EDITORIAL

Begone Cursed Alias!

***Strether Smith**, Contributing Editor*

One of my all-time-favorite ads appeared in S&V in late 1989 and is shown in Figure 1. The headline is the grabber and certainly suggests that it would be a good idea to rid your life of aliases. However, inspection of the fine print will show that the ad did not tell us what aliases were or why we should care. The Precision Filters (PFI) folks (and I) thought it was obvious. We were wrong!

I started teaching digital data acquisition short courses about the same time. My main emphasis was the same as the ad: It is critical to reduce aliasing errors to an acceptable level. There were a variety of tools available (including the PFI devices advertised), and I naively thought that system developers, vendors, and users would employ these tools (and others developed since) by default. The problem would be solved once and for all, and I could remove that section from my courses. Wrong again!

The fact is, there are still a large number of data acquisition vendors in the dynamic measurement business selling systems that don't provide adequate alias protection. Perhaps worse, there are far too many instances where the results from good systems are compromised by improper setup because the users do not understand the consequences of inadequate alias protection. So I will, once again, try to put myself out of business.

So, What is Aliasing? In the 1940s, a bunch of smart guys were working at Bell Labs developing the technology that is the basis of today's telephones. One of the products of their studies was the fundamental rule of discrete data collection: Shannon's Theorem[1] and paraphrased here from Wikipedia:[2]

> *Exact reconstruction of a continuous-time, base-band signal from its samples is possible if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.*

This is an amazing insight. It says that we know everything there is to know about the signal if we just sample at twice the bandwidth of the signal. Simple, straightforward, great, except it is impossible to satisfy. No real signal is truly band limited, so there will always be signal components whose frequencies are higher than the Nyquist frequency (one half of the sample rate *S*). Shannon's theorem is always violated, and digitizing our data will always result in errors. These errors are called aliasing.

Reference 3 includes an extensive discussion of the aliasing phenomenon. For the purposes of this discussion, it is adequate to recognize one of the "features" of Shannon's theorem: We can only "see" data that have frequencies between zero and

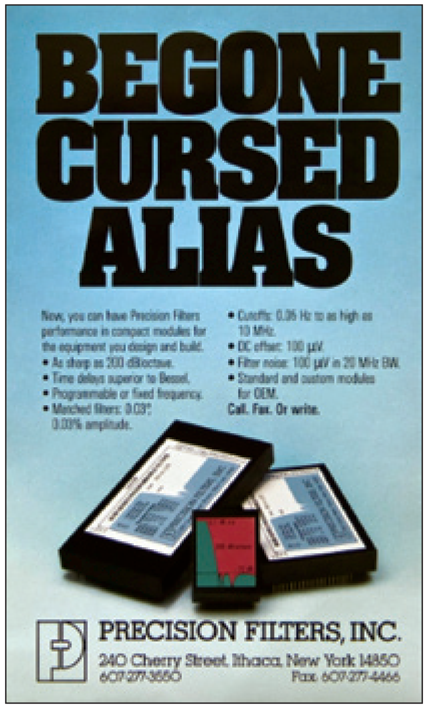

Figure 1. Begone Cursed Alias ad from 1989 S&V Magazine.

the Nyquist frequency (termed the "base-band" frequency range). So, what happens to the energy at higher frequencies? It has to go somewhere. What happens is that all of the energy above the Nyquist frequency piles itself on top of the (correct) energy in the base band. This "pile" of energy is an error. If there is significant energy above S/2, it will significantly corrupt the data in the base-band measurement range.

Therefore, our objective is to be sure that energy above the Nyquist frequency (out of band) is small enough so that it does not cause unacceptable errors in our measurement.

This is really a Catch 22. We can only see data up to the Nyquist frequency. So, how can we tell whether the out-of-band energy is adequately small? Unless we do a separate experiment with a wider bandwidth, we can't tell.

So there is really only one solution: Guarantee yourself that energy above the Nyquist frequency is adequately suppressed. We will come back to this after we look at what happens if we don't pay attention to the rules.

Significantly Aliased Data. Figure 2 shows the full time history and Fourier spectrum of a shock test acquired at one million samples/second. The spectrum has several notable features:

- It rolls off significantly at high frequency and is very small at the Nyquist frequency (500 kHz.) (More about this later).
- There is lots of energy between 10 and 30 kHz.
- The resonance of the accelerometer can be seen near 90 kHz.

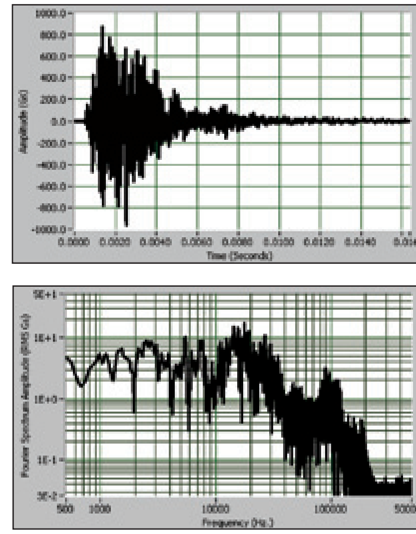

Figure 2. Data acquired at 1 MS/S.

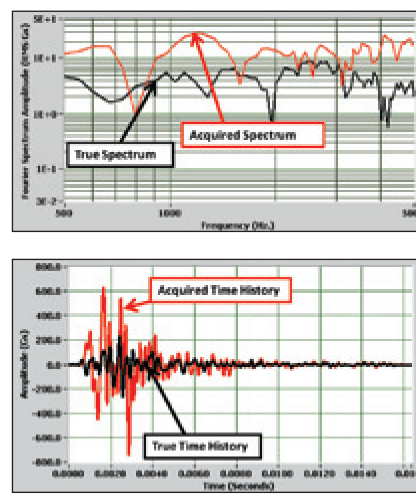

Figure 3. Data acquired at 10 KS/S.

What if we were only interested in data up to a frequency of 4 kHz? If we follow Shannon's theorem (but incorrectly use the desired bandwidth instead of real frequency range), we might sample at 10 kSamples/sec. If we don't do any low-pass filtering, the result is shown in Figure 3.

- In the spectrum, the acquired spectrum is much higher (for most frequencies) than the truth. The RMS value of the acquired signal is a factor of 2.6 higher than the truth. The energy does not roll off significantly at the Nyquist frequency.
- In the time history, the response is very high and is not well correlated with the true history.

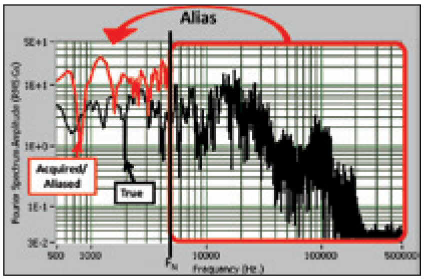

Figure 4 Signal aliasing.

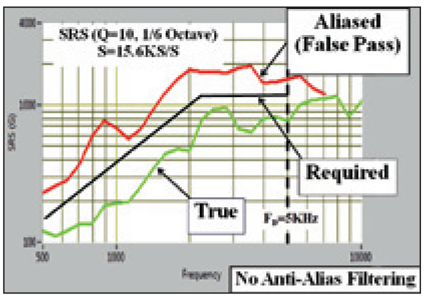

Figure 5 Acceptance false pass due to aliasing.

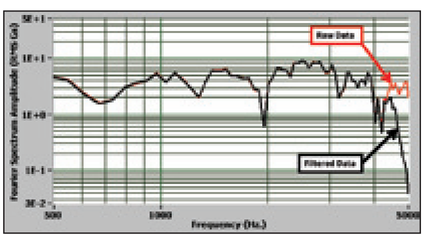

Figure 6. Evidence of low-pass filter protection.

Note that the erroneous time histories and spectra don't look unreasonable. There is no clue yet that the data set is corrupt. The errors are due to the energy above the Nyquist frequency superimposing (aliasing) itself on the baseband energy as shown in Figure 4.

Figure 5 shows a rough emulation of a recent testing disaster that was caused by aliasing. The lab was using a high-speed data acquisition system that, in a single-channel mode, was capable of acquiring the data from a typical test without filters (similar to the data shown in Figure 2). However, for the "problem" tests, the system was used in a multi-channel mode (sharing the sample rate between several channels), and the bandwidth of the signal was also increased. The result was that the data were seriously aliased. The figure shows the effect on the Figure 2 data when the acquisition rate is 15.6 kSamples/sec and is analyzed as a shock response spectrum (SRS). The aliased data falsely indicated that the test was passed (response exceeded the requirement) when, in fact, it was not.

Can We Tell Whether a Data Set is Significantly Aliased? What are the clues that the data shown in Figure 3 was aliased? The time history and spectrum look reasonable at first glance.

There is only one alias criteria I know of, and it does not work all of the time. If the

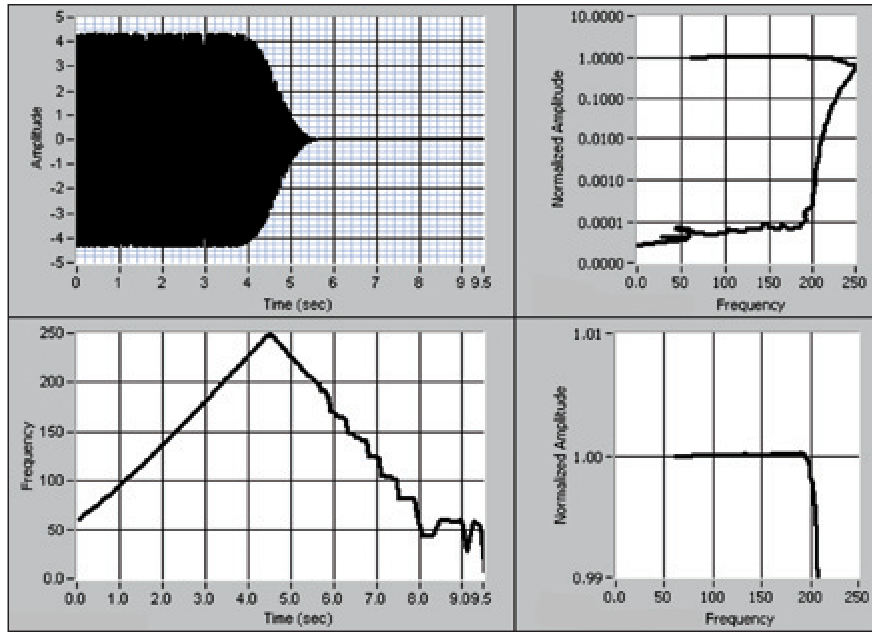

Figure 7. Sine-sweep system characterization.

Fourier spectrum is significantly attenuated at the Nyquist frequency, the data are probably OK. For example, we should insert a sharp, *analog,* low-pass filter at 45% of the sample rate in the signal path before digitizing. Then the spectrum of the Figure 3 data would look like Figure 6, and the resulting data set would be acceptable. The assumption is that if the spectrum is adequately attenuated at the Nyquist frequency, it will not come back up at higher frequency. Unfortunately, there are instrumentation errors that can produce high-frequency signals,[3,4] so this is not a 100% test. It is necessary but not sufficient.

This criterion can also produce a case of false rejection. Some of the modern sigma-delta systems have a filter whose cutoff frequency is at the Nyquist frequency. In this case, the data are OK when the spectrum is not rolled off at $S/2$, but the validity of the system (and the data produced) must be proven by the method to be discussed.

So there are two ways to satisfy the Nyquist frequency attenuation criterion: Sample really fast (as shown in Figure 2) or use a significant low-pass filter with a cutoff at or below the Nyquist frequency (as shown in Figure 6).

The only safe method is to use a sharp low-pass filter, because we cannot be certain what the data bandwidth will be.

Is a Candidate System (or the one I already have) Adequately Protected? We should look for critical specifications in the system technical description:

- Search for the word "filter." If it's not there, the system is probably not an acceptable choice. At best, the system must be proven by the test to be described.
- If they have filtering, look for one or more of the following phrases:
  - 8-pole Butterworth or Bessel low-pass filter. (Butterworth is preferred for alias protection, but Bessel is OK when used properly. Fewer poles can be used but high sample rates are required to provide adequate alias protection. In addition, analytical post-process filtering is necessary to remove distorted/aliased energy in frequencies above the cutoff.)
  - Oversampling digital filter following an appropriate analog filter.
  - Sigma-delta (or delta-sigma) filtering following an appropriate analog filter.

Specifying filtering is only the first step. The specification has to be verified, characterized and proven. This can only be accomplished with a test. An easy experiment to perform is a sine sweep. The following describes a process that uses an analysis program that can be obtained (free) from the author:

1. Set up the data acquisition system with the highest sample rate you expect to use.
2. If the filter is adjustable, set its cutoff to an appropriate value (probably less than one-half of the sample rate).
3. Perform a linear-frequency sine sweep from $S/10$ to $2 \times S$. Make an ASCII file and load it into the program. A good result is shown in Figure 7. The plots show:
   a. Upper left: time history.
   b. Lower left: frequency calculated from time history. It goes up (as expected) to a point (the Nyquist frequency), and then goes down (aliased). It turns to trash when the sine signal gets too small.
   c. Upper right: cross plot of magnitude vs. frequency with wide dynamic range. You want the fold-back energy to be small. In this case, it is 80 dB below 200 Hz (nominal system bandwidth

for this sample rate.)

   d. Lower right: vertically-expanded cross plot. You want it to be close to 1 (or whatever the gain is) to the system bandwidth.

   This test shows that the system will provide data that is relatively undistorted and alias protected up to 200 Hz (40% of the sample rate).

4. Repeat the test with the lowest sample rate/filter setting that you intend to use. (Some oversampling/sigma-delta systems use a different filter algorithm for low sample rates).

Satisfactory performance on this test does not necessarily mean the system is good in all aspects. However, poor performance means that data from the system are highly suspect and that they must be enhanced by the addition of adequate low-pass filtering functions.

That still does not get us completely off the hook. Many good systems with programmable filters allow the cutoff frequencies and characteristics to be set inappropriately, or worse – disabled. Improper filter setup is as bad (or worse) than no filters at all.

Conclusions. As noted in the introduction, methods to reduce aliasing to acceptable levels in shock and vibration tests have been available since the 1980s. Despite this, severely aliased data still surfaces far too often. Why?

- Systems that are marketed to the dynamic measurements community do not have adequate alias protection. *Caveat emptor*!
- Test lab personnel don't understand the aliasing phenomenon. If you don't understand the problem, it is far more comfortable to ignore it.
- It is not obvious that aliased data are corrupt. Aliased data look real.

The only reliable strategy for a digital data acquisition system is to use a "significant" low-pass-filter, alias-protection strategy. If you don't, Murphy's law decrees that sooner or later you will get burned.[5]

References

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3. Smith, Strether, "Chapter 13: Shock and Vibration Data Acquisition," *Harris' Shock and Vibration Handbook*, pp 13.1-13.16, Sixth Edition, McGraw-Hill, New York, 2010.
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5. Smith, Strether, "The DA Time Bomb," *Sensors Magazine,* Special Issue on Data Acquisition, 1997.

The author can be contacted at: strether.smith@comcast.net.