Using the IMAT Toolbox in the Modal Test Process

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Planning and executing a modal test involves many steps in many different disciplines in the process of selecting accelerometer locations, acquiring test data, and processing it to produce data necessary for characterizing and understanding the dynamic behavior of a structure under consideration. Many software packages are available to assist the engineer through the various steps of this process. ATA has developed IMAT¹, a MATLAB[®]-based toolbox that contains extensive capabilities for all phases of the modal test process. IMAT was born about 16 years ago out of a desire to experiment with data processing techniques on test data in a rapid and flexible manner with minimal information loss round-tripping to other software. It has since expanded greatly in capability to bridge the test and analysis worlds. IMAT gathers the many collective years of experience ATA has gained as the leading independent modal test company in the world. Much of its functionality was first developed in house for use in our consulting business and then incorporated as standard capabilities in the IMAT software. In this article we will step through the process of designing and performing a modal test using the IMAT software throughout.

A modal test is typically performed on a test article to characterize its dynamic behavior. This usually encompasses extracting a set of linearly independent mode shapes. On a complex test article such as an aircraft or satellite, a finite-element model (FEM) is often used to design the modal test to ensure that the test goals are successfully achieved. The results of the modal test are typically used to correlate the FEM for further analysis such as aerodynamic stability in the case of an aircraft or coupled loads in the case of a spacecraft.

Much is involved in designing a successful modal test. We must first decide what modes are important to characterize. Next, we must select a set of accelerometer degrees of freedom (DOF) that accurately capture these modes as a set of linearly independent mode shapes. During the test, we must excite the structure to measure frequency response functions (FRF) from which the modal parameters (natural frequencies, damping, mode shapes, and scaling) can be estimated. Finally, we must verify the quality of the results.

From here, the test-extracted data are typically compared to the original finite-element model so that the FEM can be correlated to better match the test data. Typically, the correlation process involves some amount of remodeling and tuning material moduli, bending stiffnesses, and spring properties to better match test data. ATA has a software product called Attune² that facilitates this process; Attune is not a part of IMAT and a discussion of its capabilities is not included in this article.

To illustrate the modal test process, we will use ATA's iron bird test article. The iron bird was fabricated by ATA as an internal development and training tool that simulates the dynamics and form factor of a fighter jet. A photograph of the test article undergoing a modal test is shown in Figure 1.

IMAT Overview

Designed for test and analysis engineers, IMAT provides a framework for easily importing finite-element (FE) simulation data and measured test data into the MATLAB environment to leverage MATLAB's versatile programming, visualization, and mathematical strengths. Test and simulation data such as mode shapes, time histories, spectra, and other functions can be viewed and manipulated in MATLAB while preserving all of the native data attributes.

IMAT facilitates the manipulation of test and simulation data in MATLAB by defining several data types. IMAT defines MATLAB classes



Figure 1. Instrumented iron bird test article.

for XY functions (*imat_fn*), mode shapes (*imat_shp*), coordinate traces or degree of freedom lists (*imat_ctrace*), results such as stress (*imat_result*), FEM entities (*imat_fem*) such as nodes (*imat_node*), elements (*imat_elem*), and coordinate systems (*imat_cs*), groups (*imat_group*), and several others. These custom data types allow IMAT to seamlessly store and track descriptive attributes along with the data.

These attributes include function type, abscissa and ordinate data types and qualifiers, function identification descriptions, and many more. IMAT also includes standard methods such as multiplication, addition, and plotting that understand and correctly handle the attributes, allowing the user to operate on these data types the same way they would on standard MATLAB data types. These data types interact with each other. In addition, MATLAB's object-oriented capabilities allow the user to extend IMAT's data types to easily add attributes and capabilities; users can then easily extend the already significant capabilities IMAT has to offer, customizing IMAT to suit their needs.

IMAT is subdivided into several components, each of which contains the functionality related to a specific subset of tasks. Core IMAT contains data type definitions and import/export capabilities between the universal files and binary associated data files (ADF) used by I-deas Test and NX Response Simulation.

IMAT+Modal gathers the capabilities needed by the modal test engineer before, during, and after a modal test. The genetic algorithm (GA) for accelerometer selection selects optimal accelerometer locations for a modal vibration test. The Test-Analysis Model Toolkit (TAMKIT) provides Nastran-based procedures for selecting the instrumented DOF and for reducing the FEM matrices to these DOF, as well as methods to back-expand test shapes to the full FEM or an intermediate display set for visualization. The Modal Test Toolkit (MTK) contains routines that are useful when performing modal survey tests, such as methods for placing sensors and exciters, extracting modes from test data, verifying shape extractions through FRF synthesis comparisons, providing shape independence and completeness checks, and other useful routines. AFPoly[™], the patent-pending alias-free polyreference modal parameter estimation algorithm, is a dramatic improvement over the classic polyreference technique.

IMAT+Signal provides the user with many capabilities for signal processing, which in general encompasses processing time-domain data into frequency-domain functions. In addition to command-line functions for tasks such as computing PSD and CSD from time histories, computing FRF, and time-domain filtering, IMAT+Signal also provides two graphical user interfaces (GUIs) that greatly simplify the process. The spFRF application provides a fully functional GUI for converting time histories to FRF and other frequency-domain functions, while RTK provides a GUI that facilitates analysis and order tracking of rotational events using the Vold-Kalman filtering technique.^{3,4,5} Recent computational improvements to the Vold-Kalman filter have been incorporated into the software.^{6,7}

IMAT+FEA extends IMAT's capabilities into the simulation realm for the analysis engineer. It provides the ability to interface with Nastran, Abaqus, and Femap, in addition to I-deas. The imported data is placed into the IMAT data types, so it is easy to use in MATLAB, and all of the existing IMAT functionality is readily available. FEMs and associated results can also be displayed using the Visualization Toolkit,⁸ which can display very large models with ease.

The capabilities found in IMAT and its toolboxes greatly enhance the test and analysis user's effectiveness in performing modal tests. We will see how it is used in the test process on the iron bird test article.

Pretest Accelerometer Selection

The first step in modal test planning is to select a set of target modes to which the FEM will be correlated. Standard analysis software outputs such as effective mass or natural frequencies are generally used to select target modes so that the final, correlated model accurately represents the response in the desired frequency range.

With a defined set of target modes, the next step is to select a set of accelerometers that will result in linearly independent mode shapes when viewing only the response of the instrumented locations. Selecting robust instrument locations poses several challenges to the test planner: a typical FEM includes many hundreds of thousands or even millions of DOF. But a modal test has only up to a few hundred DOF that must represent the dynamic behavior of the entire structure and allow differentiation of the target modes.

Complicating this approach even further, the selected accelerometer locations must be robust enough to capture the relevant dynamic behavior of a test article, which may have significantly different dynamic characteristics from the FEM. What's more, the industry-standard metric for computing linear independence – the pseudo-orthogonality matrix – requires that the test DOF account for the distribution of mass of the entire FEM. In many cases, multiple test article configurations (such as aircraft fuel states or store configurations) must be tested, so using a common instrumentation set for all configurations would be advantageous.

For a structure with well-understood dynamic behavior such as an aircraft, experienced test or analysis engineers may be able to intuitively select appropriate accelerometer locations based on past experience. The relevant modes of an aircraft will invariably be the bending and torsional modes of the lifting surfaces and the fuselage. With more complex or modally dense structures such as satellites, a manual process is not sufficient. ATA has encountered many complex structures where manual or even sophisticated deterministic methods of accelerometer placement have not achieved adequate results in terms of linear independence or the necessary accelerometer count. ATA has pioneered the development of various automated methods to help select accelerometer locations both to improve the pseudo-orthogonality metric and to reduce the number of accelerometers. These methods have been captured in IMAT+Modal.

ATA's standard approach for accelerometer selection has adopted functionality in TAMKIT and GA, two of the components of IMAT+Modal. First, TAMKIT routines are used to reduce the full-size FE model down to a candidate set of accelerometers locations – typically 1000 to 2000 DOF. This candidate set must be an excellent representation of the full FEM modes and frequencies, because any further reduction will not be as good as the initial candidate set. Then GA is used to select the accelerometer locations from the candidate set. The intermediate candidate set is necessary to ensure that the randomness of GA is only selecting from a pool of "good" accelerometer locations, which increases the efficiency of the process.

TAMKIT includes Nastran-based solution routines that imple-

ment deterministic methods such as effective independence,⁹ iterative Guyan, and kinetic-energy-based methods to select accelerometer locations that either maximize linear independence or instrument the active mass in the target modes. It also includes Nastran-based matrix-reduction routines to perform test-analysis model (TAM) reductions using modal,¹⁰ hybrid,¹¹ or improved reduced system (IRS)¹² methodologies to supplement the Guyan (static) reduction available in Nastran by default.

GA¹ is a GUI that uses the principles of survival of the fittest to select optimal accelerometer locations. The iterative process replicates the evolution of a population through cloning, crossbreeding, and mutation to find the optimal set of accelerometers that capture the dynamic behavior of the target modes. Many accelerometer sets are evaluated and ranked according to their fitness using the pseudo-orthogonality metric. The reproductive behavior of an entire population is mimicked using a randomized process to preferentially select those sets with the highest fitness to form the next generation.

As generations progress, the fitness of the entire population tends to improve, and since the initial population includes accelerometer sets selected using several deterministic methods such as those included in TAMKIT, the final result is guaranteed to be as good as or better than any of the individual deterministic methods that seeded the initial generation. After a specified number of generations, the single best accelerometer set is retained for additional scrutiny using one of several result-plotting capabilities from the GA GUI. Randomness built into the process ensures that the final GA accelerometer set has cast a wide net over all available accelerometer locations to ensure that the evolution path is not biased by a local minimum. It also means that two identical GA runs may produce slightly different accelerometer sets, although many of the critical locations will invariably be selected by the best set of every GA simulation.

ATA has found many benefits of the GA approach over deterministic accelerometer selection methods. The GA-based approach to accelerometer selection can drastically reduce the accelerometer count of a test, minimizing the test equipment and duration of the test setup and associated costs. With deterministic methods, the rule of thumb for accelerometer count is that eight to 10 accelerometers are typically required per target mode to achieve good orthogonality, while experience has shown that four to five accelerometers per mode is usually sufficient when using GA. GA can also simultaneously select accelerometer locations for multi-configuration tests, minimizing accelerometer count and setup time by selecting locations that are appropriate for the target modes across all tested configurations. It can also optimally select triaxial accelerometer locations, accounting for the benefit of all three DOF in the selection process.

The user may also enforce accelerometer placement at specific locations if a particular frequency response function is desired, and GA will account for the benefit of the manually selected location in determining the fitness of each set. More recently, a manual accelerometer placement form was added so that users can interactively modify the final GA-selected accelerometer set and quickly run trade studies to see the influence of moving, adding, or removing accelerometers.

We will use the iron bird test article to demonstrate the accelerometer selection process used by ATA. For this example, we assume that the iron bird will be tested in two configurations – with and without wings stores (see Figure 2) – so we can select a common accelerometer set suitable for both. While the modes of these two configurations are likely very similar, this approach is equally applicable for a structure with very configuration-dependent mode shapes, such as a stowed and deployed solar array on a satellite.

The first step in the accelerometer selection process is to use a TAMKIT routine – iterative residual kinetic energy – to select a candidate set of 150 DOF from each 40,000 DOF FEM that reflects the dynamic response of each FEM up to 30 Hz. This routine starts with a small user-defined DOF set and iteratively adds DOF to minimize the residual, or excluded, kinetic energy. The candidate set of 150 DOF was confirmed to be sufficient to represent the target modes of each configuration using another TAMKIT routine, *ortho*, which



Figure 2. Iron bird FEM with and without loaded wings.



Figure 3. Final GA form for multiconfiguration accelerometer placement for iron bird.

computes the full FEM-to-candidate set pseudo-orthogonality. The bare and loaded iron bird FE models have 16 and 20 modes below 30 Hz, respectively.

Next, the characteristic matrices of the two reduced models, which can be exported directly from Nastran using standard Nastran commands, are loaded into GA. The target modes and total number of accelerometers are specified in the GUI. For the setup shown in Figure 3, a total of 40 accelerometers were selected across the two configurations to enable a single set of accelerometers to capture the target modes of both configurations. After solving for 100 generations, the cost (error) of the pseudo-orthogonality metric over both configurations was reduced by more than 50% from the starting point (blue curve). GA reduces the entire pseudoorthogonality matrix into a single value for set-to-set comparison, and the minimum cost from each generation is plotted on the main GA form. The pseudo-orthogonality and frequency errors can be viewed directly from the GUI window to assess the effectiveness of the current accelerometer set (top panel, Figure 4). A small sample of some other outputs available directly from the GA GUI is shown in Figure 4.

Since the pseudo-orthogonality is close to identity and the frequency error is mostly less than 1%, this accelerometer set could be retained for use in test. If these results were not sufficient, then additional generations could be simulated to allow the set to continue to evolve. To document the current set and the error metrics, a Microsoft Excel XML report may be exported from the GA form for each FE model configuration; portions of this report are shown in Figure 5. For this simple example, 36 target modes were accurately captured with only 40 accelerometers. Even taking into account that many of the two FEMs' modes are similar in shape, the GA accelerometer set was able to replicate, at minimum, 20 distinct modes with 40 accelerometers, a ratio of two accelerometers per mode.

ATA has found that before finalizing the accelerometer locations, it is often desirable to modify some of them to make the locations more symmetric, simplify the installation, or make changes for other reasons. The *Edit Accelerometer Selection* form, shown in Figure 6, can be opened from the main GUI to facilitate these types of modifications. This form shows the relative importance of each selected accelerometer and allows quick trade studies with full access to the entire suite of error-metric plotting. The final, manually modified accelerometer set or an XML report can also be exported directly from this form. This form can also be used to manually select the entire accelerometer set from scratch, provid-





Figure 4a-c. Interactive outputs available from main GA form for each FE model configuration: a) pseudo-orthogonality matrix; b) frequency error of accelerometer set model; c) interactive overlay of accelerometer locations on FEM.

ing a convenient means to interactively check the orthogonality and frequency error metrics without using the genetic algorithm functionality.

Modal Test Setup and Execution

Once pretest sensor selection has been completed, the next step in the modal testing process is to determine exciter locations and

	DOF	Rank	
1	7622Y+	21.370	
2	7616X+	11.285	
3	7622Z+	8.513	
4	2366Y+	7.672	
5	2162Y+	7.668	
6	7630Y+	6.217	
7	7629Y+	6.156	
8	12151Y+	2.639	_
9	12127X+	2.552	
10	11607Y+	2.152	
11	12149X+	1.963	
12	11120Y+	1,958	
13	12275Y+	1.870	
14	11907Y+	1.853	
15	7118X+	1.656	
16	7431X+	1.644	
17	7629X+	1.457	-

orientations that are best able to excite all of the target modes of the structure. For most aircraft, the optimal shaker locations are usually determined by inspection and are located at the ends of the wings and tails and possibly each engine. For other structures such as satellites, locating exciters may not be as straightforward, and using an automated routine is helpful. To address this need, ATA developed an automated shaker selection algorithm that is based on using the normal mode indicator function (NMIF) as a figure of merit.14,15

Figure 4d. Relative ranking of importance of each accelerometer.

An example NMIF plot is shown in Figure 7. At resonance, the NMIF dips toward zero, and the closer the dip is to zero, the more observable that mode will be. Given an initial set of candidate exciter locations and orientations, the function will output an optimal set of shaker locations and orientations that generates an NMIF value below a user-defined level for every target mode. Also, by comparing the operating deflection shape to the mode shape, repeated or closely spaced modes are accounted for. The algorithm undergoes several iterations of potential shaker combinations until a minimal number of shakers can achieve the target goal. The algorithm was rapidly developed using IMAT and is now part of IMAT+Modal in the function called *shaker_locator*.

The next step in the modal test process is to excite the structure at the input locations and measure time histories of the inputs and the responses. After the time responses are reviewed for data quality, they are transformed into the frequency domain and evaluated once again.

Traditionally, either modal shakers or modal impact hammers are used to excite the structure. Oftentimes, for a given structure, a combination of shaker and impact excitation is appropriate – modal shakers are used to characterize the overall structure, and impact methods are used to identify modes that were not observable from the shaker excitation. ATA has also pioneered different excitation techniques including the patented multi-sine¹⁶ excitation and simultaneous impact testing methods.¹⁷ These methods were rapidly developed and tested using IMAT.

One of the original reasons for creating IMAT was to be able to process test data from I-deas in the MATLAB environment with minimal loss of information in the translation. However, IMAT software is independent of the data acquisition platform. Test data can be collected using any data acquisition system that supports a data file format readable by IMAT, such as universal files or ADF files.

While IMAT's functions are available to the user from the MATLAB command line or in user-written programs, a few full-featured GUI applications have been built on IMAT's functionality to create a convenient and efficient environment for performing many of the data processing and plotting tasks typically required when conducting a modal test. One of these IMAT applications is spFRF, which stands for signal processing for frequency response functions. Its primary purpose is to process time-response data into spectral functions such as frequency response, as well as coherence, auto spectra, cross spectra, and principal components.

IMAT function calls are used to read time-response functions from ADF files and universal files and to write time-response and spectral functions to the same. The spFRF application can also read from and write to an *imat_fn* object in the MATLAB workspace. This allows the MATLAB workspace to be used as a temporary storage location for data functions that are accessible to the user from the MATLAB command line and can be manipulated with the available IMAT functions.

The spFRF Setup module is shown in Figure 8 displaying the excitation force channel and one acceleration channel from a sinesweep test run. The time responses are plotted in the upper axis,

	FBM Mode Freq (H		łz)	TAM Mode				Freq (Hz)			%Difference			PORTHO			XORTHO					
		1 1.228				1				1.22	28	0.0			99.99			100.00				
	2					1.341		2			1.341			0.0			99.92			99.96		
		3 1.47			1.473		3			1.473			0.0			99.98			99.99			
	4					3.065				4	3.066			0.0			99.86			99.93		
	5					3.674				5	3.677		7	0.1		99.73			99.86			
	6				4.618			6			4.64	13			0.5		97.87		98.9		8.92	
	7				6.131				7		6.14	13			0.2		99	.20		99	9.60	
	8				1	9.086	;			8	9.128		28	0.5			98.16			99.07		
	9				1:	2.886					13.024			1.1				95.78		97.86		
	10			1:	3.022		10			13.142			0.9			96.39		98.16				
	11			1-	4.525		11			14.586			0.4				98.34		99.16			
	12			1	5.948		1:			16.041			0.6				97.69		98.83			
	13			2	0.337		1				20.62	8	1.4				94.45		97.04			
	14			2	0.544			1	4	20.829		9	1.4		94.62		.62	97.12				
	15			2	8.083			1	5		28.42	4	1		1.2		94.98		96.94		6.94	
	16			2	9.093				6	29.511		1	1.4		1.4	94.37		.37	96.44			
	17			34	4.106			1	7	34.352		2	0.7		0.7		96.95		98.40		3.40	
	18			43	3.032				9	52.073		3	21.0		1.0	39.60		.60	59.29			
	19			43	3.210			20		52.897		7	22.4		2.4	38.56		.56	60.99			
			20		43	3.248			1	8		47.53	5		3	9.9		16	.43		24	4.39
	FEM Ps				thogona	lity Tabl	le .				-			-	-							
Н	_	00	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	PUgg 1.2	1.2	0.00	1.5	3.1	3.7	4.6	6.1	9.1	12.9	13.0	14.5	15.9	20.3	20.5	28.1	29.1	0.01	43.0	43.2	43.2
Н	2	1.3		1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	-0.02	0.00	-0.01
	4	3.1			1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	0.00	0.00	-0.01	0.00	-0.01	0.01	-0.04
Н	5	3.7					1.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00
	7	6.1						0.96	0.99	0.00	0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	0.00	0.03	0.00
Н	8	9.1								0.98	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	-0.02	0.09
	10	13.0								0.01	0.90	0.96	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.01	-0.03
Н	11	14.5											0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	-0.01
E	12	20.3						-0.01						0.98	0.94		0.00	0.00	0.00	0.00	0.00	0.04
F	14	20.5							_	_		_	_			0.95	0.05	0.03	0.00	0.01	0.00	0.02
E	15	28.1		0.01													0.95	0.02	0.00	-0.03	0.01	-0.04
	17	34.1	0.01	0.00				-0.02											0.97	0.0	-0.05	-0.03
	18	43.0	0.01	-0.02		-0.01	0.02	0.03		-0.02			-0.02					-0.03	-0.05	0.40	0.02	-0.07
	20	43.2	0.02			-0.04				0.09		-0.03		0.04		0.02		-0.04	-0.03	-0.07	0.05	0.16

Figure 5. Sample Excel XML report from GA.



Figure 6. Manual accelerometer selection form allows interactive modification of accelerometer set with full access to error metrics.



Figure 7. Valleys in the normal mode indicator function indicate mode frequencies.

and the auto spectra for one frame are plotted in the lower axis.



Figure 8. The spFRF setup module displaying excitation force channel and one acceleration channel from a sine-sweep test run.

The user selects the channels and time range to be processed and specifies the signal processing parameters, such as the block size, window, overlap, and triggering method. The spFRF application can process continuous measurements with either the Welch or Daniell spectral estimation method,¹⁸ as well as triggered measurements with the ability to preview and accept or reject each frame. Rather than processing the time histories in the data acquisition software and discarding them after transforming the data to the frequency domain, it is useful to store the time histories and then process them in spFRF, since the data can then be reprocessed to evaluate windowing, frequency spacing, or overlapping, for example.

Processing the time data in spFRF also allows some additional flexibility in setting up and running a modal test. For example, to process data from an impact test run in spFRF, the acquired time signals need not be triggered on the hammer channel. Rather, the time signals can be acquired in the free run mode with a sufficient interval between the impacts, and a software trigger can find the measurement frames. Also, the hammer could be connected to any channel in the data acquisition system, which makes the channel setup easier for a test on a large structure.

In addition to the signal processing of the time signals to spectral functions, some of the other IMAT+Signal functions are available from the spFRF GUI, such as filtering, decimation, and DC bias removal. These types of auxiliary tools are implemented by using the command-driven nature of the IMAT applications; that is, almost all of the functionality of the GUI can also be accomplished as a MATLAB command. There are also a number of query commands for retrieving data and information from spFRF. These design features make the IMAT applications user extensible. Users can create their own tools for custom-formatted plots or standardized setup templates, for example, which are accessible directly from the application menu bar. This makes it easy for users to incorporate their own custom processing methods in the standardized tool, taking advantage of other existing IMAT and MATLAB capabilities in spFRF.

After the signal processing is completed, the generated spectral functions are reviewed and saved in the results module. The results can also be sent to spVIEW, which is another IMAT application for plotting multiple datasets of time- and frequency-domain measurements and mode indicator functions (MIF). The spVIEW MIFs module is shown in Figure 9 displaying the power spectral mode indictor functions (PSMIF) from three test runs at different force levels.

The typical data flow for a modal test is depicted in Figure 10. For a large-scale modal test (such as an aircraft with several



Figure 9. spVIEW MIFs module displaying PSMIF from three test runs at different force levels.



Figure 10. Typical data flow for modal test.

hundred accelerometers and four or more shakers), several test personnel would be engaged in the overall process – with one doing the data acquisition, another doing the data review and signal processing, and one or more others doing the parameter estimation and verification. In this scenario, each of the blocks in Figure 10 represents a separate computer and test engineer, and the data files are transferred between the computers to complete the stages.

The output of the data acquisition stage of the process is a data file containing the recorded time histories. This file is read into the spVIEW time response module in which the time signals are reviewed for data quality. If deemed satisfactory, these timeresponse functions are sent to spFRF and processed into spectral functions, which are reviewed and saved to a file. The spectral functions are also sent to the spVIEW spectra module, where they are accumulated across numerous test runs and further reviewed. Mode indicator functions can also be computed from the FRF and sent to the spVIEW MIFs module for comparison to other test runs, as shown in Figure 9. The FRFs generated from spFRF are the input to the modal parameter estimation and verification stage of the process.

Modal Parameter Estimation and Verification

Once FRFs have been calculated, modal parameters can be estimated. In IMAT, modal parameter estimation is performed in AFPoly, which is another IMAT application. AFPoly stands for "alias-free polyreference" and refers to a patent-pending, multiple reference modal parameter estimation technique developed by Vold.¹⁹ It is a Laplace-domain rational fraction polynomial model that is formulated in an orthogonal polynomial basis. By formulating in the continuous frequency (i.e., Laplace) domain, the solution is not contaminated by the residual effects of out-of-band modes being aliased into the analysis frequency band. The implementation also uses a generalized orthogonal polynomial companion matrix, which gives this algorithm numerical stability and accurate results over a wide frequency range containing many modes. The benefit of this is that the entire frequency range can be processed



Figure 11. The AFPoly poles module displaying stability diagram with autonomously selected poles at top as gold stars.



Figure 12. Four-view mode shape plot of iron bird test mode shape.

in one pass for all of the modes instead of being partitioned into a number of narrow bands.

AFPoly guides the user through the process to identify and compute the modal parameters (natural frequency, damping, and mode shapes) in a step-by-step manner. In the setup module, FRFs are read from an ADF or universal file or an *imat_fn* in the MATLAB workspace. Coordinate traces can also be used to quickly sieve a few hundred responses down to the X-direction DOFs on the wings to focus on that elusive third antisymmetric fore/aft bending mode, for example. In the poles module, the modes are selected from a stability (or consistency) diagram, as shown in Figure 11, and an autonomous pole selection option has been implemented based on the common statistical subspace autonomous mode identification (CSSAMI) method.²⁰⁻²²

This procedure evaluates the large set of poles and residue vectors produced by the modal parameter estimation algorithm to identify clusters of consistent modes. While this technique is not intended to completely eliminate all user interaction in selecting the modes, it does greatly help make the first pass in the process, which the user can then evaluate and revise.

As a companion to AFPoly, there is a single-degree-of-freedom (SDOF) modal parameter estimation IMAT application called SDOFit,²³ which will estimate the frequency and damping of modes in a narrow frequency band from a single FRF using a single-input, single-output, frequency-domain rational fraction polynomial



Figure 13. Comparison of test and analysis mode shapes using crossorthogonality calculated by IMAT's MTK module.

model with generalized residuals algorithm. The residuals are included in the model to account for the out-of-band modes. Although primarily intended as an SDOF method, this will allow more than one mode to be included in the model, since there are occasions that this approach yields better results.

The poles are selected from a stability diagram that is generated not by varying the number of modes – which would defeat the purpose of an SDOF method – but by varying the order of the residuals polynomial. The results (poles, residue vectors, and participation factors) returned from SDOFit are incorporated into the AFPoly stability diagram. SDOFit is implemented as an AFPoly tool, and the FRFs, coordinate traces, test display model, and test-analysis model are sent from AFPoly into SDOFit. SDOFit can also be invoked from the MATLAB command line, in which case an *imat_fn* of FRFs is the only required input, and an *imat_shp* object containing the poles and mode shapes is the output.

In the shapes module of AFPoly, a complex and a real residue algorithm are available, and the user can select which residual terms to include in the model. In addition, a different set of response DOF can be chosen for the residue calculation than were used for the poles solutions. In the verify module, the results are evaluated by overlaying FRFs of MIFs computed from the test FRF and FRFs or MIFs synthesized from the estimated modal parameters. The available MIFs include the power spectral (PSMIF), complex (CMIF), quadrature (QMIF, or CMIF of the imaginary part of the FRF), normal, and multivariate (MMIF) mode indicator functions, all of which are computed for both the measured and synthesized FRFs using calls to IMAT functions of the same names.

For animating mode shapes, the test display model can be imported from a universal file, a Nastran Ouput2 or bulk data file, or an *imat_fem* in the MATLAB workspace. If a test-analysis model is also imported, back-expanded mode shapes can be animated directly from AFPoly. The as-measured or back-expanded mode shapes can be written to an ADF or universal file or to an *imat_shp* in the MATLAB workspace, where they will be available for further verification with some other IMAT functions. Figure12 shows a four-view plot of the iron bird test mode shape using IMAT's *vtkplot* function.

Most modal tests result in several shape extractions from the same or different test data sets. For example, a single mode shape may be extracted several times. Oftentimes, selecting the "best set" of final reported shapes from many candidate shape extractions involves selecting a set of test modes that minimizes the off-diagonal terms of a test self-MAC or self-orthogonality matrix. IMAT's mode-sort function greatly speeds up this process by first automatically grouping similar modes together and then downselecting to a single mode for each group to produce a final set of test mode shapes.

The final step in a modal test is to mathematically verify that a final set of extracted mode shapes is unique (shape independence) and that each analysis target mode shape has been extracted (shape completion). Mode shape independence and completeness can be mathematically verified either through the modal assurance criterion (MAC) or, if a mass matrix is available, through an orthogonality calculation. IMAT's ortho function can be used to calculate all of these matrices, and a corresponding *comparison_table* function can automatically create a table based on these matrices to match

test to analysis modes. These two functions have been combined in IMAT's *report_test_xml* function, which performs all of these calculations and exports them to an XML file that can be read into Microsoft Excel for easy viewing.

Figure 13 shows the test shape to FEM shape cross-orthogonality matrix. The row corresponding to the FEM CRSS value indicates how much of a given FEM shape is captured by test shapes. In this way, the user can verify whether an analysis mode has been captured by a linear combination of test shapes.

Once the user has reached this point in the process, a final set of test mode shapes is delivered to the analyst for further use – for example, to correlate the analytical FEM to the test results.

Conclusions

The modal test process involves many steps in selecting accelerometer locations, acquiring test data, and processing it to produce data necessary for characterizing and understanding the dynamic behavior of the structure under consideration. ATA has many years of experience in all of the aspects of modal test planning, execution, and result processing and verification and has captured much of this knowledge in the IMAT toolbox.

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