EDITORIAL

Thank You, Mr. Fourier

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Those of us who identify with the fields of acoustics or structural dynamics (such as the readers of *Sound & Vibration*) routinely depend on signal frequency spectra, transfer functions, power spectral densities, and more to ply our trade. If it wasn't for a man by the name of Jean Baptiste Joseph Fourier, none of these tools might have come into being. Let's explore what Fourier accomplished and how his achievements resulted in these easy-to-acquire tools that we depend upon so much today.

Fourier was born in France, the son of a tailor, on March 21, 1768. He was the ninth of 12 children. His mother died when he was nine years old, and his father died the following year. He was educated in a military school run by the Benedictine order.

At age13, he became engrossed in mathematics, and by 14, he had completed a study of six volumes of *Cours de mathematiques.* He continued his studies in mathematics, capturing many awards along the way, but in 1787, he diverted his interests to train for the priesthood.

In 1789, Fourier decided not to take his religious vows, and one year later, he became a teacher at the Benedictine school where he had earlier been a student. He took a prominent role in the local district where he lived, became involved in promoting the French Revolution, was imprisoned briefly, and subsequently was nominated to study at the Ecole Normale in Paris, where he was taught by Lagrange and Laplace.

Fourier next began teaching at the College de France (soon renamed Ecole Polytechnique). Repercussions of his earlier arrest remained; he was again arrested briefly, and in September 1795, was back teaching at the Ecole Polytechnique. Two years later, he replaced Lagrange as the chair of analysis and mechanics.

Interestingly, early academicians such as Fourier were viewed as multidimensional, and in 1798, he accompanied Napoleon's army as a scientific adviser during its invasion of Egypt. In 1801, he returned to France and was appointed by Napoleon to administer the drainage of large swamps and the construction of a new highway.

While living in Grenoble, France, and performing these administrative tasks for Napoleon, Fourier pursued important mathematical work on the theory of heat transfer. He published his results in a memoir entitled *On the Propagation of Heat in Solid Bodies* in 1807. A controversial component of this work involved expansions of functions as trigonometric, or what we now call Fourier series. A committee comprised of Laplace, Lagrange, Monge, and Lacroix expressed concern about the lack of rigor in this work, and Biot objected because the work did not cite him.

In 1811, the Paris Institute awarded Fourier a prize for this memoir, although concern lingered about this lack of rigor. Regardless, Fourier was elected to the Academie des Sciences in 1817.

Joseph Fourier died May 16, 1830, with Biot and Poisson both still expressing criticism of his work.

Today we can generate a unified mathematical theory beginning with the convolution integral and, with progressive limitations, evolve Laplace transforms, Fourier integral transforms, and Fourier series. We can express the forward Fourier integral as the first of the two equations below. Even though this integral formulation has existed for almost 200 years, it saw little practical use over most of its existence. Its implementation required an analytic function $f(t)$.

If someone had explained digital systems to Fourier, perhaps he would have alternately formulated the discrete Fourier transform (DFT) as the second of the two equations below:

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}$$

$$\bar{F}_m = T\sum_{n=0}^{N-1} f_n e^{-j(2\pi mn/N)} m = 0,1,\ldots,N-1$$

Note that the DFT formulation can accommodate a sampled time function (it doesn't have to be analytic), making it of great practical value. However, even if the DFT had existed at the advent of digital computing, the existing computers did not have the computational capability to perform it until 1965.

In 1965, J. W. Cooley of IBM and John Tukey of Princeton formulated a method to eliminate redundancy in the DFT, enabling it to become computationally compatible with computers of the era. This new algorithm, the fast Fourier transform (FFT), produced the identical result as the DFT, only much more time efficiently.

Therefore, in 1965 it became possible to digitize a signal and compute its spectrum on existing digital computers. As an interesting aside, Tukey reportedly came up with the idea for the FFT during a meeting of a U.S. presidential committee discussing ways to detect nuclear weapons tests in the Soviet Union.

In the latter part of the 1960s, early signal digitizers became available. Six-bit digitizers came first, then 8-bit digitizers, and increasingly greater bit resolution rapidly evolved. In 1967, thanks to the availability of these early digitizers, Time Data developed the first commercially available FFT analyzer. Soon the number of analyzer manufacturers expanded, two and more channel analyzers evolved, and it became possible to ratio spectra and compute frequency response and coherence functions. Today's modern, high-speed processing hardware has minimized much of the need for and the excitement over the FFT.

Like most tools we use in modern science, analyzers simply efficiently implement the discoveries of the early mathematicians and philosophers. So when test practitioners push an analyzer button requesting an instantaneous frequency spectrum, power spectrum, frequency response function, cepstrum, or more, they should reflect back and express silent appreciation to an orphaned boy named Jean Baptiste Joseph Fourier who became a great mathematician and whose work supports our professions today.

***Editor's Note:** The Fourier transform and Fourier's Law are also named in his honor. He is also generally credited with the discovery of the "greenhouse" effect. In the 1820s, Fourier calculated that the Earth at its distance from the sun should be considerably colder than it actually is if warmed only by the sun's effects. He suggested that interstellar radiation might be responsible for a large part of the additional warmth, but also considered the possibility that the Earth's atmosphere might act as an insulator. This is widely recognized as the first proposal of what is now known as the greenhouse effect.* SV

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