Calculating Stress and Strain from Experimental ODS Data

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A finite-element analysis (FEA) model can be used together with experimental operating deflection shape (ODS) data to calculate stresses and strains in a machine or mechanical structure. This allows for the on-line monitoring of structural stress and strain, which can be compared with prescribed warning levels to ensure that dangerous levels are not exceeded. Examples are included to illustrate how ODS data measured with multiple accelerometers can be used to calculate stress and strain. Also, when these data are displayed together, an ODS in animation on a 3D model of the machine or structure, high levels of stress or strain, or "hot spots," are quickly observed.

In a rotating machine, the dominant forces are applied at multiples of the machine running speed, called orders. An order-tracked ODS is assembled from the peaks at one of the order frequencies in a set of response frequency spectra of a machine. When displaying in animation on a 3D model, an order-tracked ODS is a convenient way to visualize vibration levels and therefore monitor the health of the machine.

A companion publication[2] illustrates how modes participate in an order-tracked ODS of a rotating machine and how they participate differently at different operating speeds. It is also shown how the modal participation can be used to expand an order-tracked ODS so that it is suitable for display on a model of the machine.

It is well known that most rotating machines will exhibit different vibration levels under different loads and speeds. ODSs are conveniently acquired by attaching multiple accelerometers to the machine surfaces and acquiring vibration data from the machine while it is running. In addition to visualizing the deflection of the machine in real time, an ODS can be used to calculate stress and strain by deflecting an FEA model of the machine.

Variable-Speed Rotating Machine

Figure 1 shows a variable-speed rotating machine instrumented with eight triaxial accelerometers. An accelerometer is attached to the top of each bearing block, and six accelerometers are attached to the base plate; three on the front edge and three on the back edge. This test setup was used to measure order-tracked ODSs under different machine speeds.

A laser tachometer with its beam pointed at the outer wheel of the machine was used to measure the machine speed, as shown in Figure 1. The outer wheel had reflective tape on it, so the laser measured the once-per-revolution speed of the machine.

Figure 2 shows a model of the machine that was used to display ODSs in animation. Each of the numbered test points is displayed as a cube icon.

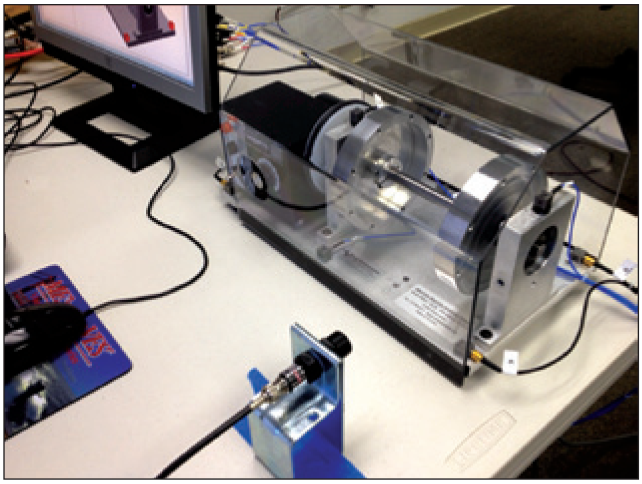


Figure 1. Variable-speed rotating machine.

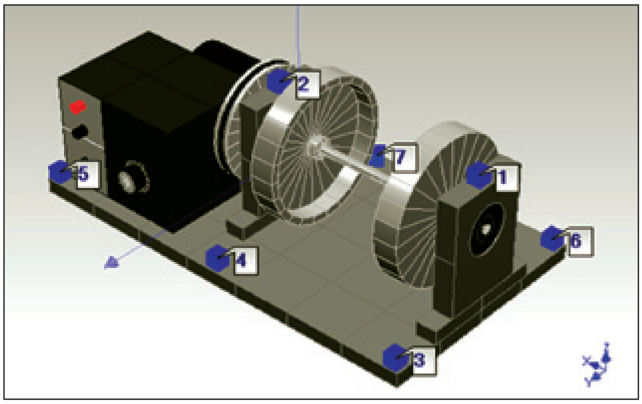
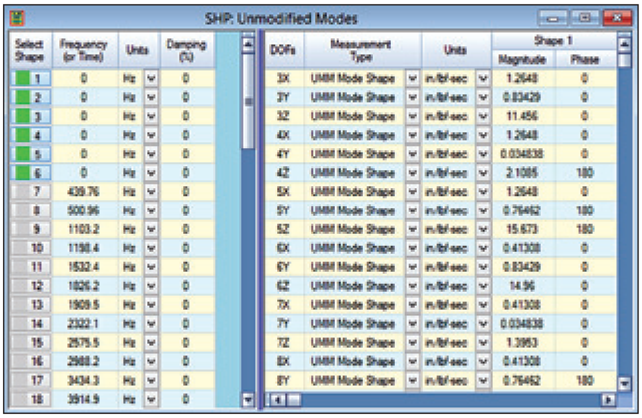


Figure 2. Rotating machine model.

Mode Shapes of the Machine

The order-tracked ODSs of the machine in Figure 1 contain participation of both rigid-body and flexible mode shapes of the base plate and bearing blocks. Since the machine is resting on four rubber mounts (one under each corner), its rigid- body modes will participate significantly in its ODS. The machine has six rigid-body mode shapes that describe its free-free motion in space, but they also participate in its ODS since it is resting on four soft springs.

The rigid body and flexible body mode shapes of the machine were obtained from an FEA model of the base plate and one of the bearing blocks. The first 20 mode shapes of the base plate, together with 20 modes for each of the bearing blocks, were used together with the SDM method[7] to solve for the modes of the combined substructures. Reference 2 contains more details of this procedure.



SHP: Unmodified Modes

Select Shape	Frequency (or Time)	Units	Damping (%)
1	0	Hz	0
2	0	Hz	0
3	0	Hz	0
4	0	Hz	0
5	0	Hz	0
6	0	Hz	0
7	439.76	Hz	0
8	500.96	Hz	0
9	1103.2	Hz	0
10	1198.4	Hz	0
11	1532.4	Hz	0
12	1826.2	Hz	0
13	1909.5	Hz	0
14	2322.1	Hz	0
15	2575.5	Hz	0
16	2988.2	Hz	0
17	3434.3	Hz	0
18	3914.9	Hz	0

DOFs	Measurement Type	Units	Shape 1 Magnitude	Shape 1 Phase
3X	UMM Mode Shape	in/lbf-sec	1.2648	0
3Y	UMM Mode Shape	in/lbf-sec	0.83429	0
3Z	UMM Mode Shape	in/lbf-sec	11.456	0
4X	UMM Mode Shape	in/lbf-sec	1.2648	0
4Y	UMM Mode Shape	in/lbf-sec	0.034838	0
4Z	UMM Mode Shape	in/lbf-sec	2.1085	180
5X	UMM Mode Shape	in/lbf-sec	1.2648	0
5Y	UMM Mode Shape	in/lbf-sec	0.76462	180
5Z	UMM Mode Shape	in/lbf-sec	15.673	180
6X	UMM Mode Shape	in/lbf-sec	0.41308	0
6Y	UMM Mode Shape	in/lbf-sec	0.83429	0
6Z	UMM Mode Shape	in/lbf-sec	14.96	0
7X	UMM Mode Shape	in/lbf-sec	0.41308	0
7Y	UMM Mode Shape	in/lbf-sec	0.034838	0
7Z	UMM Mode Shape	in/lbf-sec	1.3953	0
8X	UMM Mode Shape	in/lbf-sec	0.41308	0
8Y	UMM Mode Shape	in/lbf-sec	0.76462	180

Figure 3. FEA mode shapes of the base plate and bearing blocks.

Some of the mode shapes of the base plate and bearing blocks (60 in all) are listed in Figure 3. Notice that the first six modes are rigid body modes with frequencies of zero**.**

ODS Expansion

ODS data were acquired from the rotating machine shown in

Based on a paper presented at IMAC XXXIII, the 33rd International Modal Analysis Conference, Orlando, FL, February 2015.

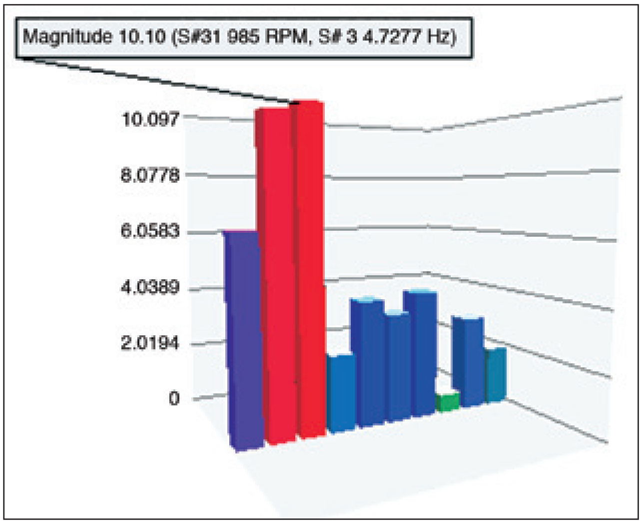


Figure 4. Magnitudes of modal participation at 985 RPM.

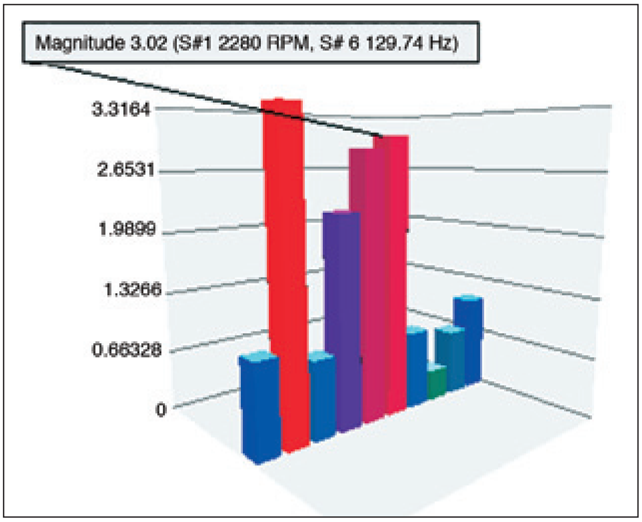


Figure 5. Magnitudes of modal participation at 2280 RPM.

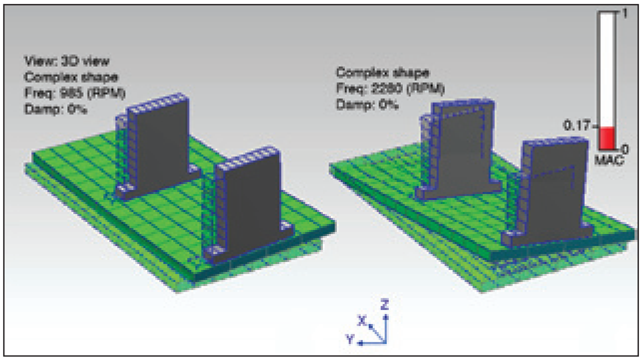


Figure 6. Expanded ODSs of base plate and bearing blocks.

Figure 1 at two different machine speeds. The mode shapes of the base plate and bearing blocks were then used to expand the experimental ODSs acquired from the eight accelerometers to ODSs for all DOFs on the base plate and bearing blocks.

The modal participation of the first 10 FEA modes in the 985 RPM ODS is shown in Figure 4. The modal participation factors show that the first three modes are the dominant contributors to the 985 RPM ODS. All three of these rigid body modes are being excited at this speed, and the machine is simply bouncing vertically and sideways on its rubber mounts.

The modal participation of the first 10 FEA modes in the 2280 RPM ODS is shown in Figure 5. The participation factors of Modes 2, 5, and 6 indicate that they dominate the 2280 RPM ODS. At this higher speed, the machine is rocking and twisting on its rubber mounts, with more motion at the outer bearing location.

Figure 6 is a comparison display of the two expanded ODSs on a model of the base plate and bearing blocks. Animation of these shapes more clearly shows that they are different, and their *low MAC value (0.17)* also indicates that they are different. The 2280 RPM ODS has much more motion at the outboard bearing block. This is due to the greater cyclic force created at the higher speed by the unbalance weight that was added to the outboard wheel.

Strain From Shape Data

Strain is the forced change in the dimensions of a structure measured as a change in dimension per unit length. Its units are typically displacement per unit displacement. Strain in an FEA element is a function of the deflections of its vertices. These deflections can be obtained from the components of an expanded ODS or from mode shapes.

When the dynamics of a machine can be adequately represented by mode shapes, an ODS can be represented as a weighted summation of mode shapes. This is the well known superposition property of modes.

The equation for calculating the *normal and shear strain* at an FEA element vertex is:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = [B] \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \vdots \\ u_n \\ v_n \\ w_n \end{Bmatrix} \qquad (1)$$

where: [B] = displacement strain matrix for an FEA element vertex

$$[B] = \begin{bmatrix} \frac{\partial}{dx} & 0 & 0 & & \frac{\partial}{dx} & 0 & 0 \\ 0 & \frac{\partial}{dy} & 0 & \cdots & 0 & \frac{\partial}{dy} & 0 \\ 0 & 0 & \frac{\partial}{dz} & & 0 & 0 & \frac{\partial}{dz} \\ \frac{\partial}{dy} & \frac{\partial}{dx} & 0 & \cdots & \frac{\partial}{dy} & \frac{\partial}{dx} & 0 \\ 0 & \frac{\partial}{dz} & \frac{\partial}{dy} & & 0 & \frac{\partial}{dz} & \frac{\partial}{dy} \\ \frac{\partial}{dz} & 0 & \frac{\partial}{dx} & \cdots & \frac{\partial}{dz} & 0 & \frac{\partial}{dx} \end{bmatrix} \text{(6 by 3n)}$$

$$\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \vdots \\ u_n \\ v_n \\ w_n \end{Bmatrix} = \text{ODS (or mode shape) components at all vertices}$$

n = number of element vertices

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \text{normal and shear strain at FEA element vertex}$$

Stress from Strain

Stress is the amount of force acting within a cross sectional area of a structure. Its units are typically force per unit area. Stresses are calculated from strains with the following equation:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = [C] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad (2)$$

where:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}$$ =normal and shear stress at FEA element vertex

$$[C] = \frac{E}{(1+v)(1-2v)} \begin{pmatrix} (1-v) & v & v & & 0 & \\ v & (1-v) & v & & 0 & \\ v & v & (1-v) & & & \\ & & & \frac{(1-v)}{2} & 0 & 0 \\ & 0 & & 0 & \frac{(1-v)}{2} & 0 \\ & & & 0 & 0 & \frac{(1-v)}{2} \end{pmatrix}$$

E = Young's modulus of elasticity
v = Poisson's ratio

Strain from Mode Shapes

The participation factors shown in Figures 4 and 5 indicate that the *flexible-body Modes* 7, 9, and 10 also contributed to both the 985 and 2280 RPM ODSs. Figure 7 is a display of the deflection of each of these mode shapes, together with the *normal strain in the X-direction* of the base plate and bearing blocks.

Note that where local bending occurs in each mode shape, positive strain (*tension*) on one side of the base plate equals the same amount of negative strain (*compression*) on the other side. Also, the cross section of the plate transitions from positive to negative strain values.

Stress from Mode Shapes

Figure 8 is a display of the deflection of Modes 7, 9, and 10 together with the normal stress in the X-direction of the base plate and bearing blocks.

Note again that where local bending occurs, positive stress (tensile) on one side of the base plate equals the same amount of negative stress (compressive) on the other side of the plate. Also, the cross section of the plate transitions from positive to negative stress values.

Strain from 985 and 2280 rpm ODSs

Figure 9 is a display of the normal strain in the X-direction calculated for both the 985 RPM and 2280 RPM ODSs. Note that the strain distributions of the two ODSs are different primarily because the ODSs themselves are different (see Figure 6). Also, the peak strain levels are higher for the 2280 RPM ODS, which is expected. Unlike the mode shape strain values, these values are realistic, because they are based on the experimental ODS values in inches of displacement.

Stress from 985 and 2280 RPM ODSs

Figure 10 is a display of the normal stress in the X-direction calculated for both the 985 RPM and 2280 RPM ODSs. Again, the stress distribution is noticeably different between the two ODSs. The peak stress levels are also higher for the 2280 RPM ODS, as expected.

Note that these stress values are dimensionally correct (in psi units), because they are based on the strain values in Figure 9, which are also dimensionally correct.

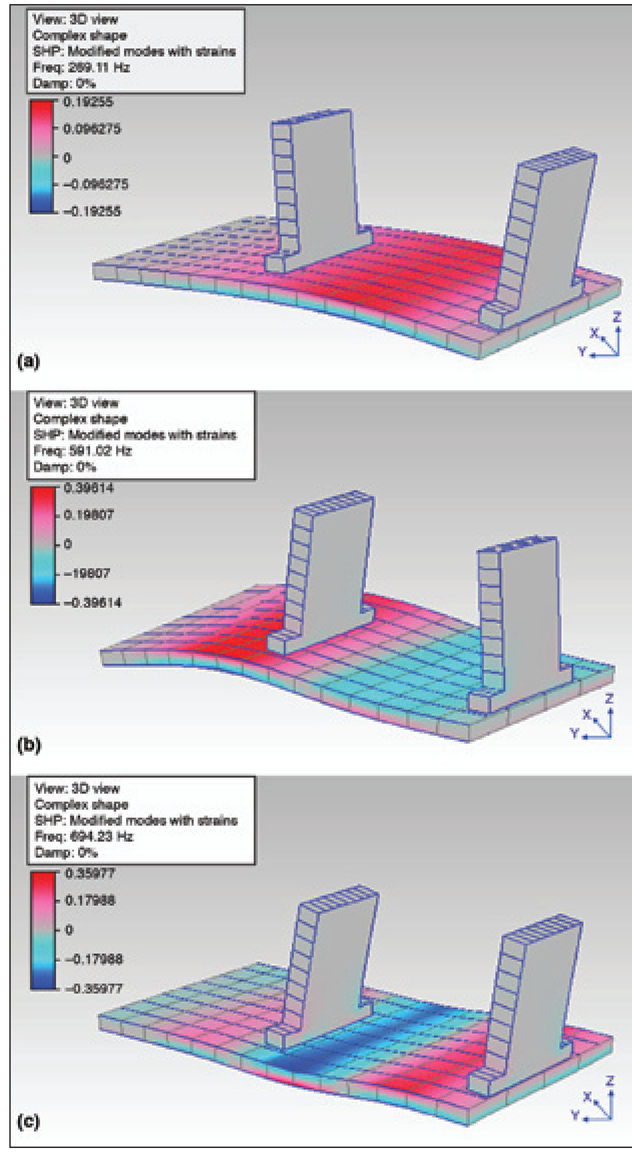


Figure 7. Normal strain, X-direction: (a) Mode 7; (b) Mode 9; (c) Mode 10.

Conclusions

We've shown how an FEA model of a machine or structure can be used to calculate stress and strain from experimental ODS data. Experimental ODSs can be acquired while a machine is running, and FEA mode shapes can be used to expand the ODS data to include all of the DOFs of the FEA mode shapes. The details of shape expansion were given in References 1 and 2. Then the FEA model was used to calculate stress and strain values by deflecting it with the expanded ODS data. Finally, we showed that both the deflection and the stress and strain of the FEA model can be displayed together on the model.

Note that only ODS data in displacement units will give valid stress and strain values when used to deflect an FEA model. A key advantage of this technique is that the same FEA model can be used both for calculating the mode shapes required to expand the experimental ODS data and also for calculating stresses and strains.

When implemented as part of a troubleshooting or long-term monitoring system, this technique can provide valid stress and strain data in real time under different machine conditions.

References

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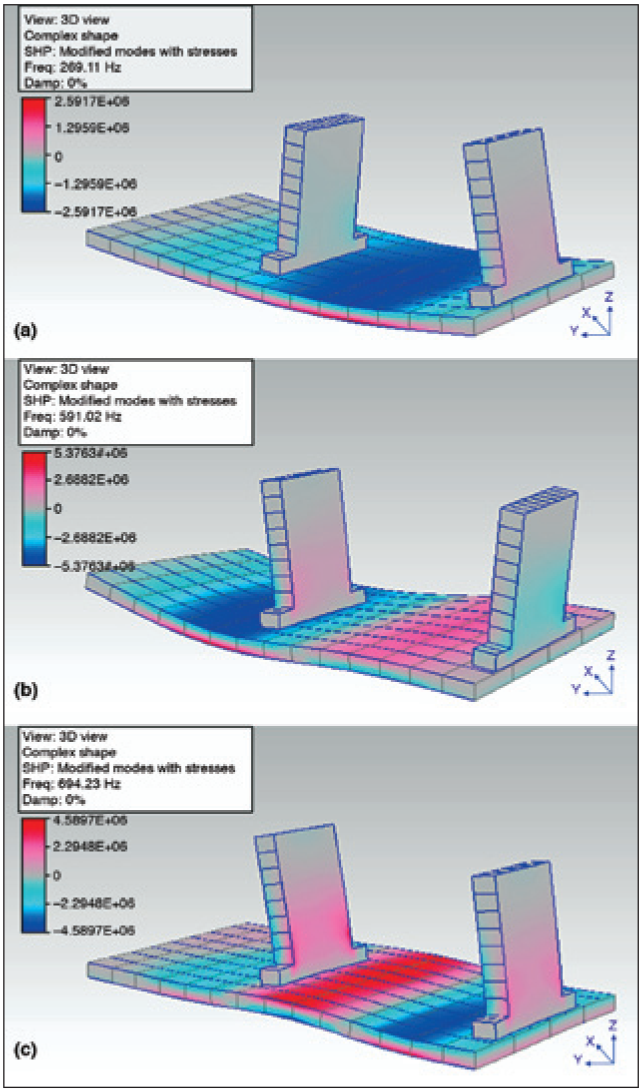

Figure 8. Normal stress, X-direction: (a) Mode 7; (b) Mode 9; (c) Mode 10.

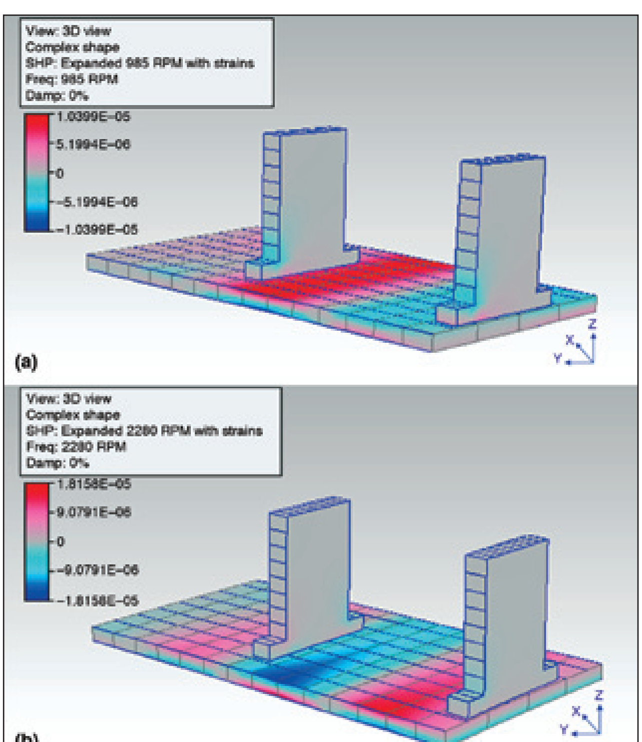

Figure 9. Normal strain, X-direction: (a) 985 RPM ODS; (b) 2280 RPM ODS.

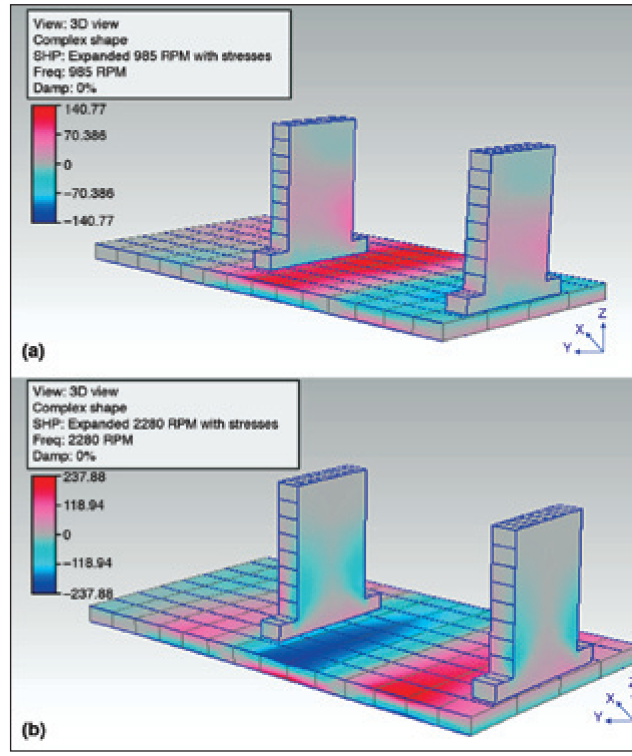

Figure 10. Normal stress, X-direction: (a) 985 RPM ODS; (b) 2280 RPM ODS

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