Visualizing Structural Vibrations Using a Novel Strobe Light Setup

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The visualization of structural vibrations has always been a key technology when teaching vibration analysis. If the structural motion can be perceived directly without numerical simulation or extensive measurements, basic modal analysis concepts like mode shapes, natural frequencies or resonances and antiresonances are intuitively understood. The application of stroboscopes has a long tradition in vibration analysis and is particularly well established in rotor dynamics. Nevertheless an inexpensive and simple device for the visualization of larger structures like car spoilers was missing. The application of several arrays of white-light, high-power, light-emitting diodes together with an electronic power supply resulted in an innovative and highly flexible distributed light source perfectly suited to visualize resonant structural vibrations in the mid-frequency range. Furthermore, the effect of aliasing is demonstrated convincingly and the results can be compared qualitatively to experimental modal analysis. In combination with an electrodynamic shaker, the setup has been applied successfully to visualize vibrations of parts of car bodies, tennis racquets, skis, piano or cello strings, as well as plate and shell structures.

Theoretical and experimental modal analysis is a demanding and complex scientific discipline, and consequently it is important to attract a student's attention by a suitable combination of complementary teaching methods. If the confidence in numerical simulations and a virtual experiment is sufficient, almost any phenomenon might be explained and understood by applying theoretical or numerical methods only. However, since vibrations are present in everyday life, it is possible to explain many phenomena by investigating real effects on commonly used structures.

In the low-frequency range, structural vibrations might be felt directly; medium or high frequencies often radiate acoustic emissions that can be heard. Unfortunately this direct perception contributes little to a fundamental insight. If, on the other hand, it is possible to fully visualize the movement of a structure, the understanding will be increased tremendously. Furthermore, if it can be proven convincingly that theoretical predictions and real experiments do compare well, this raises confidence in a virtual experiment.

These considerations were the driving force behind the development of a proposed setup that has shown to be a versatile tool for explaining and teaching vibration phenomena. It is still widely accepted to use large and greatly simplified structures with low natural frequencies to prove that theoretical predictions do compare well with experiments. Pendulum type structures, large helical springs, standing waves on water or large strings are commonly used to explain basic principles like natural frequencies and mode shapes. Also widespread are Chladni plates to display node lines or the visual demonstration of sound wave patterns in a Kundt's pipe filled with cork or polystyrene balls. These experiments have the commonality that they are based on idealized elementary structures or substantial structural simplifications when compared to real systems, a drawback that is eliminated with the proposed application of stroboscopic light.

The basic idea is to apply several arrays of high-power, light-emitting diodes (LEDs) in a stroboscopic configuration to obtain a homogenous illumination level over the entire vibrating structure. At frequencies of more than 40 Hz, the sequence of light flashes is perceived as uniform light intensity by most observers. This assumes that, harmonic motion of a relatively flexible structure with moderate to large vibration amplitudes, an arbitrary frequency shift can be achieved by violating Shannon's sampling theorem.

Based on a paper presented at IMAC XXXI, the 31st International Modal Analysis Conference, Garden Grove, CA, February 2013.

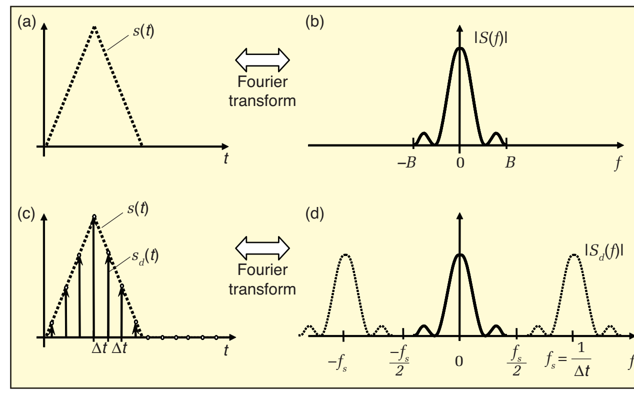

Figure 1. Continuous and impulse-sampled time signal with corresponding spectra.

Consequently, the original high-frequency motion is mirrored about the sampling frequency, rendering the visible alias frequency. Conversely, the actual mode shapes remain unaffected, and the real structure seems to vibrate in slow motion. This strategy works for any structure as long as the vibration amplitudes are in the visible range. Laboratory experiments have been carried out using many technical and common articles, and the setup has become a valuable, quick and easy system for demonstrating different vibration modes and creating a feeling for modal relations.

Theoretical Background

Whenever a continuous process is discretized by taking equally spaced samples, the effect of aliasing has to be considered. In many film productions, aliasing is the reason for car wheels turning forward and backward during an accelerated motion. In computer graphics and image processing, a similar effect results from a finite spatial resolution; often observed as appearance of a Moiré pattern. Aliasing is a frequently encountered and widely underestimated problem that can occur in all data acquisition processes whenever samples are taken from a continuous process, thereby corrupting the measured data by changing the signal or system information. In modal analysis, it can be observed as both a spatial or temporal effect. Although generally undesired, it is exactly the effect of changing the frequency content of the observed motion by aliasing that the current application is based on.

From the theory of signal analysis it is well known that any discrete time signal corresponds to a periodic spectrum, and due to the symmetry (duality) of the Fourier transform (FT), any discrete spectrum has a corresponding periodic time signal.[1] If a continuous signal $s(t)$ is discretized by impulse sampling at equally spaced time intervals Δt using the sampling frequency $f_s = 1/\Delta t$, the discrete time signal $S_d(t)$ is obtained, see Figures 1a and 1c.

Its spectrum $S_d(f) = FT\{S_d(t)\}$ is continuous with the period f_s in frequency domain and proportional to the spectrum of the original continuous time signal $S(f)$; see Figures 1b and 1d. Since the considered mechanical systems have low-pass characteristics and the excitation is limited in frequency as well, the highest signal frequency component of $s(t)$ is denoted B. If properly sampled, $S_d(f)$ contains periodic and well separated signal sections. If f_s is reduced, the periodic sections will start to overlap. Since $S_d(f)$ is the sum of all spectral images, $S(f)$ and $S_d(f)$ will start to differ in the fundamental frequency band, $-B \le f < B$ (see Figure 2a).

Because of the symmetry of real signals, $|S_d(f)| = |S_d(-f)|$, it

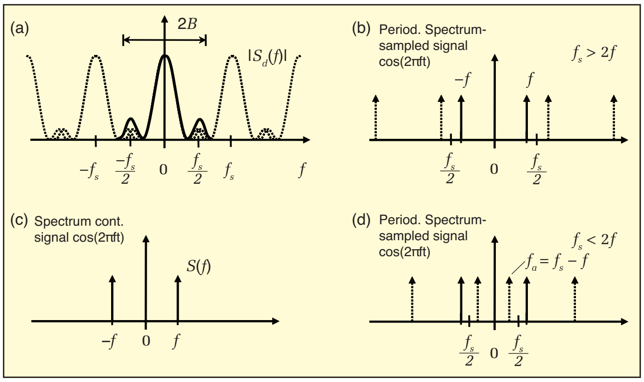

Figure 2. Spectral degeneration by frequency folding.

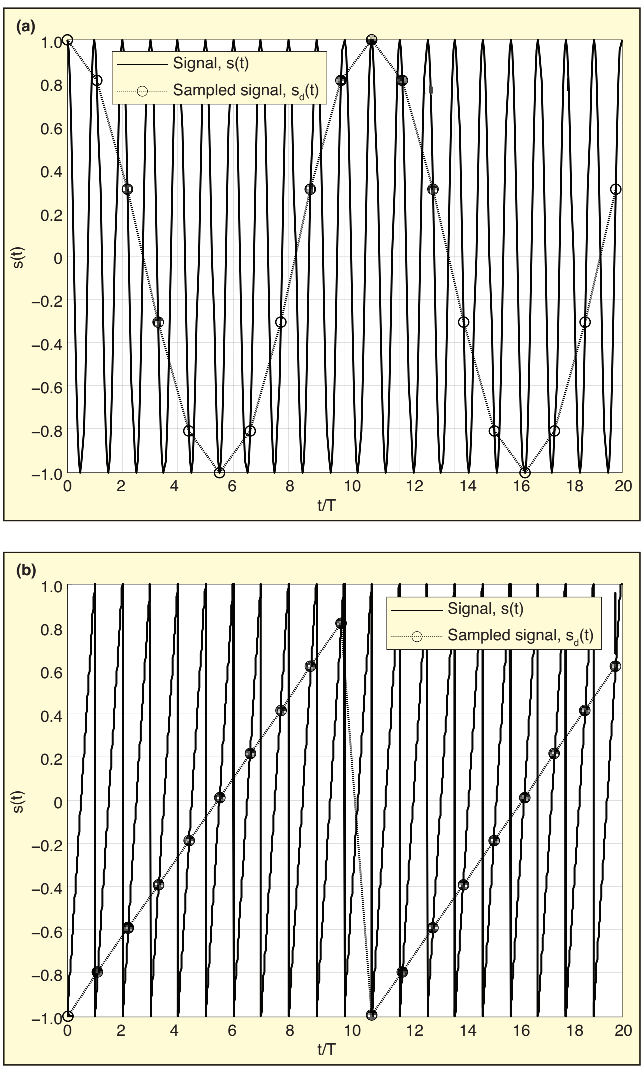

Figure 3. Aliasing effect for harmonic and periodic signal.

becomes apparent from simple geometric interpretations that the resulting spectrum seems mirrored about $f_s/2$. Consequently high-frequency components will appear in the fundamental frequency band, thereby representing alias frequencies. This spectral degeneration, also known as frequency folding, can be avoided if the condition holds.

$$f_s > 2B \tag{1}$$

Equation 1 is known as the Nyquist-Shannon sampling theorem and represents a sufficient condition for exact reconstructability of the original time continuous signal $s(t)$ from the sampled data $s_d(t)$. The Nyquist rate $2B$ is a central property of a band-limited signal, contrary to the Nyquist frequency $f_s/2$, which results from the sampling system. The effect of undersampling can be best explained by considering the single frequency signal $s(t)$=cos($2pft$). For this signal, the bandwidth is certainly limited by $B=f$, and the spectrum consists of two spectral lines at the frequencies $\pm f$. For the sampled signal this is also true as long as inequality (Eq. 1) holds. But as soon as the sampling frequency is reduced below $f_s<2B$, the true spectrum and its periodic images overlap, thereby forming a spectrum whose lowest and therefore predominant frequency is a result of the sampling process and does not exist in the original signal (see Figures 2c and 2d).

The spectrum given in the fundamental frequency band of Figure 2d corresponds to the alias signal $s_a(t)$=cos($2pf_at$), $f_a=f_s-f$. Actually only $s_a(t)$ is visible in the time domain (see Figure 3a).

Although the frequency has changed, the amplitude of $s(t)$ and $s_a(t)$ remain identical. f_a vanishes if $f_s=f/n$, and $n=1,2,3$. The condition $f_a=f_s-f=0$ is frequently used to determine the angular frequency of rotating shafts. For periodic signals with the fundamental frequency f and several frequency components, the frequency folding still appears and can be analyzed by linear superposition. However, instead of using the time scaling theorem of the Fourier transform, the effect of undersampling is directly understood by looking at the time signal (see Figure 3b). In fact any signal of period $T=1/f$ shows a scaling in the time domain and can be slowed down to almost zero if $f_s \approx f/n$. For $f_s>f/n$ the signal is running forward, but for $f_s<f/n$, it seems to be running backward in time.

Again, the amplitudes of the signal are generally not affected by the undersampling procedure. Because the aim is to visualize fast periodic vibrations in slow motion, only sampling frequencies of about $f_s \approx f/n$ are of interest. If sampling is performed by stroboscopic light flashes and $s(t)$ represents the displacement of an arbitrary point on a vibrating structure, the time scaling effect must occur for any point. Therefore, the entire structure appears to vibrate in slow motion. To ensure that the structural motion is visible, the peak displacements should be in the mm range. This requires a strong excitation or the effect of a structural resonance. For this reason, the setup is adequate to visualize resonant vibrations, which are observed as the operating deflection shape (ODS) in case of single frequency excitation. Furthermore, if the structure's natural frequencies are well separated, the ODSs are almost identical to the mode shapes.

Technical Details and Practical Aspects

The LED stroboscope is a prototype that has been designed and built at the University of Applied Sciences in Wiener Neustadt. A total of 4×8 white, high-power, light-emitting diodes of type LUW-W5AM (1A, 3W each) are arranged in four linear arrays of about 1 m long. Because of the significant heat production, the LEDs are glued to a beam-type heat sink that additionally acts as supporting element (see Figure 4a). The cooling is a measure of precaution in case the power is switched on permanently.

During normal stroboscope operation, the LED's duty cycle is around 1-2%, which requires no cooling. To achieve a constant homogenous illumination, the power supply offers four channels with linear, constant, current control. Each channel is connected to a standard BNC connector to control one array using standard TTL-signals. The input section is protected against external overvoltage up to 50 V; the power section (LM317T) is equipped with an internal protection against overheating. In Figure 4b, the schematic diagram of a single channel is given. The internal power of 24 V is provided by a simple rectifier circuit connected to a large smoothing capacitor and provides power for the adjustable voltage regulators that are used in constant current, with current set to approximately 0.7A. Each output (CH1_1_OUT,CH1_2_OUT) is connected to a series of four high-power LEDS, together forming one linear stroboscope array.

Depending on the actual application, this basic circuit can be duplicated arbitrarily. In the stroboscope prototype, it was used four times (see Figure 4a). Although it is possible to operate the arrays independently, they are generally connected together and used in phase for best illumination of the test object. It is strongly advised to use very powerful LEDs because they allow for high averaged light intensities while still using short light flashes.

Apparently, best visual effects are obtained if ambient and es-

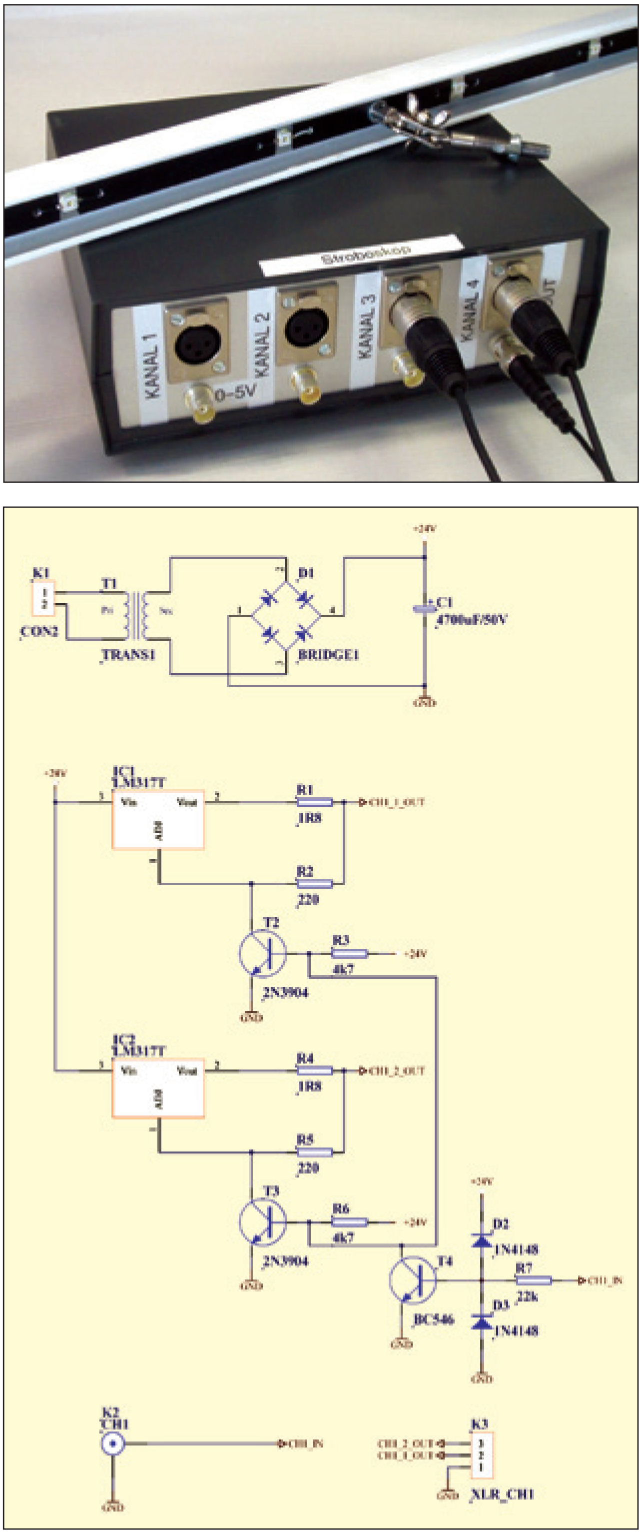

Figure 4. Amplifier prototype with LEDs attached to the cooling body (top) and electrical schematic of a single channel.

pecially artificial light is reduced to a minimum. The higher the duty cycle, the better the illumination level, but at the price of blurring the visual perception. Experience shows that, depending on the spectator, stroboscopic light flicker can cause discomfort. This is why we recommend operating the system at frequencies higher than 40-50 Hz.

This guarantees a perfect illusion of the slowly moving structure. However, note that flashing light might trigger epileptic seizures in people who suffer from photosensitive epilepsy, so people watching the experiment must know about this effect. For best results, the alias frequency should be ideally around 1-2 Hz, with maximum displacement amplitudes of more than 1-2 mm. If the structure has shiny or painted surfaces, the moving light reflections also clearly indicate the structural motion and contribute to a strong visual effect even for much smaller displacements. Since switching of the electronic device is possible up to several kHz, it is possible to investigate local vibration effects as well as very small devices. In this situation magnifying optics might be necessary to observe the vibrations.

For appropriate excitation, harmonic force or ground excitation is recommended, which is generally provided by an electrodynamic shaker. For light and almost undamped structures, a simple alternative has proven to work satisfactorily. If a small strong permanent magnet is attached to a lightly damped test structure, a simple coil connected to a commercial audio amplifier under harmonic input can be used to excite the test structure's resonance. A similar but inverse setup can be used if electrically conducting elements of string type have to be analyzed. A current-carrying conductor located in the homogenous field of a strong permanent magnet experiences an electric force proportional to the current and the magnetic field. This type of excitation is used to investigate vibration and wave phenomena on strings or excite stringed instruments in a realistic way, respectively.

Applications

It must be pointed out clearly, that the system presented here can never be used to replace vibration measurements or experimental modal analysis. However, it is a very simple and effective setup to visualize and understand some basic principles of modal analysis like modal decomposition and its corresponding deflection shapes, the effect of resonance peaks as well as anti-resonances, superposition, the influence of the modal constant and the effect of nodes and node lines. One major advantage of the setup is the possibility to apply it for the analysis of both simple teaching experiments and complex structures.

When using the stroboscope, only estimates of the structural resonances are required, because varying the harmonic excitation frequency quickly renders the resonant vibrations that are often accompanied by acoustic emissions and a blurred view of the geometry. Using the stroboscope with a flash rate of 1 or 2 Hz above the resonance frequency f_j extracts the true structural displacements in slow motion, and the mode shape vector ψ_j (precisely the ODS) is observed directly. Several different applications are discussed in the following paragraphs.

Cantilever Beam. When studying a cantilever beam experimentally, the natural frequencies, the number and position of vibration nodes as well as the deflection shapes are in perfect agreement with theoretical considerations for the lower vibration modes. Therefore, it is straightforward to explain the effect of a node with respect to structural vibration measurements or structural excitation. When looking at transfer functions in the modal superposition form, [2,3]

$$H_{pq}(j\omega) = \sum_{r=1}^{N} \frac{A_{pqr}}{j\omega - s_r} + \frac{A^*_{pqr}}{j\omega - s^*_r} \quad A_{pqr} = Q_r \Psi_{pr} \Psi_{qr} \qquad (2)$$

with s_r, Q_r and ψ_j denoting the pole and modal scaling constant for each mode r and the elements of the modal shape matrix respectively, the influence of the modal constant (residual) A_{pqr} becomes apparent and can be proven experimentally by varying the excitation point. Of course the experienced practitioner knows about the danger of missing natural frequencies and mode shapes using single-point excitations, but when studying modal analysis the demonstration of this effect is crucial.

Anti-Resonances. This important phenomenon, often applied in passive vibration absorption, can also be understood directly by simple visualization. When adding a vibration absorber to a structure and tuning it to a resonance f_j of the host structure the vibration amplitude vanishes if damping is neglected.[4] A close look at this effect reveals that the absorber adds a degree of freedom to the structure, forming a pair of adjacent natural frequencies (see Figure 5a).

Due to the phase shift at resonance, both mode shapes vibrate in opposite directions and compensate each other at the tuning frequency f_j. From inspecting the extended mode shape vector,

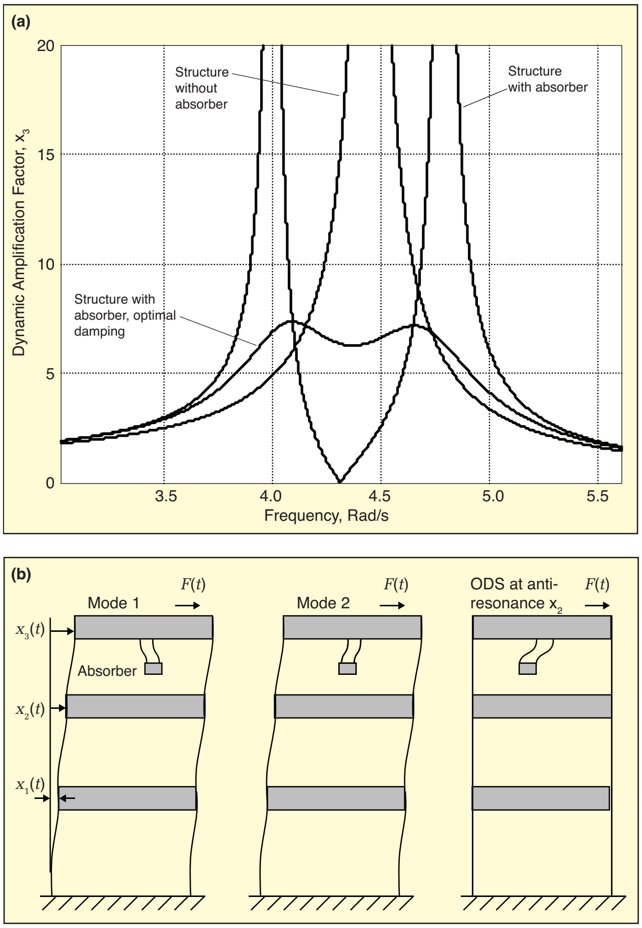

Figure 5. Frequency response of structure with/without dynamic absorber (a); corresponding mode shape vectors and ODS at anti-resonance for plane three story shear frame model with dynamic absorber attached (b).

however, it is apparent that this compensation by modal superposition is only achieved for the host structure (see Figure 5b) for a plane three story shear frame model. Although the story displacements vanish, the absorber motion is amplified, generating compensating forces. This perfect passive vibration absorption can only be achieved at the single frequency f_j. If the excitation frequency changes, the absorbing effect decreases and, even worse, another resonance phenomenon is obtained. This behavior can be avoided by adding viscous damping to the absorber which is common practice in wind and earthquake engineering, (see Figure 5a).

Complex Operational Deflection Shapes. When working with a shear frame laboratory model, for example, complex ODSs can be observed. They occur when the structural displacements are not in phase for harmonic forcing, giving the impression of a traveling wave, while real ODSs appear as standing waves. This phenomenon results from the relative phase differences between adjacent modes that dominate the ODS.

Mass Loading Effect. When investigating light structures, it is frequently observed that the mass loading of the vibration transducer influences natural frequencies and mode shapes, an effect that can also be shown using the presented setup.

Complex Structures. Out of the variety of ways to present modal concepts and phenomena, possibly the most impressive is the application to real structures whose modal deflection shapes cannot be derived intuitively. Among others are real-size car spoilers and doors, skis, tennis racquets, blank CDs and other arbitrarily shaped plate structures or loudspeaker membranes. Bending modes, torsional modes and even compression modes have been made visible directly. Some of the structures have also been analyzed numerically for qualitative comparison and it has turned out that confirming the computer results with real-scale experiments contributes a lot to the understanding and confidence of modal analysis.

Transient vibrations. Transient processes, like free vibrations, beating or mode-jumping experiments, are difficult to visualize unless they can be repeated periodically. For example, if a structure is excited with a periodic impulse type excitation, a direct observation of the transient response is possible. This effect has been tested on an electrically conducting string of length L=0.8 m, rigidly fixed at both ends, with a fundamental frequency of f_0=48 Hz. A suitable excitation is generated by applying a strong current impulse of about 30 A to the string, which is partly placed in the homogenous magnetic field of a strong permanent magnet.

The resulting force is close to an ideal impulse, which induces traveling waves on the string. A periodic transient motion is obtained when repeating the impulsive excitation with frequency f_0. Flashing the stroboscope with $f_s \approx f_0$ *Hz* displays the traveling wave in perfect accordance with the theoretical prediction. D'Alembert's solution (traveling-wave approach) but equally Bernoulli's solution (modal decomposition), can be used to fully describe the propagating wave.[2,5] The latter is given by where the coefficients u_n are defined by the initial displacement, and the string is released with no initial velocity.

Since standing waves can be generated with the same setup, this simple experiment can be used to demonstrate that modes and waves are dual theories that explain structural displacements for a specific forcing in this context. From an engineer's perspective, however, the modal approach is characterized by its versatility, because the modes can be measured or calculated, contrary to the wave approach, where a closed-form solution is required.

Rotordynamics. Although the application of stroboscopes has a long tradition in rotordynamics, it is mainly used to determine the speed of a rotating shaft. Experiments on a simple small-scale rotor test rig show that it is straightforward to visualize the forward whirling of a cantilever elastic shaft close to the critical speed. Consequently, a similar result can be expected for the backward whirl if it is excited properly.

Conclusions

This work presents an experimental setup to visualize structural vibrations using LED-based, distributed, stroboscopic-light technology. The LED technology has turned out to be inexpensive, robust, fast, energy saving, flexible and easy to use.

For the experimental setup, 32 high-power LEDs are arranged in four mobile arrays to homogenously illuminate a test structure. For any periodic excitation, the structural response appears in slow motion if the vibrational displacements are in the visible range and the flash frequency is chosen close to the excitation frequency.

Several basic modal analysis concepts like mode shapes, natural frequencies or resonances and anti-resonances become apparent through direct observation. Furthermore, the results can be compared qualitatively to classical experimental modal analysis methods or computational results. The system has been applied to a variety of structures, including piano and cello strings, skis, tennis racquets, as well as plate- and shell-type structure of various sizes. For large structures, the illumination and light intensity can be increased by simply adding more light arrays all operating in phase, which allows observing local and global vibration modes at the same time.

Acknowledgments

The author gratefully acknowledges Johann Leinweber and Helmut Frais-Kölbl, Department of Electrical Engineering, University of Applied Sciences, Wiener Neustadt, for their help in developing the electronic part of the project and for various discussions during the experimental setup and the results.

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