Full-Field Dynamic Stress/Strain from Limited Sets of Measured Data

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Often times, occasional events may cause significant displacement and corresponding stress strain damage to a structure. Using limited sets of measured data, expansion of real-time data has been shown to provide accurate full-field displacement results. This displacement data can be used in conjunction with a finite-element model to identify full-field dynamic stress-strain results. This approach is demonstrated for an analytical model to show the methodology proposed. Examples illustrating different configurations of measured data sets along with simulated noise are presented to illustrate the technique.

Failure of a structure due to dynamic loading invariably involves occurrence of dynamic stresses and strains in the structure. The induced dynamic stresses and strains may be higher than the maximum permissible limit. Periodic estimation of such induced dynamic stresses and strains is extremely crucial for averting any dangerous accidents. Different loading conditions will impact the structure's health differently. General design processes use a factor of safety to counter the problem of keeping the induced stresses and strains within permissible limits. However, there is no accurate way to estimate the remaining fatigue life of the structure especially when occasional, unexpected loading conditions occur. To accurately predict the remaining useful life of a structure, correct estimation of dynamic stresses and strains induced in the structure is of prime importance.

There are primarily two approaches traditionally used to evaluate such transient stresses in structures. The first approach would be to collect the transient response information at limited locations. Such a process predicts the real-time response of the structure under severe dynamic loadings but does so at only limited locations. Massive structures like bridges, ships or wind turbines would need thousands of sensors to get the required data; this is highly improbable and unfeasible.

The second approach would be to predict the transient forces that the structure experiences and then use the analytical model to find the dynamic stresses the structure experiences. The problem here lies with the force estimation process that is highly sensitive to the number of chosen degrees of freedom and their distribution on the structure. Apart from this, the other issues, such as approximations while developing a finite-element model, invariably induce errors in estimating dynamic stresses on the structure.

The approach used in here has been introduced in Reference 1, where full-field real-time displacements were obtained successfully from limited sets of displacement data using a real-time operating data expansion technique recently studied by Chipman.[2] The limited set of sensors would provide displacement data, and the expansion algorithm would expand the limited data set to a full-field displacement solution, completely eliminating the force estimation step required in the previous approach. In this article, the full-field displacements are used to obtain full-field dynamic stress-strain information. The estimated dynamic stress-strains using this approach are then compared with reference stress-strain solutions to validate the proposed approach.

Theory

The approach presented here is conceptually explained in Figure 1. Traditional finite-element design processes involve discretization of the component into elements, assembling the system matrices and applying appropriate approximations of boundary conditions and loading to estimate the dynamic stress-strain that the component sees throughout the designed life of the component/

Nomenclature

$\{X_n\}$ = full set displacement vector
$\{X_a\}$ = reduced set displacement vector
$[M_a]$ = reduced mass matrix
$[M_n]$ = expanded mass matrix
$[K_a]$ = reduced stiffness matrix
$[K_n]$ = expanded stiffness matrix
$[U_a]$ = reduced shape matrix
$[U_a]^a$ = generalized inverse of reduced shape matrix
$[U_n]$ = expanded shape matrix
$[T]$ = transformation matrix.
$[T_U]$ = SEREP transformation matrix.
$[REF_n]$ = reference data at all degrees of freedom (DOFs)
$[RTO_a]$ = real time operating data at measured DOFs
$[ERTO_a]$ = expanded real time operating data at all DOFs

structure. The approach presented involves collecting experimental displacement data at limited measurable locations, then a real-time operating data expansion technique is used to expand the limited displacement data set to full-field displacements. The expansion technique uses mode shape information obtained from the analytical model of the structure.

Finally, the full-field displacement data set is used to recover the dynamic stress-strain information by using a back-substitution process shown in Figure 2. The finite element process is interrupted to incorporate the expanded full-field displacement solution in order to obtain the full-field dynamic stress-strain solution. In Figure 2, the full-field strain solution is shown at one time step; however, the full-field strain solution is obtained for all the required time steps through the recovery process.

Model Reduction

Model reduction is necessary to develop expansion approaches for modal data for the unmeasured translational DOF as well as for rotational DOF. For this work, the expansion is needed for augmenting the limited-set real-time operating data to provide a full-field displacement solution. The reduction techniques are the basis of the expansion discussed here. These techniques have been presented in an earlier work cited in the references; only summarizing equations are presented below. Several model reduction methods have commonly been used for expansion of measured data. Four common methods are Guyan,[5] dynamic condensation,[6] SEREP,[7] and a hybrid method.[8] In these methods, the relationship between the full set of degrees of freedom and a reduced set of degrees of freedom can be written as:

$$\{X_n\} = [T]\{X_a\} \quad (1)$$

All of these methods require the formation of a transformation matrix that can project the full mass and stiffness matrices to a smaller size. The reduced matrices can be formulated as

$$[M_a] = [T]^T [M_n][T] \quad (2)$$

$$[K_a] = [T]^T [K_n][T] \quad (3)$$

For this article, only the SEREP method has been used for the expansion of mode shapes.

The system equivalent reduction expansion process (SEREP) produces reduced matrices for mass and stiffness that yield the

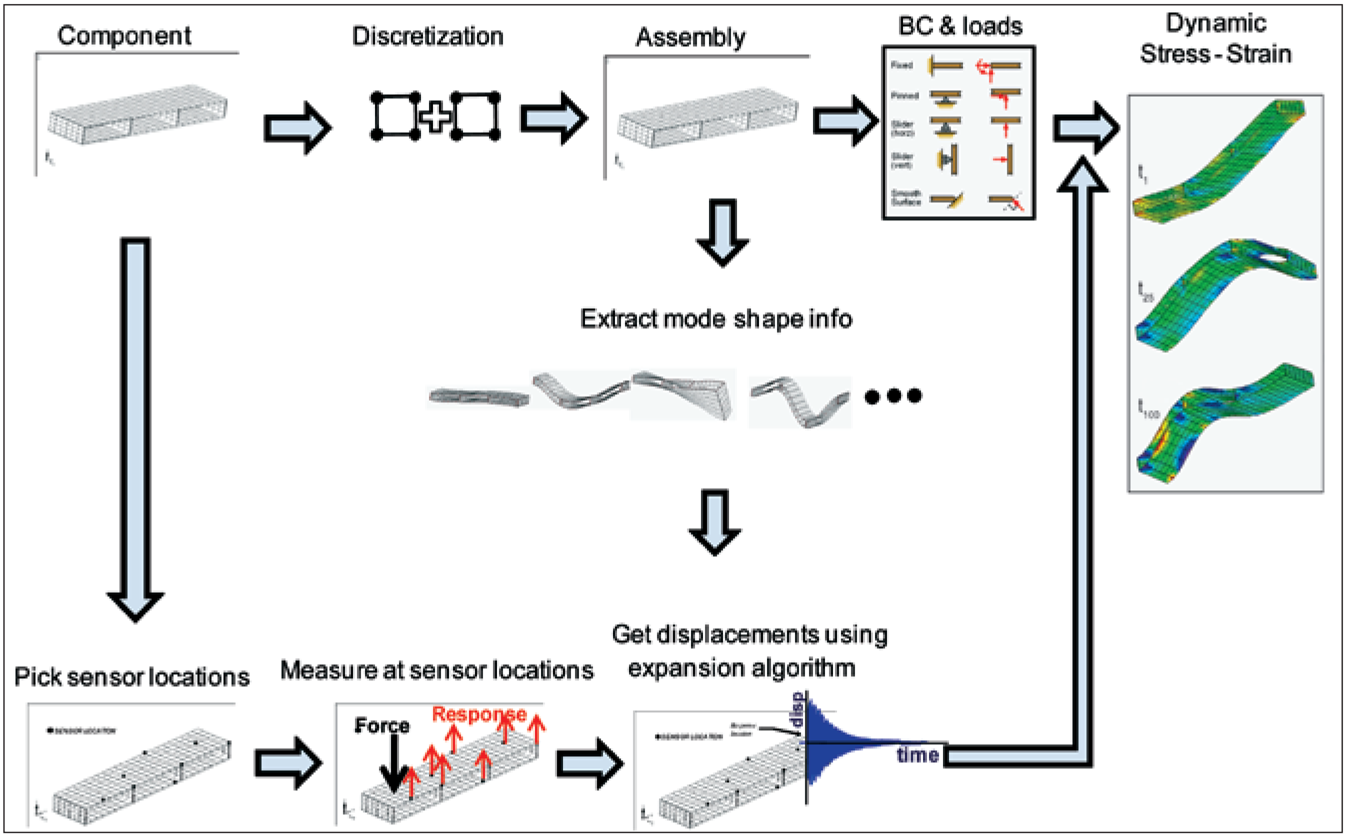

Figure 1. Various design approaches to estimate dynamic stress-strain.

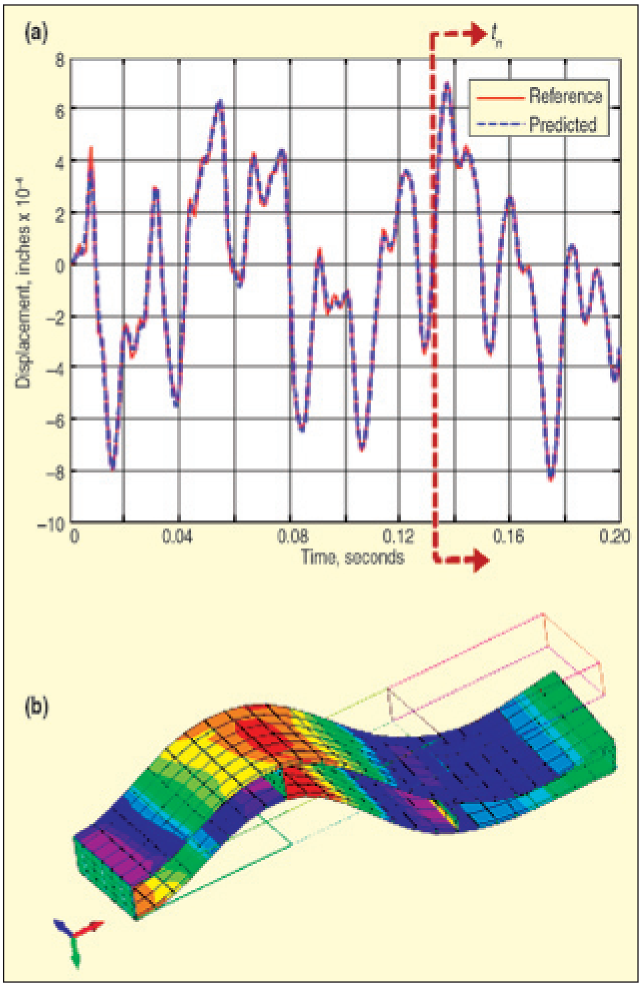

Figure 2. Recovery of dynamic stress-strain from full-field displacement data.

exact frequencies and mode shapes as those obtained from the eigensolution of the full size matrix. The SEREP transformation is formed as:

$$[T_u] = [U_n][U_a]^g \tag{4}$$

The SEREP transformation is developed with analytical mode shapes for the structure, but can also be evaluated using measured modal vectors as was done in Chipman's work.[2] Equation 1 is used for expansion of real-time operating data and is written as:

$$[ERTO_n] = [T][RTO_a] \tag{5}$$

Correlation Tools

The correlation tools used to compare the results of the expanded *RTO* $[ERTO_n]$ and "reference time solution" $[REF_n]$ are briefly discussed here to clarify the differences between the techniques. Correlation tools such as, the modal assurance criterion (MAC)[9] and the time response assurance criterion (TRAC)[10] will be used to verify the accuracy of the expanded *RTO* solution in each case. These functions are summarized here with details found in Reference 2. The MAC as used for this work will identify correlation of the expanded real-time operating displacement solution obtained with the reference solution. The MAC can be computed at each time step *t* to compare the transient displacement solution. The MAC is written as:

$$MAC\_RTO = \frac{\left[\{REF_n\}^T\{ERTO_n\}\right]^2}{\left[\{REF_n\}^T\{REF_n\}\right]\left[\{ERTO_n\}^T\{ERTO_n\}\right]} \tag{6}$$

Similar to the MAC, the TRAC is a tool used to determine the degree of correlation between two time traces. For the cases presented here, the TRAC is the correlation for one DOF over all time for the expanded time data $[ERTO_n]$ compared to the actual measured data $[REF_n]$. The TRAC is written as:

$$TRAC\_RTO = \frac{\left[\{REF_n(t)\}^T\{ERTO_n(t)\}\right]^2}{\left[\{REF_n(t)\}^T\{REF_n(t)\}\right]\left[\{ERTO_n(t)\}^T\{ERTO_n(t)\}\right]} \tag{7}$$

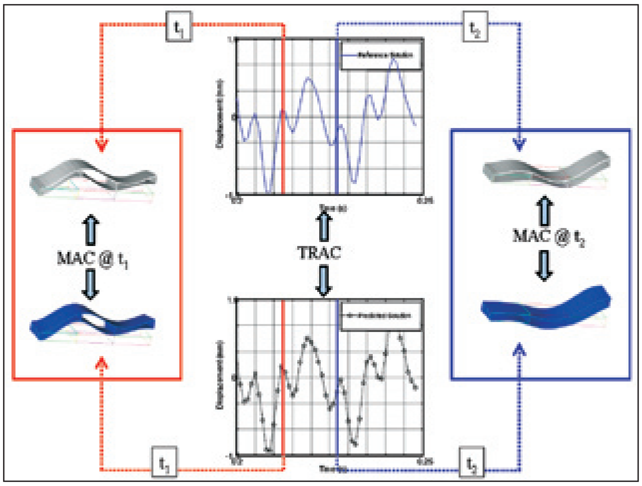

Figure 3. Schematic showing MAC and TRAC.

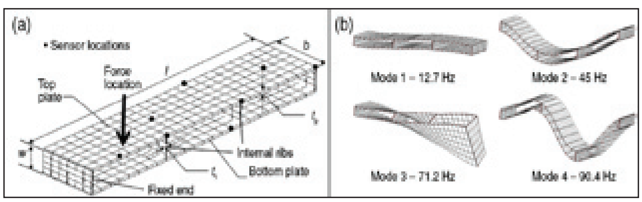

Figure 4. (a) Box-beam model used for analytical study of expansion algorithm along with typical set of sensor locations; (b) first four mode shapes of the structure.

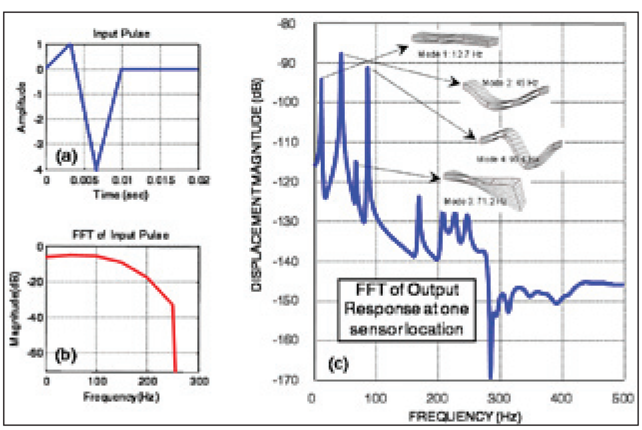

Figure 5. Details of excitation force and response of structure.

The values produced by both the MAC and TRAC will range from 0 to 1; values approaching 1 indicate good correlation. Figure 3 shows the MAC and the TRAC correlations. Typically, the TRAC is comparison of the displacement-time response solutions (predicted and reference solutions), while the MAC is a comparison between the deformed shape of the structure at each time instant (Figure 3).

Model Description

To demonstrate the expansion technique to be used for determining displacement for dynamic stress and strain from limited sensor locations, an analytical study was performed on a model that resembles typical blade construction of a wind turbine. The box-beam model used for analysis is shown in Figure 4a. The dimensions of the model are:

- Length (l) = 60 inches
- Breadth (b) = 12 inches
- Width (w) = 6 inches
- Thickness of top and bottom plates (t_1) = 0.5 inches
- Thickness of internal ribs (t_2) = 0.25 inches
- Material – aluminum

The finite-element model of the box-shaped structure contains a total of 434 nodes. The natural frequencies and mode shapes of the beam are shown in Figure 4b. The frequencies typically are well separated. The model has an assortment of modes containing transverse and lateral bending and torsion modes that will be excited using a time pulse where, primarily, the first four frequencies are excited.

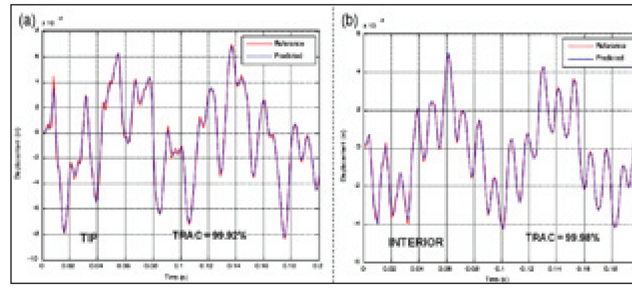

Figure 6. Overlay of displacement response from reference and predicted solution using first six modes for expansion; (a) response location on the top plate, (b) response location inside the rib.

Modal damping typically seen in these types of structures was added to the model. The excitation used was a time pulse applied vertically downward, 10 inches from the bottom right corner of the box beam. The input time pulse is shown in Figure 5a. The time pulse is a combination of two triangular functions, which when seen in the frequency domain show a very uniform excitation over the frequency range of interest (see Figure 5b). The time pulse primarily excites the first four modes of the structure as seen in FFT of the output response as shown in Figure 5c, but there is also some additional response of smaller magnitude from several higher frequencies.

Cases Studied

The comparison between the reference solution and expansion solution is made in this section. Several different analytical cases have been studied to evaluate the effectiveness of the proposed expansion algorithm. The cases studied are based on some important factors that affect implementing the technique. The cases studied here are based on three important factors that need to be considered for successfully implementing the proposed approach:

- Number of modes used in the expansion process.
- Effect of different noise levels on the solution obtained through the proposed technique.
- Different sensor configurations used for obtaining a full-field expanded data set.

Foremost among the prominent factors affecting the expansion solution is the number of modes used in the expansion algorithm. Theoretically, there is no upper limit for the number of modes to be used for expansion, but from a practical standpoint, only a limited number of modes can be used for expansion. Case 1 deals with the study of the number of modes used for expansion.

Different noise levels have been studied in Case 2 to gage the performance of the expansion algorithm in the presence of noise. Noise is another important issue that has shown to have an affect on successfully implementing the expansion algorithm.

Case 3 discusses the different sensor configurations and their effect on the solution obtained through the expansion algorithm. The sensors used for obtaining the real-time data (aDOF measurements), invariably play an important role in the accuracy of the solution.

Case 1. For obtaining full-field displacement response from limited sets of displacement data, using the real-time operating data expansion technique, the formulation of the expansion transformation matrix is known to be an important factor. The transformation matrix for the SEREP expansion technique is formulated using mode shape information. The expansion transformation matrix should contain at least those structure modes that are primarily excited. For reference, the displacement response solution obtained using limited sets of displacement data is shown for typical response locations (tip and interior of the box-beam) in Figure 6. Note that the full-field displacement solution obtained from limited sets of displacement data has been detailed in Reference1. The first six modes were used in formulation of expansion transformation matrix. As observed in Figure 6, the expansion solution matches the

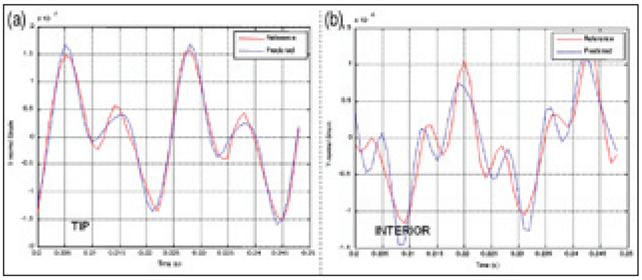

Figure 7. Overlay of strain response of box beam at (a) tip and (b) interior location; predicted solution used first six modes in expansion process.

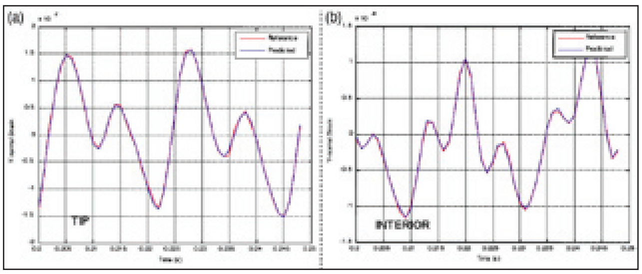

Figure 8. Overlay of strain response of box beam at (a) tip and (b) interior location; predicted solution used first 10 modes in expansion process.

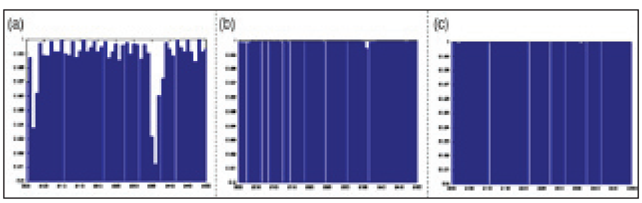

Figure 9. MAC between displacement response vectors of reference and expansion algorithm solutions with different number of modes used for expansion; (a) using first 6 modes; (b) using first 8 modes; (c) using first 10 modes in expansion process; MAC performed for time steps 200-250 and shown for 0.9-1.

reference solution well with TRAC values of 99.9+%. This shows that the prediction of full-field displacement response through the expansion process is very accurate.

The full-field displacements obtained from using the first six modes are used for obtaining a dynamic strain solution. This predicted dynamic strain solution (Y normal strain) is compared with the reference strain solution in Figures 7a and b, with response location at the tip of the box beam and in the rib of the box beam respectively. Recall that the force pulse primarily excites the first four modes of the system. Only 14 y (vertical) direction points of a total of 434 finite-element model nodes were used as sensor locations measuring vibration displacement response. The strain response is shown for a sample time period of 0.05 second.

Although the full-field displacements obtained using the first six modes provided excellent correlation with the reference displacement solution, the full-field strain results obtained after incorporating the six-mode full-field displacement solution is not as good as expected (Figures 7a and b), because the strain solution is more sensitive than the displacement solution to the contribution from higher modes. When the full-field displacement solution obtained from using first 10 modes is used to obtain the full-field dynamic stress-strain solution, however, then excellent correlation is seen with the reference strain solution as seen in Figures 8a and b. This trend is more noticeable for higher strain regions (strain in the rib) as seen from Figures 7b and 8b.

To study the sensitivity of the strain solution in detail, the displacement solution correlation was performed. Figures 9a-c show the MAC for the displacement solution between the predicted and reference model responses for the time period corresponding to the strain time period (0.2-0.25 seconds). The MAC plots are shown for three different mode configurations – using first six, eight and then 10 modes in the expansion solution. The MAC for all three mode configurations is very good, because they all over-specify the required number of modes for expansion. (Recall that the forcing pulse excites the first four modes primarily). So to show the difference between the six-, eight- and 10-mode configurations, the MAC plots are shown from 0.9-1, indicating that the MAC for that particular time period is better than 90% always. Notice that even with the six-mode configuration, the displacement responses show good correlation (better than 90%); but the correlation improves with the eight- and 10-mode configurations, confirming that a greater number of modes used in the solution process will always yield better expansion results. So, as a general rule, more modes will be required especially when considering strain rather than displacement.

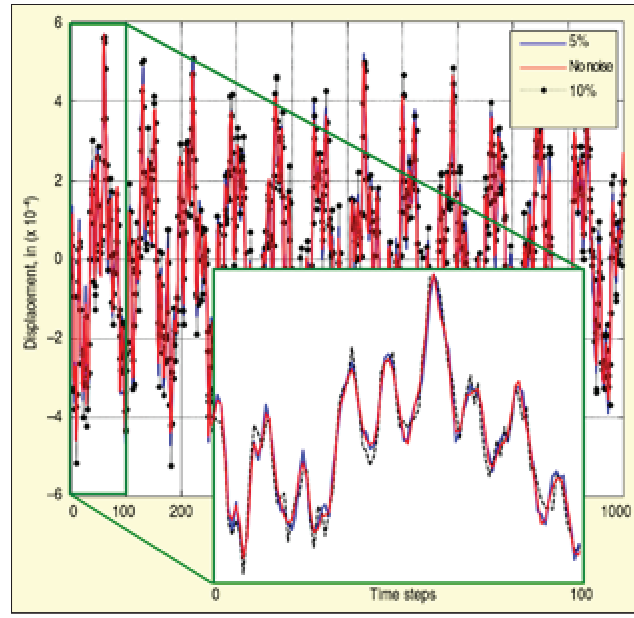

Figure 10. Effect of different noise levels (5% and 10%) on displacement-time response.

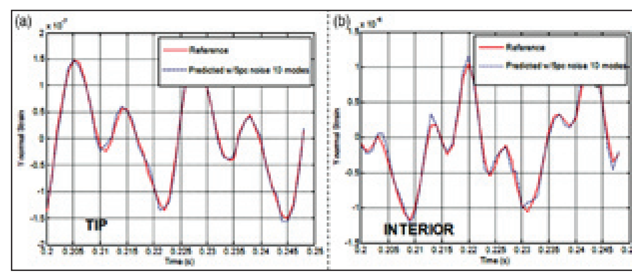

Figure 11. Overlay of strain response of box beam at (a) tip and (b) interior point; comparison made between reference solution and predicted solution with 5% noise.

Case 2. Experimental noise is a very common phenomenon that corrupts data. While collecting experimental data, the noise component is always inevitable. To study the expansion technique realistically, some random noise is added to baseline measurements using :

$$\{\text{Noise-induced measurement}\} = \{\text{Original measurement}\} + r \bullet \{\text{Maximum amplitude of measurement}\} \quad (8)$$

where, r is a random set of numbers ranging from –1% to +1% of the maximum amplitude of measurement. Figure 10 shows the noise induced in the model to simulate experimental conditions.

Case 1 showed the expansion algorithm to be an accurate tool when the appropriate number of modes are used for expansion. However, the expansion algorithm could also be affected by noise, as shown in Figure 10.

A varying percentage of noise was added to the model used in Case 1 to simulate the experimental conditions of real-time data acquisition; noise levels ranging from typical levels of noise (5%) and higher levels of noise (10%) were studied. The full-field displacement solution was obtained from the noisy aDOF measurements through expansion. The full-field displacements obtained were

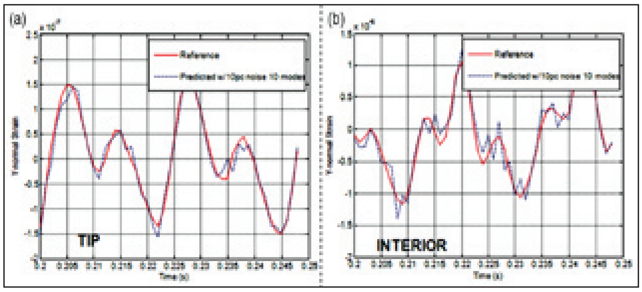

Figure 12. Overlay of strain response of box beam at (a) tip and (b) interior point; comparison made between reference solution and predicted solution with 10% noise.

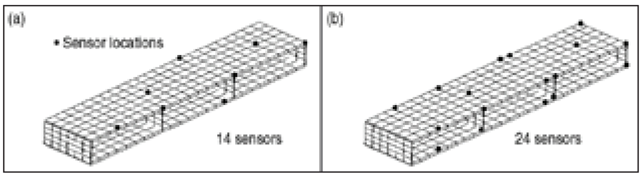

Figure 13. Different sensor locations used to determine response of structure using expansion algorithm at all 434 FEA node locations with (a) 14 sensors and (b) 24 sensors.

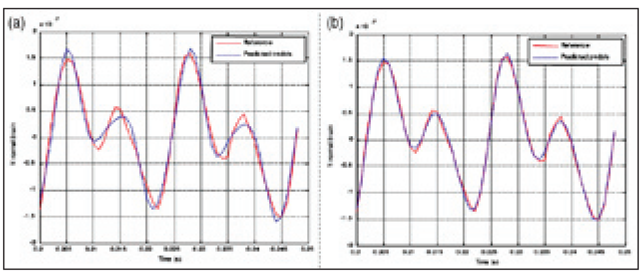

Figure 14. Strain response comparison at tip of box beam by reference and predicted solutions using first six modes for expansion with (a) 14 sensors and (b) 24 sensors.

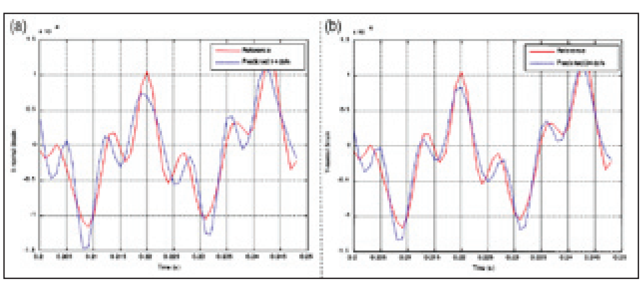

Figure 15. Strain response comparison at interior location of box beam by reference and predicted solutions using first six modes for expansion with (a) 14 sensors and (b) 24 sensors.

then back-substituted into the finite-element process to recover the stress-strain solution.

The dynamic strain solution for 5% noise on aDOF measurements is shown in Figure 11 and the strain solution for 10% noise case is shown in Figure 12. Comparison is made between the predicted strain solution with the reference solution at two typical response locations (at the tip and at an interior location in the rib of the box beam). As seen in Case 1, more than six modes are needed for accurate expansion results for predicting strain. So the first 10 modes of the structure were used in the expansion process for obtaining the full-field displacement solution. Figure 11shows that adding low levels of noise (up to 5%) did not significantly impact the strain prediction using the proposed technique. However, the solution starts to degrade with higher levels of noise, especially for high strain regions as shown in Figure 12b.

Case 3. There were 14 aDOF-simulated measurements used for expansion up to 434 nodes for all the cases discussed in this article. In Case 3, 10 redundant sensor measurements are simulated in addition to the 14 aDOF-simulated measurements to study the effect of a greater number of sensors on the strain prediction. A total of 24 sensors are used for generating simulated aDOF measurements. The 14- and 24-sensor setups are shown in Figure 13. Notice that the 24-sensor setup has sensors located somewhat redundantly at locations that are considered to be critically important for expansion. Redundant information at strategically important locations will ensure that important information is not lost during actual experimentation.

Figures 14 and 15 show comparisons between predicted and reference Y normal strain solution for 14-sensor and 24-sensor setups at the tip and inside the box beam, respectively. Note that the 24-sensor setup shows slight improvement in the strain prediction as compared with the 14-sensor setup. However, none of the sensor setups provides accurate strain prediction, because only the first six modes are used in the expansion process. Recall that for accurate strain prediction, more than six modes were shown to be required in Case 1. This indicates that use of a higher number of sensors does not necessarily satisfy the requirement for using a greater number of modes (more than six modes) in the expansion process. But generally use of redundant sensors at strategically important locations could be beneficial from a practical standpoint.

Conclusions

Using a limited set of measured responses, full-field displacements can be obtained using an expansion procedure. The measured response can then be used in the finite-element recovery process to obtain full-field dynamic stress-strain due to any measured loading condition.

Several cases were investigated to show the accuracy and robustness of the expansion procedure. The expansion procedure will produce accurate results provided that there is a sufficient set of orthogonal mode shapes to span the space for the solution; this requires that a sufficient number of modes that participate in the response be included in the expansion process.

Noise causes variation in the results, but from the cases studied, significant noise did not seriously contaminate the results obtained. In terms of the limited set of measurement points, there must be a sufficient number of measurement locations available for proper expansion. Use of redundant sensors can be an advantage from a practical perspective; however, using more sensors does not satisfy the requirement for using a proper number of modes in the expansion process.

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