that the correlation length is shorter than the pulse spacing. If a bearing frequency is 100 Hz, but the demodulated resonance at about 10 kHz, the 1% variation will correspond to 100 Hz, or a correlation length of about 10 ms. The delay would then typically be set >30 ms. DRS can operate on partial band signals obtained by selecting only the frequency band to be demodulated using so-called Hilbert transform techniques based on a one-sided frequency spectrum with positive frequencies only. It can easily be shown that the frequency shifting involved does not change the amplitude (envelope) of the demodulated signal, which can then be used for diagnostics based on envelope analysis. However, it does give a notch filter of fixed width that can have detrimental effects at high and low frequencies. At low frequencies (low harmonics of the bearing frequencies if they exist), the bearing harmonics may still be within the bandwidth of the comb filter, and they may be treated as discrete frequencies. At high frequencies, some random modulation of the discrete frequency components may be left in the form of a narrow-band noise and not completely removed. This can occur at harmonics of blade-pass frequencies in turbo-machines, because the transmission of the blade-pass signals to the casing is via a turbulent fluid rather than a mechanical connection.

The cepstral editing procedure (CEP) gives some advantages compared with all the techniques noted previously. It can be used to remove selected discrete frequency components in one operation, without order tracking as long as the speed variation is limited, but it can leave some periodic components if desired. It can operate on partial-band (zoom) signals, at least in the same sense as the DRS

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technique, where it is only the envelope of the residual signal that is of interest. It is based on the cepstrum of the signal that very efficiently collects spectral components that are uniformly spaced, that is, both harmonics and modulation sidebands.

### The Cepstrum

The cepstrum has a number of versions and definitions, but all can be interpreted as a “spectrum of a log spectrum.” The original definition<sup>9</sup> was the “power spectrum of the log power spectrum,” but even though one of the authors (Tukey) was a co-author of the FFT algorithm, Reference 9 predated the latter and was from the era before the FFT. The definition of the cepstrum was later revised (post FFT)<sup>10</sup> as the “inverse Fourier transform of the log spectrum,” giving a number of advantages compared with the original definition. It can thus be represented as:

$$C(\tau) = \mathfrak{S}^{-1}[\log(X(f))] \quad (1)$$

where:

$$X(f) = \mathfrak{S}[x(t)] = A(f)\exp(j\phi(f)) \quad (2)$$

in terms of its amplitude and phase so that:

$$\log(X(f)) = \ln(A(f)) + j\phi(f) \quad (3)$$

When  $X(f)$  is complex, as in this case, the cepstrum of Equation 1 is known as the “complex cepstrum.” But since  $\ln(A(f))$  is even and  $\phi(f)$  is odd, the complex cepstrum is real valued. Note that by comparison, the autocorrelation function can be derived as the inverse transform of the power spectrum, or:

$$R_{xx}(\tau) = \mathfrak{S}^{-1}[\lvert X(f) \rvert^2] = \mathfrak{S}^{-1}[A^2(f)] \quad (4)$$

When the power spectrum is used to replace the spectrum  $X(f)$  in Equations 1 and 4, the resulting cepstrum, known as the “power cepstrum” is given by:

$$C_{xx}(\tau) = \mathfrak{S}^{-1}[2 \ln(A(f))] \quad (5)$$

and is then a scaled version of the complex cepstrum, where the phase of the spectrum is set to zero. The term “real cepstrum” is sometimes used to mean the inverse transform of the log amplitude, not having the factor 2 in Equation 5. It corresponds to the complex cepstrum obtained from Equation 3, where the phase has been set to zero.

Note that before calculating the complex cepstrum, the phase function  $\phi(f)$  must be unwrapped to a continuous function of frequency. This is often possible for frequency response functions, where the phase is continuous, and quite often related to the log amplitude. (For minimum phase functions, they are related by a Hilbert transform.) However, it is not possible for response signals containing a mixture of discrete frequency components, for which the phase is undefined between components, and random signals for which the phase is discontinuous.

### Proposed Method

Editing in the real cepstrum has been used for some time to remove harmonics and/or sidebands from the spectrum, as illustrated in Figure 1.<sup>11</sup> Note that in this respect, periodic notches in the log spectrum also give components in the cepstrum, so there will be a tendency for the residual spectrum to be continuous at the former positions of discrete frequency components after removal.

In cases where it is desired to remove discrete frequency components but leave a random residual signal with the same envelope (no essential change in phase), we realized that it would be possible to remove the discrete frequency components using the real cepstrum (as in Figure 1) but then generate the time signal of the random residual using the original phase spectrum. This would be in error only at the frequencies corresponding to the removed components, but these would be relatively few in number, and the corresponding amplitudes reduced to the same level as the adjacent random components. As mentioned, the act of setting cepstrum values to zero means that the residual log spectrum will tend to be continuous across the gap where the discrete frequencies have been removed. For a random signal, this gives the best estimate of

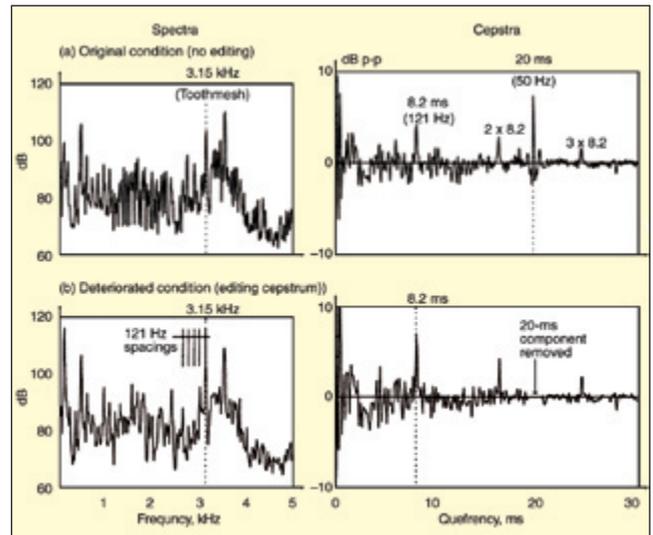


Figure 1. Editing the cepstrum to remove a particular family of harmonics: (a) spectrum and cepstrum with two families of harmonics/sidebands; (b) spectrum with only one family retained after editing the other family from the cepstrum.

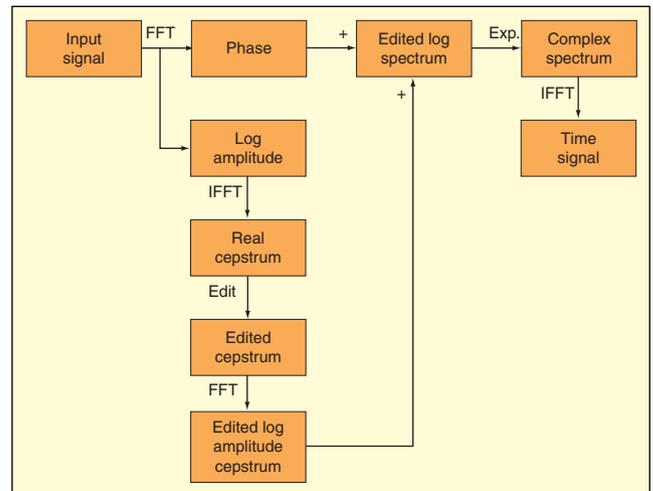


Figure 2. Schematic diagram of the cepstral method for removing selected families of harmonics and/or sidebands from time signals.

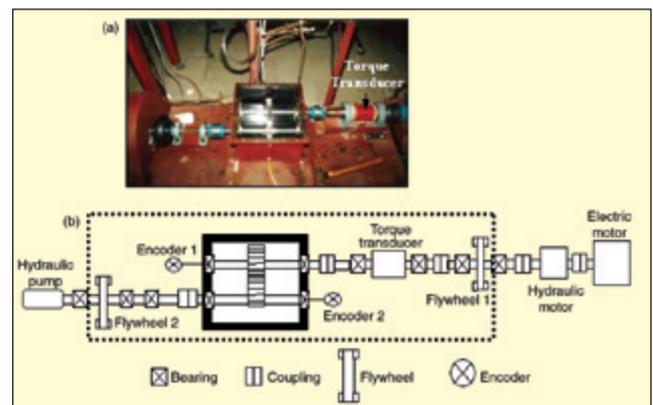


Figure 3. (a) Spur gear test rig; (b) schematic diagram of spur gearbox rig (components of interest contained in dotted box).

the amplitude of the spectrum at those frequencies.

The proposed method is shown schematically in Figure 2. Sections of the input signal are transformed to the frequency domain using the FFT algorithm. The phase is stored while the log amplitude is processed using the IFFT (inverse fast Fourier transform) to obtain the real cepstrum. Families of “rahmonics” (uniformly spaced components in the cepstrum) corresponding to the families of harmonics and sidebands to be removed from the signal are set to zero as in Figure 1, and the edited cepstrum is

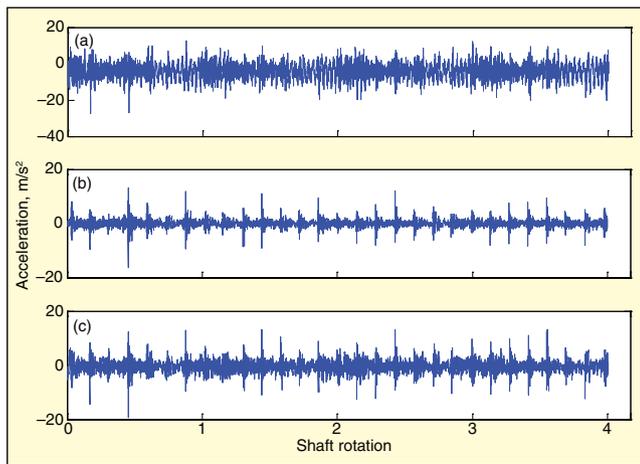


Figure 4. Time domain signals for gearbox test rig: (a) raw signal; (b) residual signal after removing synchronous average; (c) residual signal after editing cepstrum to remove the shaft harmonics.

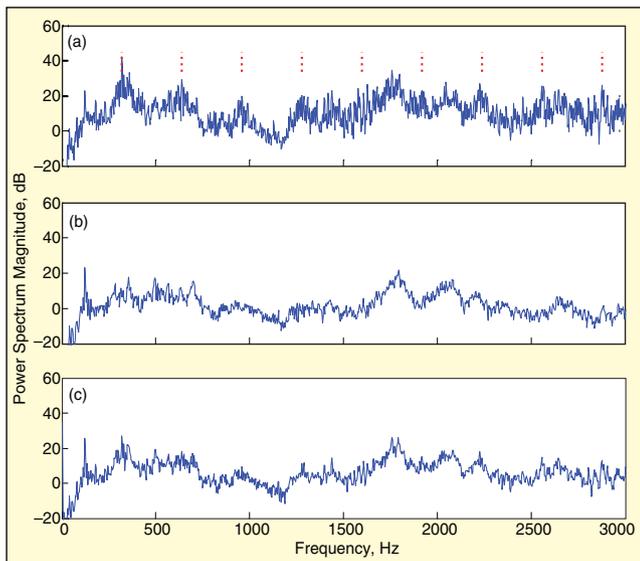


Figure 5. Power spectra for time signals of Figure 4: (a) raw signal; (b) residual signal after removing the synchronous average; (c) residual signal after editing the cepstrum to remove the shaft harmonics.

transformed by a forward FFT to the edited log amplitude spectrum. This is then recombined with the original phase spectrum to form the complex log spectrum, which can be exponentiated to rectangular form (real and imaginary parts) rather than polar form (log amplitude and phase). This complex spectrum can then be inverse transformed to obtain the time signal of the residual random part.

## Results and Discussion

The new cepstral technique was applied to signals measured on two test rigs. One is the UNSW gear test rig shown in Figure 3. The rig is driven by a variable-speed electric motor, but torque load can be increased using a circulating power loop involving a hydraulic pump supplying a hydraulic motor. For these tests, the motor was run at 10 Hz, with 50 Nm torque load, and two 32-tooth spur gears in mesh. A bearing with an inner race fault was inserted in the bottom right location in the diagram, and signals were measured by an accelerometer on the casing immediately above it.

Figure 4 compares the time signals for two separation methods, TSA and the Cepstral Editing Procedure (CEP), with the original signal. Removal of the gear signals reveals the hidden bearing signals, with a slightly better result achieved using TSA. Note that even though this is an inner-race fault, the expected modulation once per revolution (the rate at which the fault passes through the load zone) is not very strong in this case.

Figure 5 shows the corresponding power spectra up to 3 kHz – this being the band dominated by the gear-mesh signal. Harmonics

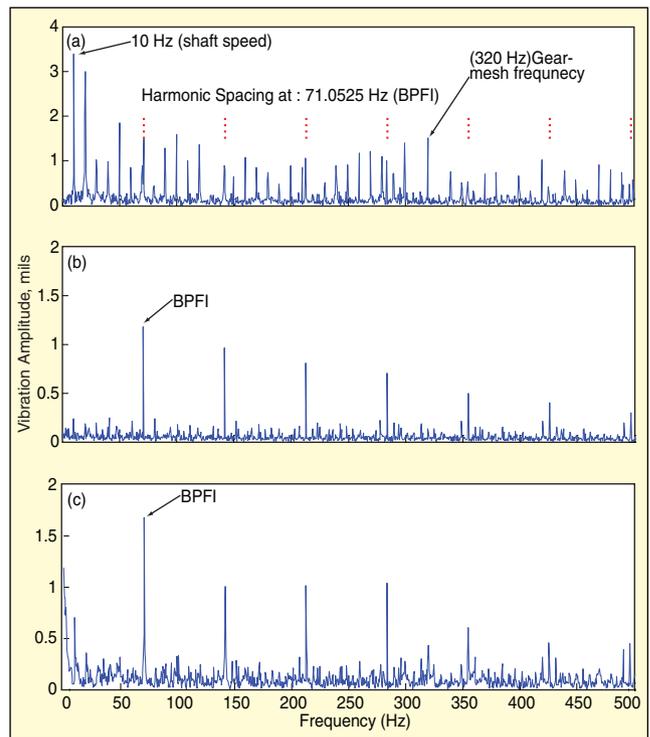


Figure 6. Squared-envelope spectra, 1-20 kHz: (a) raw signal; (b) residual signal after removing synchronous average; (c) residual signal after editing the cepstrum to remove shaft harmonics.

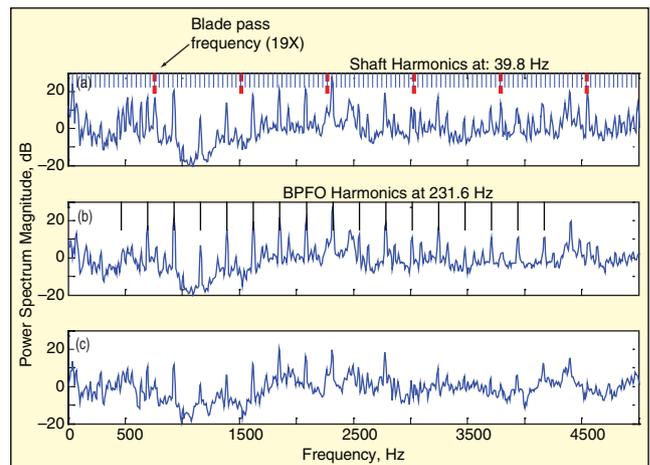


Figure 7. Comparison of spectra 0-5 kHz: (a) original signal; (b) residual after TSA; (c) residual after CEP.

of the gear-mesh frequency (320 Hz) are indicated by a harmonic cursor in the spectrum of Figure 5a, and these can be seen to be surrounded by sidebands that are spaced at shaft speed of 10 Hz, the rotational speed of both gears. It can be seen that these discrete frequency components have been removed in both residual signals for which the spectra are very similar.

Finally, Figure 6 shows the corresponding envelope spectra for the three signals. For the raw signal of Figure 6a, this is seen to be dominated by harmonics of shaft speed, including the gear-mesh frequency of 320 Hz, even though some harmonics of BPFI (ballpass frequency, inner race) can be detected with some difficulty. Both residual signals give a clear diagnosis of the inner-race bearing fault, though modulation sidebands spaced at shaft speed are more evident around the higher harmonics above about 350 Hz. This corresponds with the fact that the modulation is not very strong in the time signals.

Measurements were also made on another test rig designed to study the signals from a rotating bladed disc that can be run over a range of speeds. The shaft is supported by two self-aligning, double-row ball bearings, one of which an outer-race fault inserted and on which acceleration measurements were made. The rotor has 19 flat

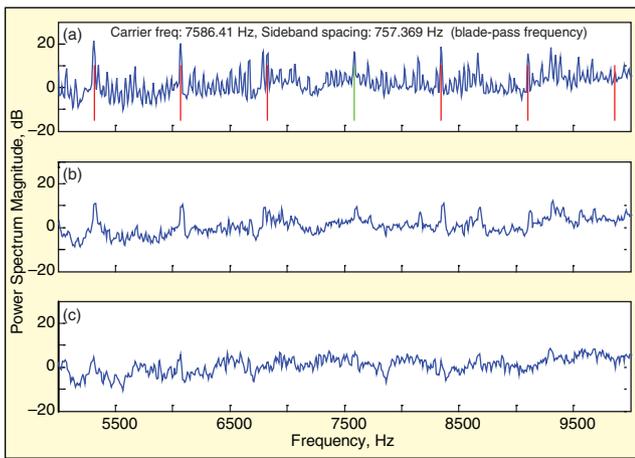


Figure 8. Comparison of spectra 5-10 kHz: (a) original signal; (b) residual after TSA; (c) residual after CEP.

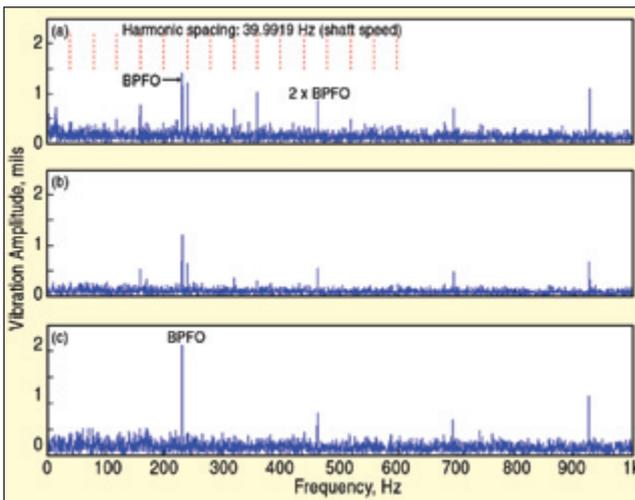


Figure 9. Envelope spectra obtained from three residual signals: (a) TSA; (b) DRS; (c) CEP.

blades and, because of its inefficiency as a fan, produces multiple harmonics of the blade-pass frequency as well as sidebands around these spaced at shaft speed (harmonics of shaft speed).

Analyses were made over a frequency range up to 30 kHz, but only results up to 10 kHz are shown here. The new cepstral method was compared primarily with TSA for the efficiency of removal of harmonics of shaft speed, including blade-pass harmonics, but the DRS method was also tried.

Figure 7 compares spectra up to 5 kHz, and Figure 8 from 5-10 kHz. The order in both figures is the same, with the spectrum of the original signal at the top (a), the middle spectrum (b) representing the results after extraction of shaft speed harmonics using TSA, and the bottom spectrum (c) showing the results after extraction of shaft harmonics using the cepstral method. Figure 7a has two sets of harmonic cursors, showing the harmonics of shaft speed (39.8 Hz), and those of the blade-pass frequency at 19x. Figure 7b shows the harmonics of BPFO (ball pass frequency, outer race), and these are seen to be the dominant components left after removal of the shaft speed harmonics. The cepstrum result in 7c appears to give similar results to the TSA.

Figure 8a has a sideband cursor centred on the 10th harmonic of the blade-pass frequency (approximately 7580 Hz) with sidebands spaced at the blade-pass frequency. In this frequency range, the TSA has left some components spaced at the blade-pass frequency, though these are obviously not discrete frequency components. We surmise that they are residual, narrow-band random sidebands around the blade-pass harmonics caused by the fact that the detection of blade passing on the casing is via pressure pulsations transmitted through a turbulent fluid. Because this pattern is still periodic in the frequency domain, it has been removed by the cepstral method (see Figure 8c).

Envelope analysis carried out on the residual signals shows even more clearly how the cepstral method removes all periodicity in the log spectrum, even that coming from uniformly spaced narrow-band noise peaks. The envelope spectra in Figure 9 show periodic patterns in the signal envelopes. In the TSA result of Figure 7a, harmonics of BPFO can be found, but there is some disruption from residual harmonics corresponding to the shaft speed. These can possibly be explained by the same mechanism as the residual blade pass pattern in Figure 8b – periodic narrow-band noise peaks. But another possible explanation is that there are sidebands distributed throughout the spectrum caused by modulation of nonsynchronous vibration components by the shaft speed. TSA removes only harmonics of the fundamental frequency and not sidebands caused by modulation of nonsynchronous components. Figure 9c, using the cepstral method, shows only the harmonics of BPFO, with all periodicity related to the shaft speed having been removed.

As a matter of interest, Figure 9b shows the results of applying the DRS technique. In principle this should remove all discrete frequency components, including sidebands, but it also has some remnants related to shaft speed, although not as strongly as TSA. This is possibly because high-order harmonics of shaft speed have been smeared sufficiently by minor speed variations (order tracking was not applied before the DRS) so as to be broader than the fixed-width notch filter given by DRS.

## Conclusions

A new method has been proposed for separating periodic and random components in a signal, such as those arising from bearings and gears. It is based on the cepstrum, which collects all periodicity in the log spectrum into a small number of components which can then be edited to remove selected families. This method has a number of advantages compared with alternative methods such as TSA and DRS. Unlike the former, it can remove sidebands as well as harmonics and also periodic narrow-band noise peaks rather than just discrete frequency harmonics. It can also operate over limited band regions, where the TSA must include all harmonics down to zero frequency.

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