Shock Severity Estimation

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The recently issued specification ANSI/ASA S2.62-2009, *Shock Test Requirements for Equipment in a Rugged Shock Environment*, specifies shock severity levels according to the plateau level on the PVSS plotted on 4CP or displayed as a 4CP, (pseudo-velocity shock spectrum plotted on four coordinate paper or displayed as a four coordinate plot). The levels run from 1-10 meters per second or 40 to 400 ips. These ANSI levels provide an example of how to identify shock severity. Severity is the plateau level with the frequency range; e.g., 210 ips from 8 to 200 Hz. Examples are plotted of both PVSSs and SRSs divided by 2πf, to form an acceleration PVSS or APVSS on 4CP. These show that the plateau of the APVSS is the same as the PVSS. So published SRSs can be evaluated with constant velocity lines drawn on the SRS that also show the severe frequency range. The article shows severity levels of many SRSs. Several pyroshock references that refer to plateau severities of 50 ips as very mild, 100 ips as moderate, and 300 ips as very severe are examples.

A shock is a violent transient motion delivered to the base of the equipment, and our concern is when it is severe enough to cause equipment failure. I'll say a shock is a violent velocity change. The pseudo-velocity shock spectrum (PVSS) of a simple shock plotted on four-coordinate paper (4CP) looks like a flattened hill, as shown in Figure 1. The plateau or top of the hill shows the severe frequency range of the shock. The hill slopes down and to the right with an asymptote equal to the maximum acceleration. Maximum acceleration usually defines the high-frequency extent of the plateau. The hill slopes down and to the left with an asymptote equal to the maximum displacement, and maximum displacement defines the low frequency plateau limit. The height of the plateau of the PVSS on 4CP and its frequency range is the severity of the shock. The frequency range over which the plateau is at this high level is the range of equipment modal frequencies that can be excited to this velocity. Equipment and multi-degree-of-freedom systems (MDOFs) accept shock energy only at their modal frequencies. The velocity induced in each MDOFs mode is equal to the PV at the modal frequency times the modal participation factor. Peak modal velocity is proportional to peak modal stress. Thus peak modal stress is proportional to PV.[1,2,3] The plateau of the PVSS on 4CP is the severe shock region.

Severity Definition and Precedents

Because we know the PVSS-4CP plateau is the severe shock region, we can classify the severity of a shock by its plateau level and frequency range. I'll be saying the severity of a shock is, as an example, 200 ips from 15-450 Hz. My job here has three parts:

- Convince you that this is the right definition of shock severity
- Show how you can measure the plateau on existing SRS plots
- Show some example severity estimations on published SRSs

The primary reason for defining the plateau level to be the severity is that modal velocity is proportional stress. Since maximum stress limits severity, it's natural that the velocity level that a shock can deliverer to equipment is the severity level. Stress is proportional to modal velocity: $\sigma = K\rho c v$, where $1 \le K \le 10$,[4,5] or probably more.[6] MDOF systems, both lumped and continuous, have modal responses given by the product of the participation factor and the modal PVSS value.[1,2] In my training and course lectures,[3] shock analysis for beams results in the maximum modal velocity of each mode equal to a participation factor times the PVSS value at the modal frequency. Severe velocities that cause yield point stresses in mild steel beams turn out to be about 130 ips. So 100 ips becomes a common floor for shock severity. Appendix A contains a table of theoretically severe shock modal velocities calculated from this theory. It gives velocities of rods and beams that cause yield or fracture stress; they are shockingly low for many materials.

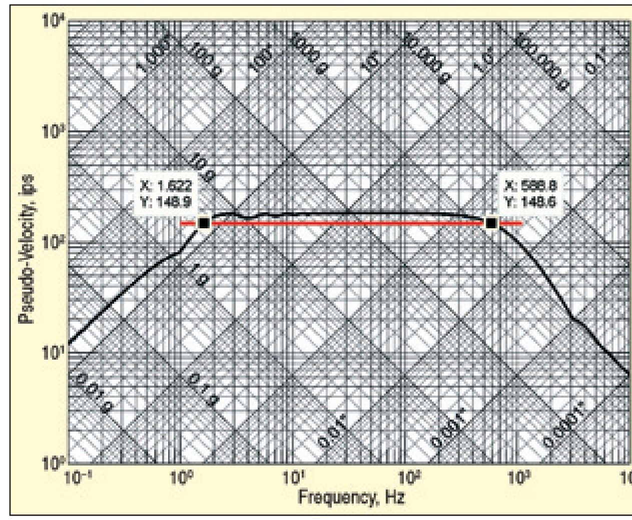

Figure 1. Five-percent damped PVSS on a four-coordinate display of the 1000-g, 200 ips, half-sine, drop-table shock. Shock severity is 185 ips from 2 to 400 Hz.

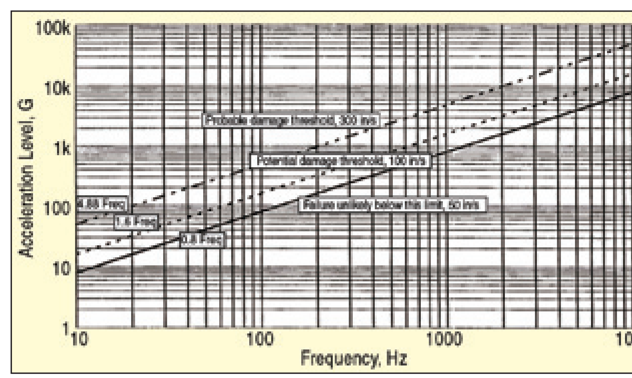

Figure 2. Morse's chart labeling velocity on an SRS as a "potential pyro-shock damage indicator for electronic equipment."[13]

In 2009, ANSI issued ANSI/ASA S2.62-2009: *Shock Test Requirements for Equipment in a Rugged Shock Environment*.[7] This specification defines 10 severity levels in terms of velocity change at impact, Level 1 being a mild shock of 1 m/s velocity change and Level 10 a very severe shock of 10 m/s. Annex D defines the PVSS and links its plateau to the velocity change with its frequency limited by drop height and peak acceleration. Since 1 meter is approximately 40 inches, a Level 2 shock has a plateau velocity of 80 ips. The specification does not specify a low frequency, which I consider a deficiency. On a PVSS, the impact velocity change is the plateau level; the maximum acceleration and maximum displacement range define its frequency range.

STANAG 4559[8] defines a shock level in terms of the PVSS on 4CP with three numbers: d_0, v_0 and a_0, which form the traditional flattened hill-shaped simple shock PVSS on 4CP.[2] On May 18, 2008, NATO Standardized Agreement (STANAG) 4549: *Testing of Surface Ship Equipment on Shock Testing Machines* was adopted. The NATO standardized level notation is defined to take the form: NS LEVEL (m, m/s, m/s^2). With this, the shock environment is completely defined in a compact notation.

Previous data[9] indicated PV to be the best severity indicator. with a PV of 150 ips as the failure level for the six fans tested. Eubanks and Juskie[10], and Gaberson and Eubanks[11] defined equipment fragility as the highest PVSS on 4CP that the equipment is known to have survived.

Piersol[12] commented on shock severity in terms of velocities, and

I'll quote his statement exactly. "Specifically, experience suggests that structural damage to equipment is rare when the equipment is exposed to a shock producing a peak modal velocity of less than 2.5 m/sec (100 in./sec) but is common when the peak modal velocity is more than 5 m/sec (200 in./sec)."

I recently discovered Morse's chart[13] and copied it here as Figure 2. It was presented at an Aerospace Corporation conference in 2000. He clearly shows that the aerospace community understood the significance of shock spectrum velocities then, and I was told that the ideas were known in the early 1970s. Note that Morse is saying that 50 ips is a velocity below which failure seldom occurs, and 300 ips is a level of probable damage. In summary, ample precedent and evidence supports the definition of severity as the plateau level and frequency range of the PVSS on 4CP of the shock.

4CP Equations

I hate to bore you, so skip down five equations if you already know Equations 2a, 2b, 2c, 2d. For those who aren't sure, I'll present the details. Four coordinate paper is sine wave paper. A sine wave with maximum value, z_{max}, and frequency as ω, ($\omega = 2\pi f$) in radians per second, and its two derivatives can be written as in Eqs. 1a, b, and c. (One dot over the z represents velocity and two dots, acceleration.)

$$z = z_{max} \sin \omega t \tag{1a}$$

$$\dot{z} = \omega z_{max} \cos \omega t \tag{1b}$$

$$\ddot{z} = -\omega^2 z_{max} \sin \omega t \tag{1c}$$

Considering the maximum absolute values, we can write Eq. 1d from 1b, and 1e from 1c:

$$\dot{z}_{max} = \omega z_{max} \tag{1d}$$

$$\ddot{z}_{max} = \omega^2 z_{max} \tag{1e}$$

If we divide both sides of Eq. 1e by ω, we get Eq. 1f, and if we compare Eq. 1f to 1d, we get Eq. 1g:

$$\frac{\ddot{z}_{max}}{\omega} = \omega z \tag{1f}$$

$$\dot{z}_{max} = \frac{\ddot{z}_{max}}{\omega} \tag{1g}$$

Finally, if we eliminate ω from Eq. 1g with 1d in 1h and simplify, we get Eq. 1i:

$$\dot{z}_{max} = \frac{\ddot{z}_{max}}{\left(\frac{\dot{z}_{max}}{z_{max}}\right)} \tag{1h}$$

$$\dot{z}^2_{max} = z_{max} \ddot{z}_{max} \tag{1i}$$

The 4CP equations are Eqs. 1d, 1f, 1g, 1i. I'll write them as Eqs. 2a, b, c, d:

$$\dot{z}_{max} = \omega z_{max} \tag{2a}$$

$$\omega z_{max} = \frac{\ddot{z}_{max}}{\omega} \tag{2b}$$

$$\dot{z}_{max} = \frac{\ddot{z}_{max}}{\omega} \tag{2c}$$

$$\dot{z}^2_{max} = z_{max} \ddot{z}_{max} \tag{2d}$$

Four-coordinate paper or a four coordinate plot, 4CP, is a nomogram that shows these relations. At every point on the 4CP, these relations relate the four coordinates. If you know two of the coordinates at a point, the 4CP equations allow you to calculate the other two. The equations also hold on an SRS plot. I use Eq. 2c frequently to calculate velocity on an SRS.

Pseudo-Velocity and Acceleration Pseudo-Velocity

The shock response spectrum (SRS) has come to mean the maximum absolute acceleration of the mass of a base-excited, single-degree-of-freedom (SDOF) system exposed to the shock. A pseudo-velocity shock spectrum, a PVSS, presents the calculated maximum absolute value of the relative displacement of the SDOF for each frequency multiplied by that frequency (which gives it the units of velocity), and plotted on 4CP. I now define an APVSS, an acceleration pseudo-velocity shock spectrum, as the maximum absolute acceleration of the SDOF's mass divided by the frequency (which gives it the units of velocity) and plotted on 4CP. I will show you that it is badly in error for 5% damping at low frequencies.

The shock spectrum equation is the equation for the response of a base-excited SDOF system excited by a shock, y. It is given here as Eq. 3a. (ζ is the damping ratio.)

The excitation or shock is y, x is the absolute motion of the mass, and z the relative motion between the mass and the base, $x - y$. Substituting $z = x - y$ in Eq. 3 gives Eq. 3b:

$$\ddot{z} + 2\zeta\omega\dot{z} + \omega^2 z = -\ddot{y} \tag{3a}$$

$$2\zeta\omega\dot{z} + \omega^2 z = -\ddot{x} \tag{3b}$$

$$2\zeta\dot{z} + \omega z = -\frac{\ddot{x}}{\omega} \tag{3c}$$

Divide Eq. 3b by ω to obtain Eq. 3c, which is important; it says $2\zeta\dot{z}$ plus the pseudo-velocity equals negative maximum acceleration over ω. If the damping is zero, the pseudo-velocity is equal to the acceleration of the mass divided by ω, and for an undamped analysis, the PVSS will equal the APVSS, or the SRS divided by ω. Many authors have guessed wrongly that for small damping, Eq. 2b or 3c with zero damping, relates PV and $\ddot{x}/\omega$. I'll show you that when we compare the PVSS and the APVSS, a significant error occurs at low frequencies, and that they are close or equal in the plateau and the high-frequency region.

In the next section I compare the PVSS and the APVSS plotted on 4CP for four very different shocks to show they agree in the plateau and high-frequency region, and that the APVSS doesn't follow the PVSS at low frequencies with 5% damping. The reason I'm doing this is to show that if you divide an SRS by frequency (calculate an APVSS) you get a good approximation of the PVSS plateau and high-frequency region and can then evaluate the PVSS plateau level (severity) and frequency range of the huge quantity of SRS data contained in the literature.

Comparisons for Four Example Shocks

Here we will examine the differences and similarities between the PVSS and the SRS divided by $2\pi f$ to form an acceleration pseudo-velocity shock spectrum (APVSS). Examples are plotted on 4CP of four very different shocks, and I point out the severity. The reason this has to be done is because PV is directly proportional to maximum modal velocity that in turn is proportional to maximum modal stress. Simultaneous plotting of the PVSS and APVSS shows them equal in the plateau and high-frequency regions. This means that the severe plateau can be identified on the SRS, and shock severity can be evaluated on all published SRS plots. The APVSS provides a way to use the huge quantity of existing SRS data to approximate a PVSS and thereby show the severity of the SRS analyzed shocks.

I'll analyze four demonstration shocks – a drop table shock machine shock, a Navy explosive shock, a pyrotechnic explosive shock, and an earthquake shock. I want to show how the severe plateau on the PVSS shows up on the SRS and the APVSS. I ultimately want show you that we can see the PVSS plateau on our SRSs, and that it's easy to draw severe velocity lines on an SRS to show the PVSS plateau. But first I think you should see how the PVSS and the APVSS compare. We find the plateau on the PVSS and then identify it on the SRS.

Consider a 200-ips, 1000-peak-g, half-sine, drop-table shock.

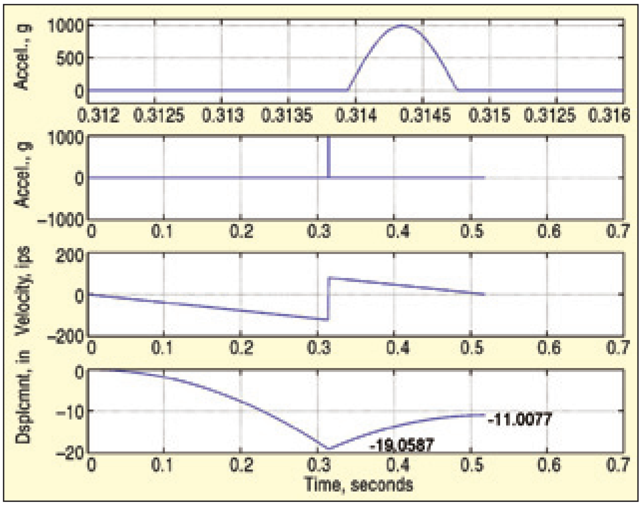

Figure 3. Time history and integrals for a drop-table, shock-machine, half-sine shock with 200 ips velocity change and 1000 g maximum acceleration. Acceleration during drop and rebound included. Note expanded time scale on top subplot to show half-sine detail.

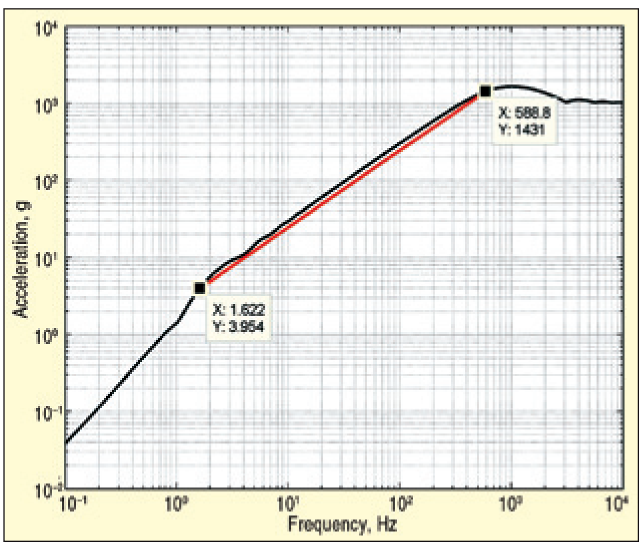

Figure 4. SRS of 200-ips, 1000-g, half sine including drop and rebound with coefficient of restitution of 0.65. Red line runs from same two frequencies as 80% plateau line on the PVSS of Figure 1. (The reason plateau or high-velocity region stands out clearly is because I have added drop to time history and made it a shock that begins and ends with zero velocity.)

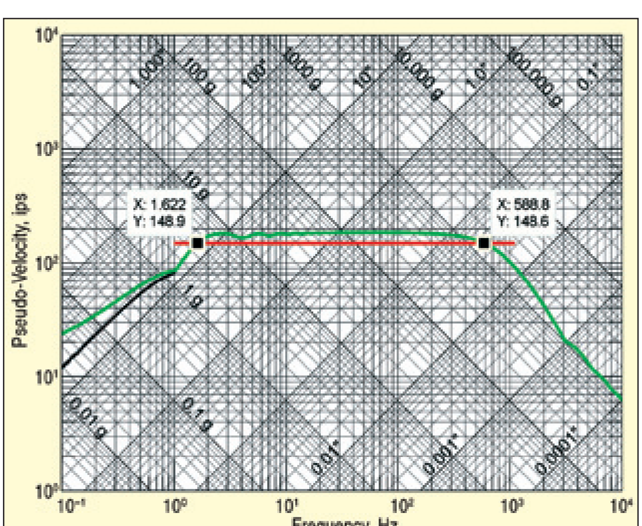

Figure 5. Superposition of the APVSS of 1000-g, 200-ips half sine on its PVSS on four-coordinate paper. APVSS is green and PVSS black. The two appear coincident until 1 Hz, when APVSS wanders upward while PVSS heads for its maximum displacement asymptote of 19 inches.

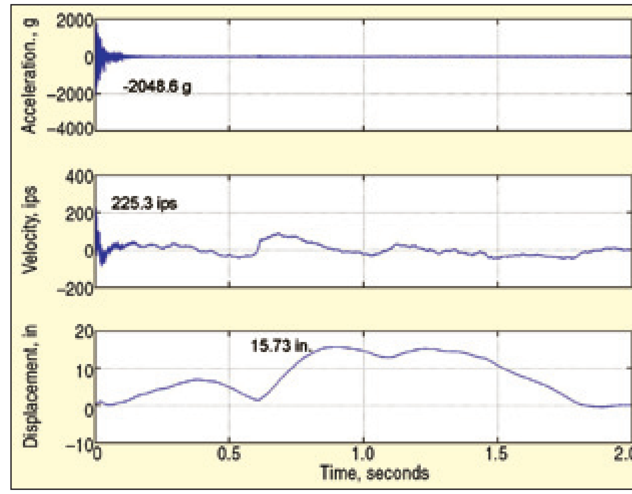

Figure 6. Explosive Navy shock test time-history with a peak g level of 2049 and a peak velocity of 225 ips and a maximum displacement of 15.7 inches.

We'll select a coefficient of restitution of e = 0.65. It is necessary to include the acceleration during the drop to understand the low-frequency limits of the shock severity. Figure 3 shows the acceleration time history and its integrals. On the top plot I show an expanded view of the impact interval so you can see the half-sine shape. Figure 1 shows its 5% damped maximax PVSS on 4CP. The flat-top plateau occurs over a range of about 2 to 400 Hz, and it's at a level of 185 ips; so its severity is 185 ips from 2 to 400 Hz. That is the frequency range for which the shock is severe, where it can induce the highest modal velocities. To more accurately show the range, I've drawn a red line at 80% of the plateau level of 185.34 ips. For 5% damping, the simple shock plateau level is at 0.9267 of the undamped plateau[14] that is at the impact velocity change of 200 ips. The 5% damping reduced it by about 6%. The line intersects the plateau at about 1.62 and 589 Hz, which is determined by using Matlab's datatip function that can read the values on the curve. This is the frequency range where the shock can deliver 80% of its peak velocity into equipment modal responses.

This is going to get very instructive; at least it was to me. Let's now look at the SRS of the shock and compare it with the PVSS. This is going to demonstrate how to read the severity from the SRS, which few understand. Figure 4 shows the SRS of our 1000 g, 200 ips half sine. If you imagine the PVSS of Figure 1, tilted 45 degrees to the left, you can somewhat see the features on the SRS. What is interesting here is that the PVSS plateau shows clearly when it is marked. I've drawn a straight red line from the same frequencies that were found on the PVSS. We identify the plateau from the same two frequencies we found on the PVSS.

To test the comparison of the APVSS (acceleration pseudo-velocity shock spectrum, formed by dividing each SRS value by $2\pi f$) to the PVSS, Figure 5 shows the superposition of the two graphs with the APVSS shown in green. What I see is that the APVSS and the PVSS are the same from about 1 Hz on in this case. This demonstrates that it's OK to draw in the velocity lines on SRSs, and it's OK to plot SRSs on 4CP by dividing by $2\pi f$. An SRS divided by $2\pi f$ and plotted on 4CP is the APVSS. It gives exact accelerations on the 4CP, while the PVSS gives exact displacements and PV on the 4CP.

The APVSS is shown as a green line on top of the black PVSS line of Figure 5. The lesson of this example is that if you have the calculated SRS values, you can divide them by $2\pi f$ and show them on 4CP to read the shock severity. If the shock has been edited to end at zero velocity, the low-frequency plateau limit will be seen. An additional important point is that we calculated to a high enough frequency to see the acceleration asymptote and a low enough frequency to see the SRS fall away from the plateau.

Now let's do the same with an explosive shock test to show how the APVSS superimposed on the PVSS works out for another case. Figure 6 shows the acceleration time history for a Navy heavyweight explosive floating shock platform shock for the first 2 seconds of the record. I edited the data by detrending, which

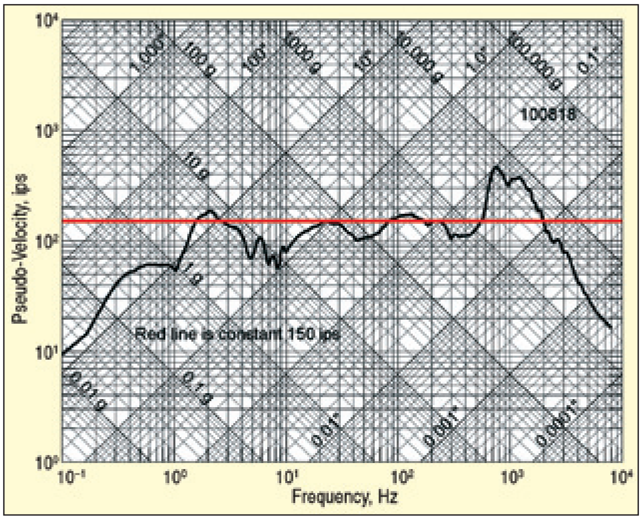

Figure 7. PVSS of explosive shock test shown in Figure 6. Severity could be 150 ips from 1.5 to 2000 Hz. (Note that where I have drawn in the plateau is a matter of judgment. I guessed it to be 150 ips in this case.)

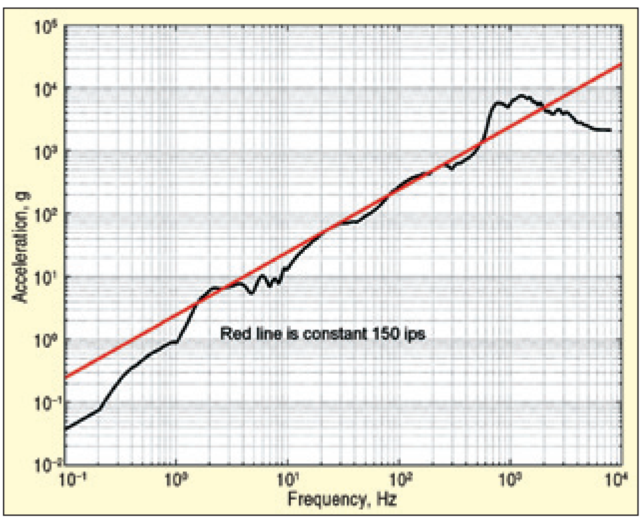

Figure 8. SRS of explosive shock test of Figure 6. Notice similarity of PVSS and SRS features and that plateau identified on PVSS can be clearly seen on SRS when 150-ips constant velocity line is drawn in.

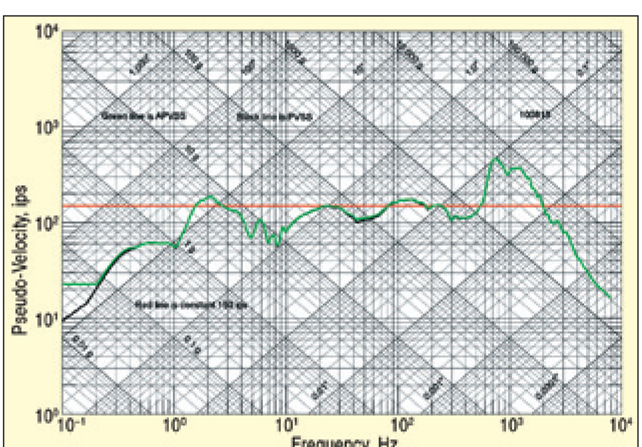

Figure 9. APVSS of Navy Explosive Shook Test of Figure 6 superimposed on its PVSS. This time we see a slight difference between the two in plateau region near 40 Hz. Low-frequency split between the two spectra at 0.4 Hz is also interesting. I have tested both, calculating algorithms at zero damping and found both yield identical results; so discrepancies we see are due to damping term in Eq. 3c.

forces the final velocity and displacement to be zero. Its rather lumpy plateau is shown in Figure 7, where I have drawn in a plateau line at 150 ips, which is debatable. I calculated the SRS for this shock and plotted it in Figure 8; this time I drew in the 150

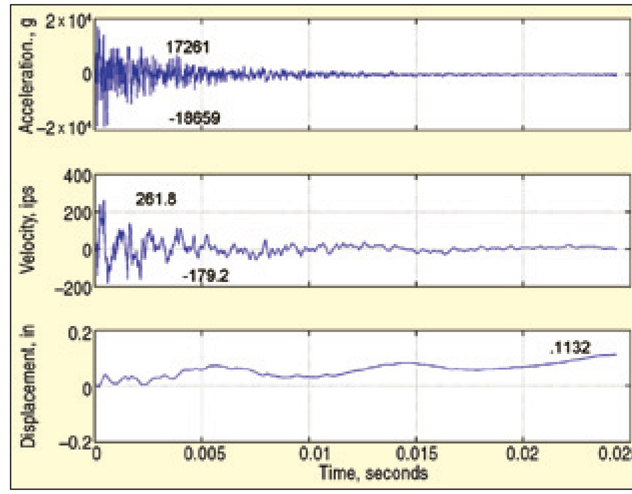

Figure 10. Time history of sever pyroshock named Frapple 32.

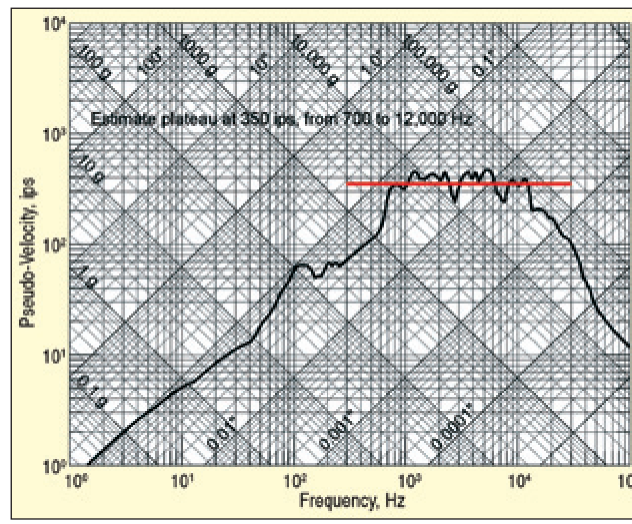

Figure 11. PVSS of very severe Frapple 32 pyroshock with plateau debatably drawn at 350 ips. (It could just as well have been placed at 400 or 450 ips.) Severity is 350 ips from 700 to 12,000 Hz.

ips line by calculating its ordinates at 0.1 and 10000 Hz from Eq. 2c. This seems to work well and intersects the SRS at the same frequencies that it intersects the PVSS. I have plotted the APVSS for the shock on top of its PVSS in Figure 9. Now we do see a slight discrepancy in the mid frequency region around 43 HZ, but you have to admit that the APVSS and the PVSS agree well in the plateau. The severity is 150 ips from 1.5 to 2000 Hz with a high frequency burst of 300 ips from 650 to 1000 Hz.

Pyroshocks

Next I want to show a very severe, extremely high-frequency explosive pyroshock example, named Frapple 32. The time history is shown in Figure 10. The maximum g levels are 17,281 and –18,659; peak velocities are 262 and –179 ips. The maximum displacement is 0.113 inches. The PVSS is shown in Figure 11, and is very severe. The shock severity is 350 ips from 700 to 11,0000 Hz. Notice I have had to shift the abscissa to one decade higher frequency to accommodate the higher frequency.

I calculated its SRS and drew in a 350 ips line from 10 to 100,000 Hz using ordinates calculated with Eq. 2c; this is shown in Figure 12. Notice that the velocity plateau line intersects the plateau at the same points as it does in the PVSS. Again I plotted the APVSS of Frapple 32 on its PVSS in Figure 13. The low-frequency discrepancy is surprising, but perhaps this is what we have to learn to expect. I don't have much experience in simultaneous plotting the PVSS and the APVSS. (Neither does anyone else. I'm sure I'm the first to publish this comparison.) And so I will just have to accumulate experience as time goes on. (I hope you will help and publish more examples.) I added the APVSS calculation to my SRS

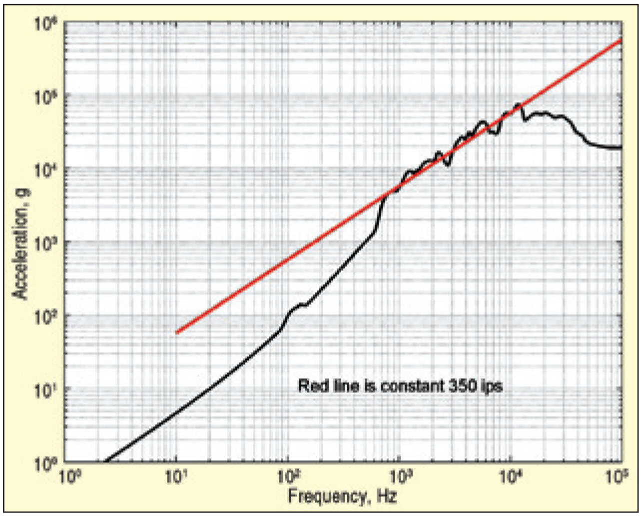

Figure 12. SRS of pyroshock Frapple32 with 150-ips red line indicating plateau.

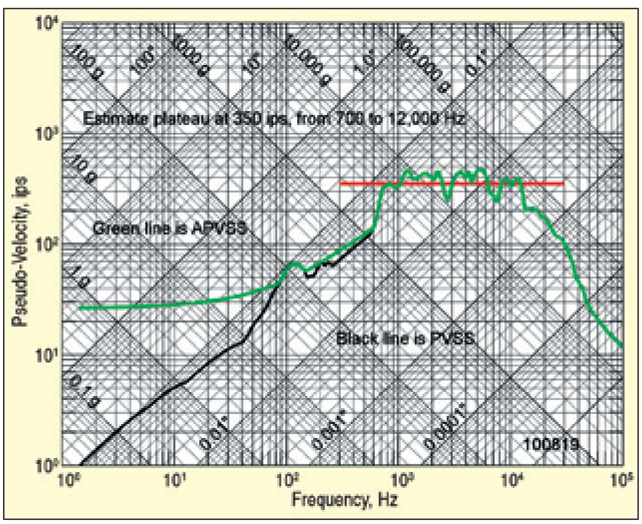

Figure 13. Superposition of APVSS on PVSS for pyroshock Frapple32. Notice the two are identical from 600 Hz but diverge markedly below 80 Hz.

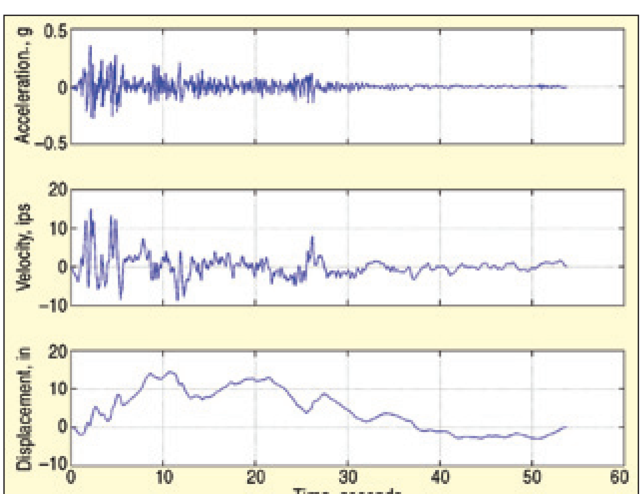

Figure 14. Earthquake time history of one of the El Centro Earthquake records. Note low acceleration levels, moderate velocities and large displacements.

program, so it's no trouble to accumulate plots like this, and I will from now on. Maybe later it will make more sense to me and I can write something additional about it.

Earthquakes

Now finally a low-frequency earthquake: I apologize for my lack

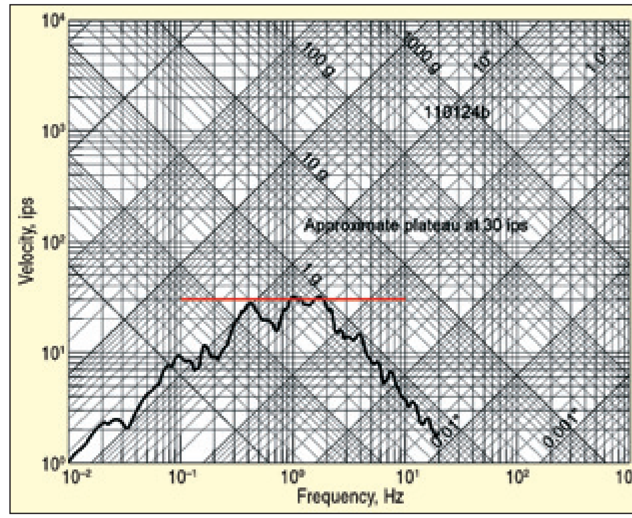

Figure 15. PVSS of El Centro earthquake example. Note I shifted the abscissa of the four-coordinate paper a decade lower to show low-frequency asymptote. I have not yet added the interdecade lines on this low-frequency four-coordinate paper.

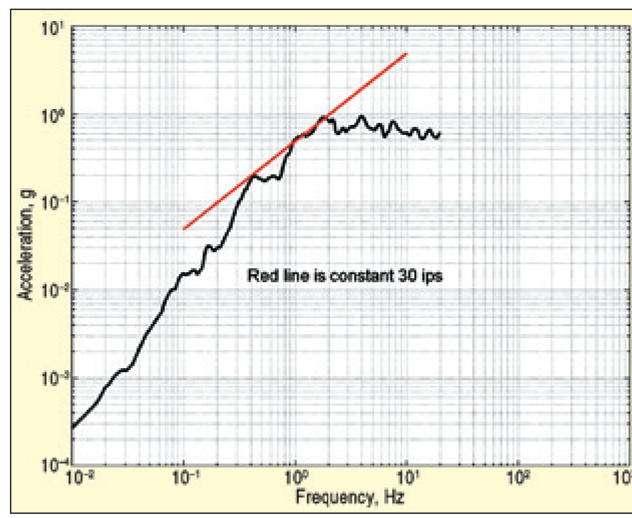

Figure 16. SRS of El Centro 3arthquake example. Notice the 30-ips plateau line fitting upper-left region of SRS. Plot not calculated to high enough frequency to show flat horizontal SRS peak acceleration asymptote

of severe earthquake motion examples. I have not been able to manage my time to permit me to find a suitable example. But as a pittance representation of this great class of shocks, I have only El Centro (IIA001 40.001.0 El Centro Site Imperial Valley Irrigation District, COMP S00 & 2688). It's a different motion and should add to the experience. Figure 14 shows the time history. Peak accelerations of about 0.3 g, peak velocities of 30 and –9 ips, with a maximum displacement of about 15 inches. Figure 15 shows its PVSS with a plateau drawn in at 30 ips. We could say the severity is 30 ips from about 0.4 to 2.0 Hz. Figure 16 shows the SRS with the plateau drawn in at 30 ips. It seems to fit in place quite reasonably and supports the idea that we can draw in a plateau on an SRS. Figure 17 shows the APVSS superimposed on the PVSS, and again we see the low-frequency discrepancy. The severity is 30 ips from 0.5 to 2 Hz.

This completes the PVSS and SRS comparisons and the first example severities. What you have seen is that in each case, the PVSS and the APVSS agree excellently in the plateau and high-frequency region. You have also seen that in all cases the low-frequency limit of the plateau is shown by the APVSS as well as the PVSS, and this is important. I will show examples where the plateau limits do not show distinctly on many published SRSs. This is due to editing and mostly to not assuring that the final integrated velocity is zero. I have defined a collision and a kickoff shock[1,2,3] as a shock that begins at a high velocity and ends at zero velocity and a shock that begins at zero velocity and ends sailing off into space at a high

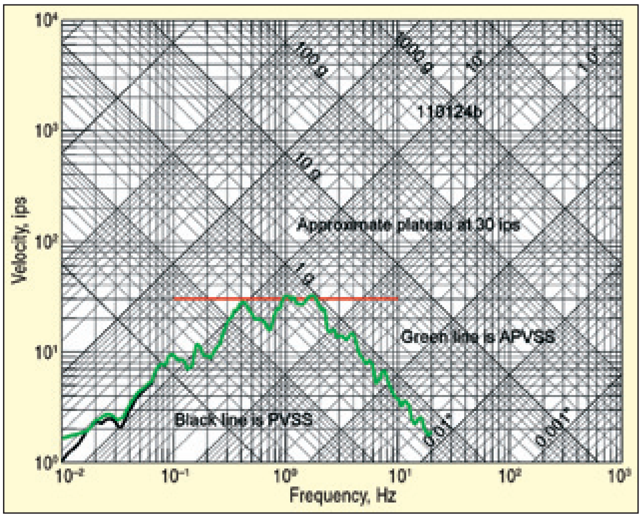

Figure 17. Superposition of the APVSS on PVSS for El Centro earthquake. Low-frequency discrepancy is there but not pronounced, since calculation did not go to low enough frequency.

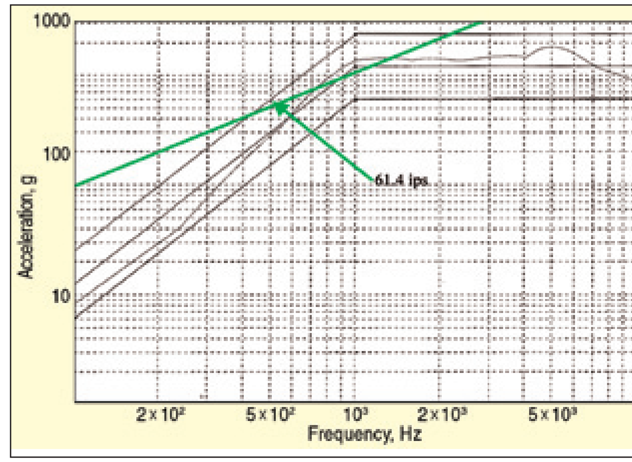

Figure 18. This is from Dr. Bateman's 79th (2008) Shock and Vibration Symposium paper.[15] In this case, grid was clearly drawn and was easy to use to draw the g = f line. Severity is 84 ips from 900 to 1000 Hz.

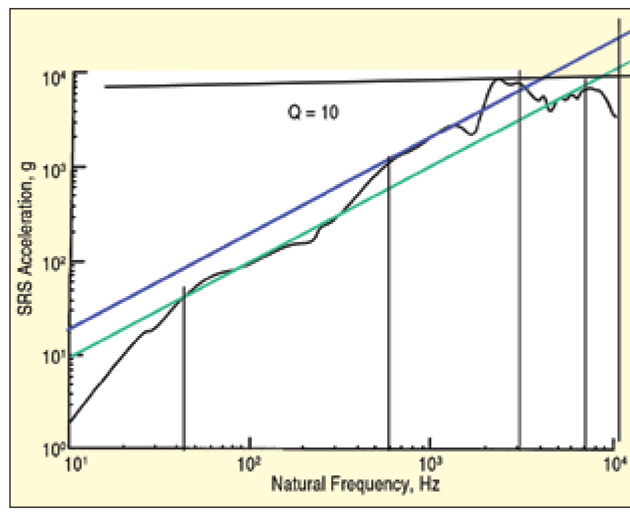

Figure 19. SRS of a famous pyroshock[16] and has been reproduced several times. I'll disclose its severity as moderately severe at 123 ips from 600 to 3100 Hz and at least mild at 61 ips from 45 to 7100 Hz. Vertical thin black lines were drawn to estimate frequencies.

velocity. In both cases, the PVSS does not have a low-frequency constant displacement asymptote. These have a low -frequency asymptote at or slightly less than the constant velocity that the collision began with, or the kickoff ended with, depending on the damping.[14] I haven't tried to discuss these cases here for lack of space. However, any shock that does not end with zero velocity must be considered a kickoff shock.

I also must call your attention to the fact that in all these four cases, the plateau was the furthest up and to the left of the SRS values or data. That's how it must be from the observation of the tipping of the PVSS counter clockwise to make an SRS. It is also true because these four shocks were edited to have a zero final velocity. Many of the cases shown below are cases where the low-frequency plateau limit does not exist; this is due to a final velocity. Most real shocks begin and end with zero velocity; it may take longer than data were collected for the motion to stop, but most shocks have a zero final velocity. The drop-table shock machine shock starts at zero velocity prior to the drop, and the drop is normally not included in the data, so that integrating only the impact will not show the low-frequency peak displacement asymptote. This must be drawn in at the drop height by the analyst on the 4CP to show the-low frequency limit of the plateau.

Estimating Severity of Published SRSs

Severity estimation can be done by drawing constant velocity lines on scanned copies of published shock SRSs. The first thing I do is to draw the diagonal line where $g = f$; i.e., the acceleration in g equals frequency in Hz which is the 61.4 ips line. To visualize or draw constant velocity lines on the log-log SRS plots, we have to do the following. I have shown you that in the plateau and the high-frequency regions for the four example shocks the SRS divided by the frequency or the APVSS agrees well with the PVSS on 4CP. Its value is given by dividing by the frequency as shown in Eq. 2c, repeated here:

$$\dot{z}_{max} = \frac{\ddot{z}_{max}}{\omega} \qquad (2c)$$

Let N_g mean the numerical value of the maximum acceleration expressed in g and let N_f be the numerical value of the frequency in Hz, (cps). Agreed-upon values of the constants are: g = 980.665 cm/sec² = 386.087 in/sec² = 32.18739 ft/sec² per.[18] The values we use in Eq. 2c are in Eq. 4:

$$\ddot{z} = N_g 386.087 \frac{in}{sec^2}, \text{ and } f = N_f \frac{1}{sec} \qquad (4a)$$

By substituting these values into Eq. 2c, we get (remember $\omega = 2\pi f$):

$$\dot{z} = N_g \frac{386\ in}{sec^2} \frac{1}{N_f} \frac{sec}{2\pi} = \frac{N_g}{N_f} 61.4477\ ips \qquad (4b)$$

Or in metric terms:

$$\dot{z} = N_g \frac{0.80665\ meters}{sec^2} \frac{1}{N_f} \frac{sec}{2\pi} = \frac{N_g}{N_f} 1.5608 \text{ meters/sec} \qquad (4c)$$

So if $N_g = N_f$, the velocity is 61.4 ips or 1,56 meters per second. The line where $g = f$ is easy to draw in Powerpoint. Published SRSs can be scanned into JPG or TIF files and inserted into a Powerpoint slide, where lines can be drawn at $g = f$, and these are lines of constant 61.4 ips velocity.

Figure 18 is the first example. This mild shock only exceeds 61.4 ips from 800 to 1400 Hz. The PV content beyond 1400 Hz decreases steadily. Imagining lines parallel to the green 61.4 ips line, the data looks like constant velocity between 900 and 1000 Hz. I scaled the acceleration at 1000 Hz to be 1360 g. Which by Eq. 2c comes out to be 84 ips. The severity could be stated as 84 ips from 900 - 1000 Hz, or above 61.4 ips from 800 to 1400 Hz. This plot with clear grid lines, abscissa and ordinate values is a pleasure to evaluate.

Figure 19 from the pyroshock design manual description paper[16] is the next example. It is typical of what I have seen. The figure shows the vertical construction lines I drew to estimate the frequencies. Power Point has a feature for drawing vertical, horizontal, and 45-degree lines by holding down the shift key while you move the mouse to draw the lines. First I had to draw construction lines: vertical from 10,000 Hz and horizontal from 10,000 g to get the other end of the green line (61.4 ips). With the green line in place, it is easy draw in the blue line parallel to it. I needed the frequency (10 Hz) and amplitude (20 g), where the blue line intersects the ordinate axis, to calculate the velocity from Eq. 2c, which comes out to be 2 × 61.4 = 123 ips.

Figure 20 (from Reference 17) also shows how the plateaus

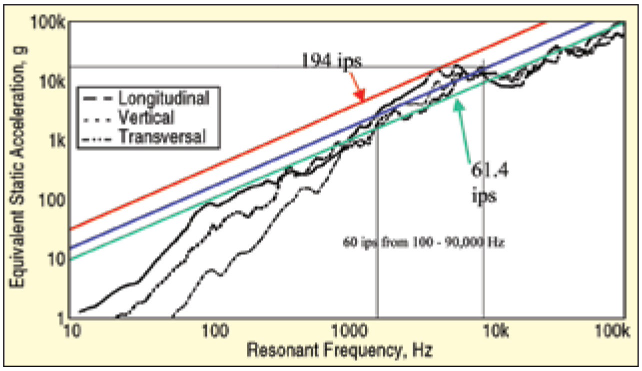

Figure 20. Good example of plateaus that show up as they should from Reference 17. Three SRSs are labeled longitudinal, transverse, and vertical. Blue line shows longitudinal shock was a 92 ips from 2000 to 9000 Hz, and it stays at the mild level of 60 ips from 100 to 90,000 Hz.

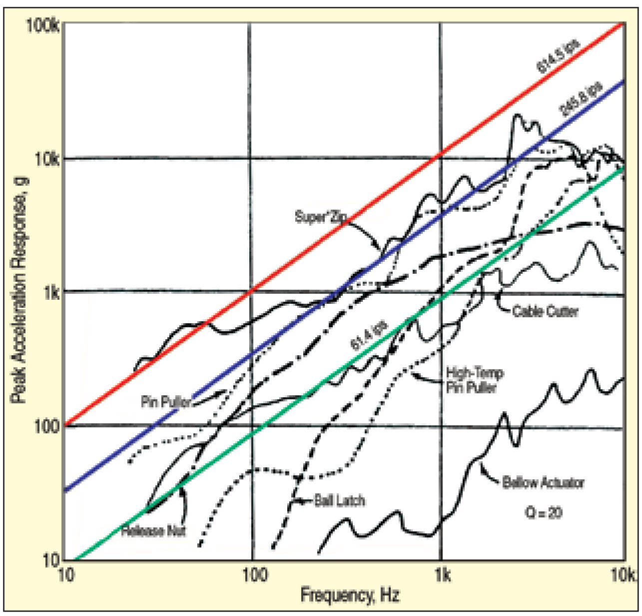

*Figure 21.Busy and very interesting SRS collection containing plots of seven different pyrotechnic shock devices. Names of SRSs in order of decreasing severity are: Super*Zip, Pin Puller, Release Nut, Ball Latch, High-Temp Pin Puller, Cable Cutter, and Bellow Actuator.[17]*

should appear. The 61-ips line follows the SRSs of the three shocks well from 100 to 90,000 Hz. The blue line shows that the longitudinal shock was 92 ips from 2000 to 9000 Hz

Figure 21 certainly shows that pyroshock events can be high-level shocks. The figure shows seven shocks. The Super*Zip has a plateau well above 250 ips, but its low-frequency region looks corrupted and ends at an unrealistic level. The trace probably had a zero shift that caused the low-frequency problem. But it has a plateau above 250 ips from 250 to 3000 Hz. The Pin Puller is just under it also at the 250 ips level. The Release Nut has a shape that serves as a nice example. If you imagine the 10 g-100 Hz to 1000 g-10,000 Hz line, which is at 6.14 ips, you can see that the Bellow Actuator is not a shock, being below the 6 ips line. This is the important part of being able to estimate severity from an SRS. I have several shocks below 6 ips that are foolishly published, because probably neither the author nor the readers know what they are looking at. Picking off the velocity of any point on an SRS is done by using the acceleration and frequency in Eq. 2c.

Conclusions

I am convinced that the plateau level (approximated as the peak elevation of the highest constant velocity line) on the PVSS on 4CP with its frequency range is the severity criterion and defined it that way. I cited a great deal of precedent to support this definition. I defined an acceleration pseudo-velocity shock spectrum, the APVSS as an SRS divided by $2\pi f$. I showed that the APVSS agrees with the PVSS in the severe plateau and high-frequency regions. This provides evidence that we can read severity from published SRS plots. I showed you the APVSS is in error at low frequencies on damped shock spectra.

I demonstrated how to read severity information from an SRS. Eq. 2c tells you the velocity of any point on an SRS curve. I explained drawing constant velocity lines on SRSs by starting with the $g = f$ line, which is 61.4 ips. Lines parallel to this line are easy to draw (in Power Point) and can be evaluated with the frequency and acceleration at any convenient point with Eq. 2c. I showed one example (Figure 21) of many I've seen, where a constant velocity line does not define an expected plateau. The spectrum does not fall away from any highest constant velocity line. I don't know for sure, but I suspect that the time history for those SRSs would not have integrated to reasonable values and was in error in some respects.

Experts and I think that 100 ips is the severe shock threshold and that 50 ips is a very mild shock. But in teaching professional test engineers this material, I met a test manager who is certain he has seen a system failure from a shock with a plateau below 50 ips. Plateaus at 200-300 ips are very severe. Damage at those levels is probable. I have copies of SRSs that have a plateau at 600 ips. The levels are guidelines, and there have to be exceptions.

The analysis requires significant calculation – but it's only a calculation. If we build it into a calculator, it will be a button push. I expect that Excel could do it. I have learned to do it in MATLAB and offer you my programs. There is a freeware calculating program called Octave. Students have told me it has run my programs, but for one reason or another, the students have not been able to teach me. Octave can't plot well, but the $50 dPlot can. Some of you have to learn it and teach the rest of us. The PVSS calculation solves the mystery of shock. It costs $3000 with MATLAB; it could be $50 with Octave. Please send me your comments and suggestions.

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Appendix: Maximum Stress Velocities

In Reference 4, we discovered and explained that stress was proportional modal velocities in rods and beams. For the stress in a rod vibrating in a mode we found:

$$\sigma_{rod} = \rho c v = v\sqrt{E\rho} \tag{A1}$$

For a beam, it is multiplied by a shape factor, which is the distance to the neutral axis divided by the radius of gyration. For a rectangular beam this is $\sqrt{3} \approx 1.73$. Thus the maximum allowable modal velocities for a rod and a beam are given in Eq. A2:

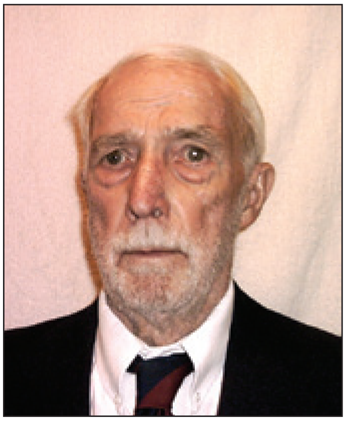
$$v_{rod} = \frac{\sigma}{\sqrt{E\rho}},\ v_{beam} = \frac{v_{rod}}{1.73} \tag{A2}$$

$$\sigma = K\rho c v \tag{A3}$$

We later found that Hunt[5] nine years earlier had done our job more thoroughly, and he presents a plate analysis saying K_ρ is between 1.1 and 2.0, in Eq A3. Crandall,[6] in a very difficult and very important note, pointed out that these stress values are the minimum to be found with those modal velocities. Due to stress concentrations, the stress for given velocity can be greatly higher. The upshot is the velocity values of this appendix table indicate the smallest stress maximum that exists. The actual maximum is probably much greater. Crandall makes this proof in an extremely roundabout way; I don't like his similitude energy proof. I want to see the proof using $F = ma$. Crandall is brilliant; I've always admired him. This proof is too important to be only available to brilliant people. Average intelligent skeptics have be able to believe it as well.

Table 1. Maximum stress modal velocities.

	E	μ	ρ	σ_{ult}	σ_{yp}	v_{rod}	v_{beam}
(Values from Sloan, 1985, Packaging Electronics)							
Aluminum 5052	9.954	0.334	0.098	34	24	477.4	275.9
Aluminum 6061	9.954	0.34	0.098	42	36	716.2	413.9
Aluminum 6061	9.954	0.334	0.1	77	66	1299.8	751.3
Be	42	0.1	0.066	86	58	684.5	395.7
Be-Cu	18.5	0.27	0.297	160	120	1005.9	581.5
Cadmium	9.9	0.3	0.312	11.9	11.9	133.0	76.9
Copper	17.2	0.326	0.322	40	30	250.5	144.8
Gold	11.1	0.41	0.698	29.8	29.8	210.4	121.6
Kovar	19.5		0.32	34.4	59.5	468.0	270.5
Magnesium	6.5	0.35	0.065	39.8	28	846.4	489.3
Nickel	29.8	0.3	0.32	71.1	50	318.1	183.9
Silver	10.6	0.37	0.38	41.2	41.2	403.4	233.2
Solder 63/37	2.5	0.4	0.30008	7	7	158.8	91.8
Steel 1010	30	0.292	0.29	70	36	239.8	138.6
Stainless	28.4	0.305	0.29	80	40	273.9	158.3
Alumina al203	54		0.13	25	20	148.3	85.7
Beryllia Beo	46		0.105	20	20	178.8	103.4
Mira	10		0.105		5.5	105.5	60.0
Quartz	10.4	0.17	0.094	27.9	27.9	554.5	320.5
Magnesia Mgo	10		0.101	12	12	234.6	135.6
EPO GLS G10 X/Y	2.36	0.12	0.071	25	35	1680.1	971.1
EPO GLS G10 Z	2.36	0.12	0.071	25	35	1680.1	971.1
Lexan	0.379		0.047	9.7	9.7	1428.1	825.5
Nylon	0.217		0.041	11.8	11.5	2395.6	1384.8
Teflon	0.15		0.077		4	731.3	422.7
Mylar	0.55		0.05	25	25	2962.2	1712.3
(Values from Roark, 1965, p 416)							
Aluminum Cast Pure	9	0.36	0.0976	11	11	230.6	133.3
Al cast 220-t4	9.5	0.33	0.093	42	22	459.9	265.8
2014-t6	10.6	0.33	0.101	68	60	1139.4	658.6
Beryllium Cu	19	0.3	0.297	150	140	1158.0	669.4
Cast Iron, Gray	14	0.25	0.251	20	37	357.8	224.2
Mg AZ80A-T5	6.5	0.305	0.065	55	38	1148.7	663.0
Titanium Alloy	17	0.33	0.161	115	110	1306.5	755.2
Steel Shapes	29	0.27	0.283	70	33	226.3	130.8
Concrete	3.5	0.15	0.0868	0.35	0.515	18.4	10.6
Granite	7	0.28	0.0972		2.5	59.6	34.4



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He worked on the ANSI committee preparing the new Equipment Ruggedness Standard; the committee quickly adopted the PVSS on 4CP as the basis for ruggedness standardization. The European STANAG shipboard ruggedness standard, which uses the PVSS plateau and its frequency range as the standard, became a model which led to the adoption of the PVSS plateau level and frequency range as the ruggedness classifier. Ruggedness levels 1, 2, 3, . . became the plateau PVSS level in meters per second.

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