# Some Educational Vibration Measurement Exercises

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**Four exercises are presented to teach experimental vibration measurements: mass calibration; accelerometer mounting; single-degree-of-freedom vibration measurement and analysis; and full-scale experimental modal analysis.**

Making accurate vibration measurements may involve choice of sensor, sensor mounting and suspending the structure under test properly. This article describes some exercises designed to teach students good vibration measurement practices. In addition, the exercises also introduce important aspects of experimental work, such as being patient, to double-check everything, and to always question one's results.

These exercises are used in a third semester course on the graduate level. The students have completed a course on general signal processing but are unfamiliar with vibration analysis techniques. In this course, both wave theory of continuous structures and discrete mechanical systems are taught. However, the exercises covered here focus on the discrete description of mechanical systems. The course goals are:

- Learn experimental methodology to ensure good results such as checking repeatability and checking everything that could affect the measurement.
- Mount accelerometers correctly.
- Investigate accelerometer effects such as mass loading.
- Correctly suspend a structure for experimental modal analysis.

An additional point of these exercises is to tie theory to experimental results. Perhaps a good way to summarize the spirit of the exercises is to use the famous quote by Albert Einstein:

> *"A theory is something nobody believes, except the person who made it. An experiment is something everybody believes, except the person who made it."*

Although the exercises primarily address the second part of this quote, experimental results must be questioned constantly and numerical models need to be evaluated.

## Exercises

The exercises require some basic vibration measurement equipment. The measurement hardware and software can be essentially any vibration measurement system. We are using a four-channel data acquisition box that can be obtained from many manufacturers today, driven by homemade MATLAB software using the Data Acquisition Toolbox and the free ABRAVIBE toolbox for MATLAB for the analysis.[1] This means that the students record time history data for subsequent analysis in MATLAB. We have found that this ensures that the students understand every step in the processing of the data; something "automatic" commercial systems often make more difficult.

We use two accelerometers, a force sensor, an impact hammer, and a shaker with amplifier and random noise generator. The sensors used are not particularly important, although the accelerometers should weigh less than 10 grams, the force sensor should be of suitable sensitivity, and the same is true for the impact hammer.

In addition to this, some relatively inexpensive measurement objects are needed. The first three exercises are made in the laboratory, during approximately 2 hours, followed by 4 to 6 hours of analysis and report writing. The last exercise, is accomplished in a second laboratory session, in approximately 4 hours, followed by 6 to 8 hours of analysis and report writing.

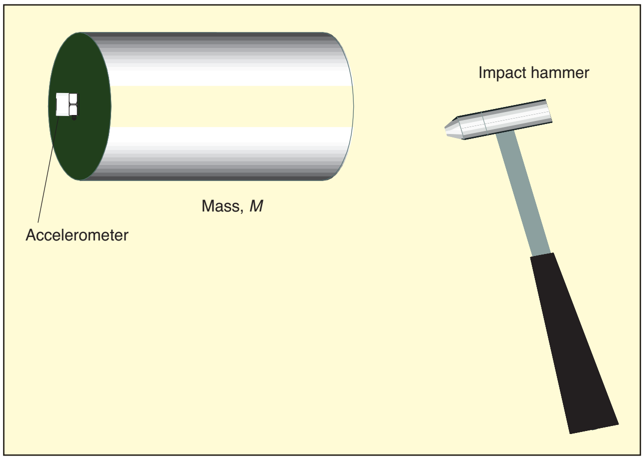

*Figure 1.Setup for mass calibration for exercise 1.*

**Mass Calibration**. The first exercise is based on calibration of an accelerometer (or a force sensor in an impact hammer) using a simple mass, as depicted in Figure 1. This well-known technique,[2] should be familiar to everyone making vibration measurements. Since Newton's second law for a mass $M$ is $F = Ma$, where $F$ is force and $a$ is acceleration, the frequency response function (FRF) of acceleration over force, should be a constant:

$$H(f) = \frac{A(f)}{F(f)} = \frac{1}{M} \tag{1}$$

That is, the FRF forms a straight line, independent of frequency.

It is worth noting that mass calibration is a good technique not only for calibration. The technique is also suitable for verifying that accelerometers are functional through the full frequency range and for checking the frequency range of a particular accelerometer. A setup for mass calibration should always be at hand in any vibration laboratory – and it should be used frequently.

The calibration procedure is simple and the parts are indicated in Figure 1. The accelerometer is attached to a mass, typically a steel rod. The weight of the mass is accurately measured and should be heavy enough, depending on the impact hammer used, so that a gentle hit with the hammer produces a suitable acceleration level.

For this exercise we use approximately a 1 kg rod with a diameter of approximately 40 mm. The mass should ideally be suspended from two strings from a supporting rig so that it can move as a pendulum in the direction of the accelerometer. It works well to also place it on a soft foam pad. The mass is then excited a few times, a few seconds apart, by the impact hammer, while the force and accelerometer time signals are recorded. The hammer tip is chosen to yield a frequency range as high as possible, typically up to 10 kHz.

After the time signals with the impact force and resulting acceleration are recorded, the students calculate the frequency response function (FRF) of acceleration with force. For this purpose they use the graphical user interface (GUI) in the ABRAVIBE toolbox based on the method described in Reference 3. This processing method allows easy processing of the time signals into a FRF.

As an additional task, the students are given the sensitivity of the force sensor in the impact hammer, but not the sensitivity of the accelerometer: They are then asked to calculate the sensitivity of the accelerometer given the measured frequency response at 159.2 Hz, for example, which is equal to 1000 rad/s, a common frequency for this purpose. Then they make a second mass calibration, using the obtained sensitivity factor, and verify that the FRF

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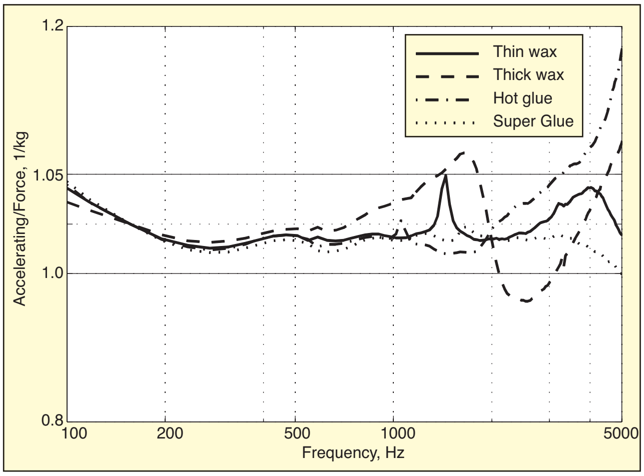

*Figure 2. Comparison of FRFs from four measurements on mass using four different mounting techniques; FRFs are normalized to the same value at 159.2 Hz (1000 rad/s).*[2]

they get is correct. This task ensures that the students think about how the sensitivity factor works.

**Accelerometer Mounting**. The next exercise is using the mass calibration method to investigate the effects of different accelerometer mounting techniques. This exercise has several important objectives – it obviously discusses different means of attaching an accelerometer to the test structure and the accuracy of these mounting techniques. It also demonstrates the difficulty of repeatable measurements since, in most cases, the students do not get the same result even if they use the same mounting technique twice. This also makes a point of discussing the concept of repeatability and the importance of this concept in engineering. Third, this exercise is used as a basis for a discussion about the importance of never trusting one's measurements. This exercise also illustrates that, like all measurement sensors, accelerometers are not perfect but have some uncertainty.

In this exercise, the students use different techniques to mount an accelerometer on the calibration mass and perform measurements as described previously. For each measurement the FRF is calculated and stored. In our case, we mount the accelerometer with the following techniques:

- Thin layer of wax
- Thick layer of wax
- Thin layer of hot glue (hot melt adhesive, using a hot glue gun
- Super glue (cyanoacrylate adhesive)

Other techniques such as a screw mount and magnetic base could also be used. They are problamatic, however, since they change the mass of the accelerometer.

Since the students already have made a measurement with a thin layer of wax in the first exercise, this means that they obtain a total of five different FRFs, of which the first two should be similar. These two FRFs based on a thin layer of wax are first compared and a good discussion on repeatability issues is held.

As the next step in this exercise, the students are asked to plot all five FRFs in one plot and determine which of the mounting techniques works best. This produces a plot similar to Figure 2, where the FRFs have first been normalized to have the same value at 159.2 Hz. Limits of ±5% around this value are plotted to indicate the accuracy limits specified by the sensor manufacturer. Note that only one of the two thin wax measurements is included in the figure, which is for clarity only. The students are finally asked to find the frequency where the uncertaintity reaches ±5% for all mounting techniques and to identify the technique giving the highest frequency limit. This often results in a dead heat between superglue and wax.

Note that this technique should be used in every vibration lab to ensure that a particular accelerometer used with a particular mounting technique is performing well over a necessary frequency range. It emphasizes one of the author's pet peeves – you should never *assume* things (like frequency range performance with an accelerometer and mounting technique combination) but rather *investigate* it.

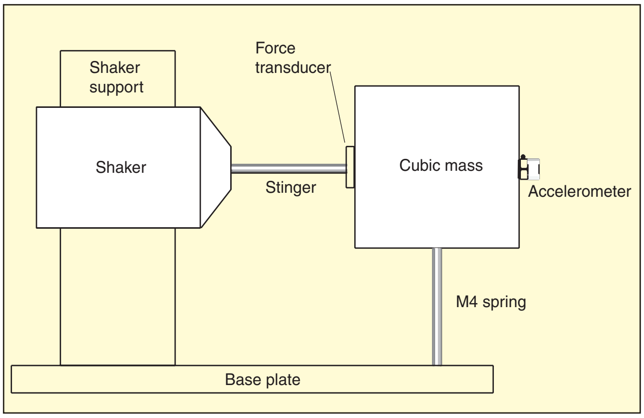

*Figure 3. Schematic illustration of the approximate SDOF system used for exercise.*

**SDOF Measurement and Analysis**. The single-degree-of-freedom (SDOF) system is a key component in vibrations and structural dynamics. In this exercise, a simple system behaving as an approximate SDOF system is investigated and used to illustrate the connection between theory and real world. The system used is shown schematically in Figure 3 and consists of a base plate of steel, approximately 100 × 300 × 10 mm; a steel cube with 30-mm side length; and a M4 bolt, approximately 50 mm long. There are two nuts locking the M4 bolt against the base plate and the mass. The mass is excited by a random force applied by a shaker through a stinger and a force sensor, and an accelerometer is mounted on the opposite side of the force sensor. The FRF between the force and the acceleration is measured and compared to analytical results.

This exercise has several objectives. First it illustrates that the SDOF system used in theory can, in a limited frequency range, be found in "real life." Second, it forms a good basis for a discussion of the difference between a model and reality, since the results obtained are rarely identical to the analytical results calculated by the students.

The stiffness of the M4 bolt is readily calculated using known formulas for moment of inertia and stiffness of a beam and is omitted here as a courtesy to professors who may want to use the this article without giving the students direct access to the answer. The students are asked to measure the various components and estimate the mass of the cube, including the accelerometer, and the stiffness, and from this estimate the SDOF natural frequency.

From the measured FRF, the students are asked to approximately estimate the mass, damping, and stiffness of the system. This can be done in two different ways:

***Method 1.*** a. First estimate the mass by looking at where the accelerance FRF levels out after the resonance. This is very approximate since there will be a second resonance affecting the leveling; but it yields a rough estimate. b. Then use the natural frequency to obtain the stiffness using the obtained mass. c. Estimate the relative damping ratio from the –3 dB bandwidth and then the damping coefficient from the known relationship between the relative damping ratio and $m$, $c$, and $k$.

***Method 2***. a. Integrate the FRF twice to a dynamic flexibility (receptance) FRF. b. Obtain the stiffness from the low frequency flat part of the FRF. c. Then use the natural frequency to obtain the mass, using the obtained stiffness. d. Estimate the damping as in step c. of Method 1.

Once these parameters are obtained, the students are asked to calculate the mass, stiffness, and damping coefficients of the system. The mass and stiffness coefficients thus obtained are compared with the mass calculated from the measurements of the cube, and the stiffness calculated for the beam. Since the results rarely come very close to the analytical model predictions, due to lack of "precision" in the setup, it forms a good discussion point for the differences between a model and reality.

**Full-Scale Modal Analysis Test**. After the initial exercises, which are made in one session, the students are well suited for the second session – a full experimental modal analysis test of a slalom ski using impact excitation with roving hammer. The students are asked to read the text of Reference 4 prior to the lab exercise so that they are acquainted with the theory and practical aspects of an experimental modal analysis.

The choice of the slalom ski is rather arbitrary and any reasonably sized, linear structure could be used. The idea of using the slalom ski instead of a simpler beam or plate is that the students find it more interesting to measure a "real" object.

The exercise is rather straightforward. First, the structure is suspended and a discussion is held on how to best suspend this long, slender structure. The "correct" answer is that it should be suspended hanging vertically, since this ensures the rotational rigid body mode is low, which is very difficult to obtain if it is supported horizontally. This is the case for all long, slender structures with small moments of inertia around the long axis.

In our case, the ski has a small drilled hole in the center of the ski in the "short" direction. Through this hole, a fishing line is threaded, forming a loop, to which a rubber cord is attached. The reason for this is to prevent the rubber cord from adding damping to the ski.

Second, the ski is instrumented with accelerometers in two corners for a minimum multi-reference test. We then discuss the potential use of even more sensors and that more redundant data can lead to better results.

Third, the impact hammer tip and suitable frequency rangeare investigated with some rough measurements. Proper FRF estimation settings such as trigger level, pretrigger condition, block size, and force and exponential windows, are then obtained using the procedures laid out in References 2 and 3, which are supported by the impact GUI in the ABRAVIBE toolbox.[1]

After these optimal settings are obtained, the experimental setup is investigated for two things – the suspension effects of the structure and mass loading effects of the accelerometers? The first point, the suspension effects, is investigated by changing the suspension, which in this case is a rubber cord, by doubling the length. FRFs measured before and after this change are compared, and if they differ in the frequency range of interest, the reasons for this are discussed. Obviously the suspension is then inappropriate. (Is it too short or is there friction between the ski and the fishing line? Are the cables for the accelerometers "pulling" the structure, adding damping?)

The second point, mass loading, is investigated by mounting an additional dummy accelerometer right next to one of the existing accelerometers and making a new measurement. If the new FRF is different from the previous without the extra mass next to the accelerometer, there is apparently mass loading with the double mass of two accelerometers. The risk of having mass loading even when using a single accelerometer is then imminent.

Actually, avoiding mass loading on the slalom ski requires very light-weight accelerometers due to the low damping. So with the 4.5 gram accelerometers used in this exercise, there is some mass loading particularly affecting the higher modes. This becomes a point of discussion, and I still have not had a single student who has thought that mass loading would occur with this small light sensor on this ski.

Finally, when everything is checked and the students are satisfied that everything is in order, the ski is excited at all points, one by one, in a 3 × 7 grid, and time data are stored at each point. Before leaving the lab, the students are encouraged to post-process all their time data and make a first parameter estimation using a Matlab script given to the students prior to the lab exercise to ensure that they get some reasonably good stabilization diagrams. This step only takes 10 to 15 minutes and ensures that the students leave with good data that allow for the rest of the analysis to be performed outside the lab.

## Conclusions

We have described four exercises; three of which are fundamental exercises that teach students good experimental vibration measurement practices and illustrate the concept of a model versus reality. The first three exercises teach the use of mass calibration using an impact hammer and accelerometer to compare different mounting techniques such as wax, hot glue, and super glue and to measure an approximate single SDOF system and identifying the mechanical properties of this system using an experimentally obtained frequency response function (FRF). Subsequently, a complete experimental modal analysis of a slalom ski is performed using impact testing, giving the students experience in this important measurement technique.

Although it is difficult, if not impossible, to teach students to be good vibration experimentalists in a few hours of lab exercises, some key points can surely be taught. The main points taught in these exercises are:

- Whether the useful frequency range of your accelerometer is sufficient or not, using the same mounting technique you are going to use in your experiment; do not trust the data sheet frequency range.
- Whether there are mass loading effects from your accelerometers, or not; do not trust your intuition.
- Whether the suspension affects your measured FRFs, or not; do not believe it does not – investigate it.
- Whether there are effects, on damping for example, from accelerometer cables, or not; again – investigate it.
- A model always has limited accuracy and could even be wrong. Therefore, it should be verified by experiments.
- Measuring FRFs with impact testing, the measurement settings need to be optimized to ensure the best possible FRFs

## References

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