Wind Turbine Substructuring Using the Transmission Simulator Method

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This work contains an example of the transmission simulator method for experimental dynamic substructuring using the Ampair 600 wind turbine. A modal test was performed on the hub with a single blade attached and then, using the hub as a transmission simulator, this subsubstructure was replicated three times, rotated into the correct orientation and then assembled together with two negative copies of the hub. Substructuring predictions of the modes and frequency response functions for the three-bladed assembly were compared to a set-of-truth test data. The article also highlights the dynamic substructuring wiki, where the test data for this structure and other helpful resources are available for researchers or engineers who wish to test these techniques using real measurements.

As manufactured systems become more complex, it becomes ever more challenging to create accurate finite-element models. Glued, bolted or press-fit interfaces lead to uncertainties, material properties may not be adequately known, and intricate geometry may require too many elements to create a practical model. Dynamic substructuring allows an analyst to predict the dynamic response of a complex system by replacing certain components with a test-based model. As long as each subcomponent can be modeled or tested, the response for a full system can be predicted, and changes can be made to one part without having to recreate the models for other parts. Experimental substructuring is particularly useful when a system is too large to test as an assembly, or when subcomponent hardware is available but the detailed design definition is not (like a subcomponent produced by an outside company).

This work contains a detailed example of dynamic substructuring on the Ampair 600 wind turbine test bed. This test bed was created by the Dynamic Substructuring Focus Group, which meets each year at the International Modal Analysis Conference. The goal of this activity is to construct the dynamics of a three-bladed rotor assembly using the results from a test on a single blade and hub. To create and evaluate this prediction, two modal tests are presented in this work. The first is of a substructure containing one blade and the rotor hub. The second is a test of the built-up rotor structure that will be used as a truth model.

Traditionally, experimental substructuring has been performed by seeking to measure the displacements and rotations of the actual point(s) at which one substructure connects to another and then assembling the substructures at those points. This typically leads to several difficulties and prompted the authors to present the transmission simulator (TS) method,[1-3] where each substructure is tested with a fixture attached. Translation measurements (accelerometers) distributed over the transmission simulator are used to characterize its motion, and then the transmission simulators can be assembled using a weakened set of constraints that minimizes sensitivity to measurement noise.

In the present context, by using the hub as the transmission simulator, the compliance and the damping of the joint connecting the blade and hub are captured within the substructure test, and the hub also mass loads the root of the blade so that blade stiffness at the root is appropriately exercised. This is a bit of a departure from previous studies, where transmission simulators were specifically designed and machined from a solid piece of material.[1-3] For this work, the actual hub was used, because it was readily available and was certain to simulate the interface conditions correctly. This provides the best possible simulation of the blade-to-hub joint, since the actual joint dynamics are contained

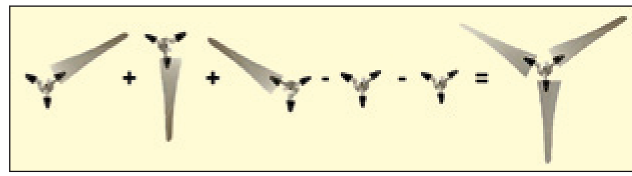

Figure 1. Substructures required to complete transmission simulator prediction.

within the experiment.

Transmission Simulator Methodology

To apply the transmission simulator method, a modal test is performed on a single-blade and hub substructure. This structure can then be replicated and rotated into proper positions for each blade in the built-up assembly. Since the transmission simulator, or hub, is included in all three copies of the experimental structure, the dynamics of two of these hubs will need to be removed from the predictions. These substructures are connected through a series of connection degrees of freedom located on the transmission simulator. This substructuring can be seen visually in Figure 1.

The same rotor hub was used as a transmission simulator in a past experiment and was found to have a first elastic natural frequency above 1200 Hz.[4,5] This elastic mode is far beyond the scope of the current test, so only the six rigid-body modes of the transmission simulator will be used to couple the substructures together.

The substructuring theory is presented in general in References 1-3, so here the substructuring process is only reviewed briefly in the context of the application to the wind turbine. For the following calculations, the subscript A represents the first blade, blade A. B and C represent the second and third blades respectively and the subscript TS represents the transmission simulator (rotor hub). The modal natural frequency and damping ratio for any subcomponent are denoted, ω and ζ respectively, and Φ represents the associated matrix of mode shapes. The physical degrees of freedom are represented by vectors denoted x, and the modal degrees of freedom are denoted by vectors q; these are related, for example, as $x_A = \Phi_A q_A$. To begin, the system of equations is written in the standard mass-normalized modal representation. Here each substructure is still distinct, so the full set of equations is written in an uncoupled block diagonal form:

$$\begin{bmatrix} I_A & 0 & 0 & 0 \\ 0 & I_B & 0 & 0 \\ 0 & 0 & I_C & 0 \\ 0 & 0 & 0 & -2I_{TS} \end{bmatrix} \begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_B \\ \ddot{q}_C \\ \ddot{q}_{TS} \end{Bmatrix} + \begin{bmatrix} 2\zeta_A\omega_A & 0 & 0 & 0 \\ 0 & 2\zeta_B\omega_B & 0 & 0 \\ 0 & 0 & 2\zeta_C\omega_C & 0 \\ 0 & 0 & 0 & -4\zeta_{TS}\omega_{TS} \end{bmatrix} \begin{Bmatrix} \dot{q}_A \\ \dot{q}_B \\ \dot{q}_C \\ \dot{q}_{TS} \end{Bmatrix} + \begin{bmatrix} \omega_A^{\ 2} & 0 & 0 & 0 \\ 0 & \omega_B^{\ 2} & 0 & 0 \\ 0 & 0 & \omega_C^{\ 2} & 0 \\ 0 & 0 & 0 & -2\omega_{TS}^{\ 2} \end{bmatrix} \begin{Bmatrix} q_A \\ q_B \\ q_C \\ q_{TS} \end{Bmatrix} = \begin{Bmatrix} \Phi_A^T F_A \\ \Phi_B^T F_B \\ \Phi_C^T F_C \\ 2\Phi_{TS}^T F_{TS} \end{Bmatrix} \qquad (1)$$

You might be surprised to see negative mass, damping and stiffness in the partition corresponding to the transmission simulator above. Indeed, when one substructure needs to be removed from an assembly, most prior works have described that as an uncoupling (or decoupling) process using frequency-based substructuring,[6] yet this is equivalent to adding a negative substructure to cancel the dynamics of the subcomponent. In Reference 1, the authors showed that uncoupling in the modal domain can be accomplished by adding a structure with negative properties, which then cancels the forces that the substructure would exert at the interface(s).

The constraints are only between those DOF that are measured on the transmission simulator, so if $x_A = x_A = [(x_{A,m})^T \ (x_{A,o})^T]^T$ denotes the set of all measurements on substructure A, then it must be partitioned into a set of DOF, $x_{A,m}$, on the TS and a set of other DOF, $x_{A,o}$. Ideally, one would like to assure that all of the measured motions on the transmission simulator are equivalent, or that:

$$\begin{bmatrix} I & 0 & 0 & -I \\ 0 & I & 0 & -I \\ 0 & 0 & I & -I \end{bmatrix} \begin{Bmatrix} x_{A,m} \\ x_{B,m} \\ x_{C,m} \\ x_{TS,m} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

Using the appropriate partitions of Φ, the constraint equation can then be rewritten as modal coordinates as seen in Eq. 3:

$$\begin{bmatrix} \Phi_{A,m} & 0 & 0 & -\Phi_{TS,m} \\ 0 & \Phi_{B,m} & 0 & -\Phi_{TS,m} \\ 0 & 0 & \Phi_{C,m} & -\Phi_{TS,m} \end{bmatrix} \begin{Bmatrix} q_A \\ q_B \\ q_C \\ q_{TS} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

Because there are experimental errors associated with the mode shapes of A, B and C in Eq. 3, using this equation usually results in disastrous substructuring results. To be robust against these errors, the transmission simulator method uses a weakened (reduced in number) set of constraints by pre-multiplying by the pseudo-inverse of the transmission simulator mode shapes partitioned to the constraint degrees of freedom. This enforces the motion of the two substructures to be equivalent only as far as those motions can be described by the set of mode shapes, $\Phi^+_{TS,M}$. Because those shapes typically come from a finite-element model, they are noise free, so this serves to filter out measurement noise and bias errors that are randomly associated with each accelerometer sensitivity or modal fit:

$$\begin{bmatrix} \Phi^+_{TS,m} & 0 & 0 \\ 0 & \Phi^+_{TS,m} & 0 \\ 0 & 0 & \Phi^+_{TS,m} \end{bmatrix} \begin{bmatrix} \Phi_{A,m} & 0 & 0 & -\Phi_{TS,m} \\ 0 & \Phi_{B,m} & 0 & -\Phi_{TS,m} \\ 0 & 0 & \Phi_{C,m} & -\Phi_{TS,m} \end{bmatrix} \begin{Bmatrix} q_A \\ q_B \\ q_C \\ q_{TS} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{4}$$

The two leading matrices can now be collected to form a single matrix, B, that contains the constraints for the modal degrees of freedom:

$$B \begin{Bmatrix} q_A \\ q_B \\ q_C \\ q_{TS} \end{Bmatrix} = B\{q\} = 0 \tag{5}$$

These constrained modal degrees of freedom can be transformed by some matrix, L, into a set of unconstrained generalized coordinates, q_g. This nomenclature comes from Eq. 7 and deserves some clarification. The EOM with DOF $\{q\}$ are termed "constrained" because an arbitrary displacement $\{q\}$ could violate the constraints. As a result, to use these DOF, one would have to simultaneously solve the EOM and enforce the constraints in Eq. 5. In contrast, the DOF $\{q_g\}$ automatically satisfy the constraints for any choice of $\{q_g\}$, so they are termed "unconstrained." They are the DOF of the assembled system, or the system after all constraints have been enforced:

$$\{q\} = L\{q_g\} \tag{6}$$

Using this substitution requires that L reside in the null space of B because $q_g = 0$ would be a trivial solution. This means that L must be reside in the nullspace of B to fulfill Eq. 5:

$$BL\{q_g\} = 0 \tag{7}$$

This substitution is then used in Eq. 1, which is also premultiplied by L^T resulting in the coupled equations of motion for the system. Note the M, C and K below are the block-diagonal matrices defined in Eq. 1:

$$\bar{M}\ddot{q}_g + \bar{C}\dot{q}_g + \bar{K}q_g = 0$$
$$\bar{M} = L^T ML \quad \bar{C} = L^T CL \quad \bar{K} = L^T KL$$
$$x = \begin{bmatrix} \Phi_A & 0 & 0 & 0 \\ 0 & \Phi_B & 0 & 0 \\ 0 & 0 & \Phi_C & 0 \\ 0 & 0 & 0 & \Phi_{TS} \end{bmatrix} Lq_g \tag{8}$$

The modal properties for the assembly can then be found by solving $\left[\bar{K} - \bar{\omega}^2 \bar{M}\right]\bar{\Phi} = 0$. The transformation above can then be used to bring the solution for the unconstrained degrees of freedom back into the physical domain.

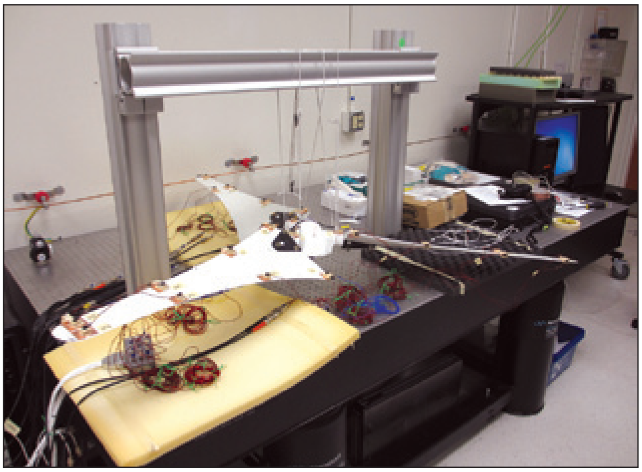

Figure 2. Rotor assembly.

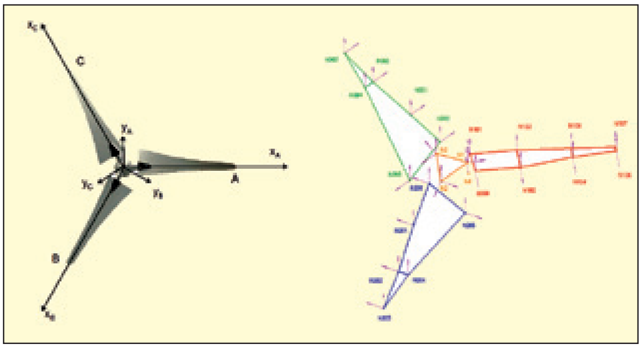

Figure 3. Substructure designation (left) instrumentation diagram (right).

Test Objective and Methods

To generate and evaluate a substructuring prediction of the rotor assembly (Figure 2), modal tests were performed on two configurations. The first is a test of the full built-up rotor assembly to act as a "truth" model, and the second is a subassembly "substructure" test to be used in the substructuring predictions. Previous tests containing these structures in similar configurations[4,5,8] have shown that the highest frequencies of interest would occur below 175 Hz, so the test range was set to 200 Hz to allow the modes of interest (and a few higher) to be captured. Rigid shapes based on the mass properties from Reference 5 are used to represent rigid-body motion of the subcomponents. After completing the modal testing, modes were extracted from the experimental data using the SMAC algorithm.[9]

The first structure was the hub connected to all three blades. Hardware assets were used from Sandia National Laboratories with the serial numbers for Blades A, B, and C being SNL009, SNL008, and SNL007, respectively. The second structure tested was the turbine hub assembly with only Blade A connected. Experimental frequencies, shapes, and frequency response functions from both

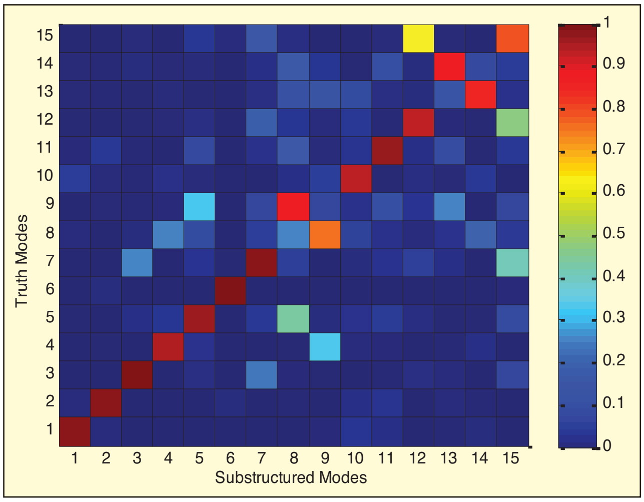

Figure 4. Modal assurance criterion between truth test and substructured predictions; first six modes are rigid-body modes (Modes 7 - 15 shown in Table 1).

of these tests are available on the Dynamic Substructuring Wiki.[10]

Each third of the rotor was given its own Cartesian coordinate system with x along the blade, y perpendicular to x in the rotation plane, and z along the axis of rotation. The origin of these coordinate systems was defined at a common point on the center of the hub. Figure 3 shows these coordinate systems as they are aligned with each blade along with the instrumentation locations for the rotor assembly system. Note that Blade A was more heavily instrumented, since it would be used in the full system test as well as the single-blade and hub substructure test.

The suspended structure was excited at several drive points in the usual attempt to find the best location to excite each individual mode. Drive points were gathered on the blades and on the rotor hub. The drive points on the rotor hub provided the best results, not because they excited the modes the most, but because they excited the modes well enough and produced FRFs with the most linear characteristics. To create a truth model for the rotor assembly, the measured response from the best drive points was used to calculate the modal parameters for each elastic mode. Modes 7, 8, 9, 12, and 15 were derived from excitations with a drive point at Node 1 in the *z* direction; while Modes 10 and 11 were derived from excitations at Node 4 in the y direction, and Modes 13 and 14 were derived from excitations at Node 3 in the *z* direction.

Substructuring Results

Using the modal testing results from the single-blade and hub test, a set of predictions for the built-up rotor assembly was obtained. This substructured model is compared to the results from the truth test. Because some modes of the system were found to be closely spaced, these modes had to be correlated based on their modal assurance criterion (MAC) values. This identification was important when looking at the 8th and 9th substructured modes as well as the 13th and 14th. These modes could be identified by MAC values as well as visual comparison. The MAC values and modal parameter comparisons can be seen in Table 1.

Table 1. Substructuring results.

Truth Mode	Freq., Hz	Damp. Ratio	Substr. Mode	Substr. Freq., Hz	Freq. Error	Substr. Damp	Damp. Error	MAC
7	20.56	1.00	7	23.49	14.26	0.73	–27.19	0.9912
8	27.78	0.98	9	28.33	2.00	0.86	–12.07	0.7655
9	29.03	0.87	8	28.03	–3.44	0.85	–1.88	0.8808
10	61.10	1.71	10	66.53	8.91	0.71	–58.31	0.9422
11	64.29	1.27	11	66.67	3.72	0.71	–44.03	0.9787
12	70.68	1.11	12	77.33	9.41	0.84	–23.71	0.9402
13	99.40	1.48	14	96.30	–1.75	1.00	–32.17	0.8618
14	102.95	1.08	13	97.66	–6.45	0.99	–8.82	0.8849
15	155.00	1.33	15	167.26	7.91	1.29	–3.05	0.7850

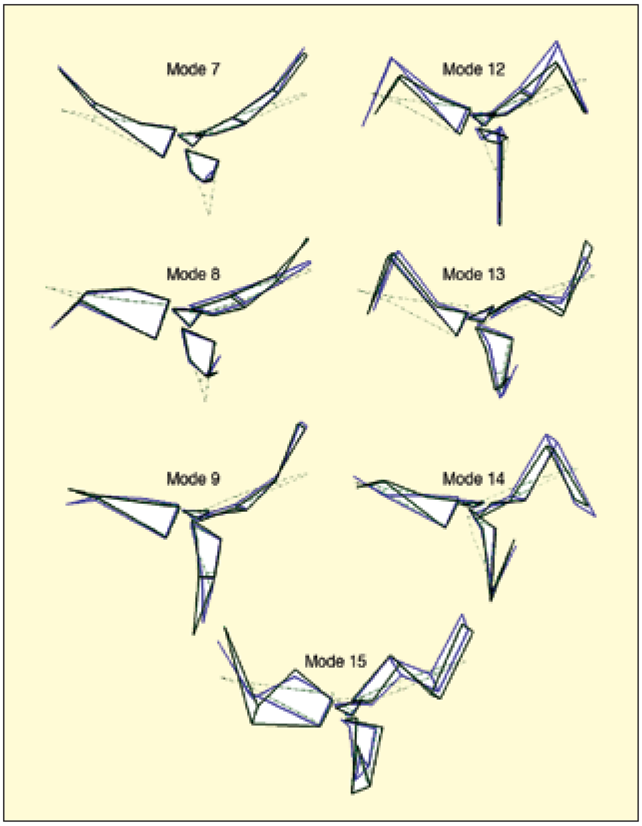

Figure 5. Bending mode shapes comparison: blue – substructuring prediction; green - truth test; broken line – undeformed).

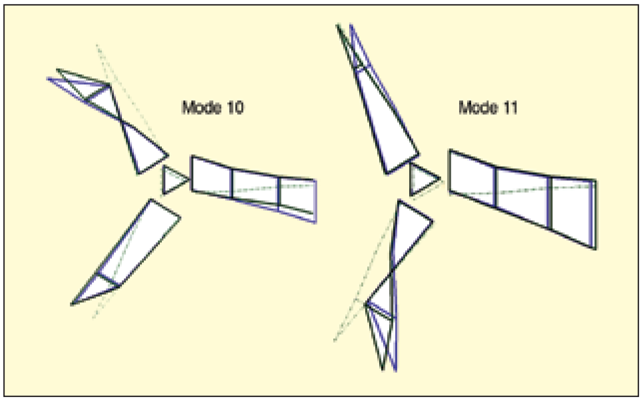

Figure 6. Torsion mode shapes comparison: blue – substructuring prediction; green - truth test, broken line – undeformed).

Table 1 identifies some trends. The substructuring approach predicts frequencies that are somewhat too high for the first, second and third in-phase, out-of-plane bending modes (Modes 7, 12 and 15). In contrast, the frequencies of three of the four anti-symmetric out-of-plane bending modes (9, 13 and 14) are too low. The edgewise modes (10 and 11) are high in frequency. These errors are somewhat larger than typical for a substructuring prediction such as this. It was noted in other studies[11] that there are significant variations among the ampair turbine blades and even more significant variations in the joints between the blades and the hub, so it is thought that much of the discrepancy between the substructuring predictions and the truth test is a result of the assumption that the three blades are identical. In Reference 12, this assumption was relaxed (each of the blades was connected to the hub and tested separately), and somewhat more accurate results were obtained.

The damping ratios predicted by substructuring show larger errors than the natural frequencies. Some modes (9 and 15) are quite close in damping, while others are as high as 58% off mark. Other studies have similarly shown that the damping ratios are more difficult to predict than the natural frequencies.

The correlation of modes between the substructured and truth models could be determined either by MAC (Figure 4) or visual comparison (Figures 5 and 6). Figure 5 contains the bending modes in an isometric view, while Figure 6 shows the edgewise modes in the *xy* plane.

Conclusions and Future Work

This effort used results from a modal test of a single blade and hub to predict the modes of a built-up rotor structure. This experimentally defined substructure was rotated and linked together using degrees of freedom on the hub to create an assembly that contained three blades and three hubs (transmission simulators). Two of the hubs were then analytically removed from the assembly. The results of this substructuring exercise were then compared to an experiment conducted on the full rotor assembly.

The rigid-body modes for these cases were constructed from mass properties. The elastic modes predicted by substructuring correlated well with those from a truth test as shown both by MACs and a visual comparison. The worst frequency error was about 15% in the first mode. The damping ratios were the most difficult to predict, with error as high as 55%. MAC values ranged from 0.77 to 0.99.

While the errors in the substructuring predictions are somewhat higher than desired, and higher than in the authors' other studies, the transmission simulator process is still far simpler than alternatives in which the motion of the actual connection points must be measured and used to assemble the components.

The results could probably be improved by including the first elastic mode of the transmission simulator. The first elastic mode of the hub was high (~1200 Hz), yet it does involve bending of the black tabs that connect the blades to the hub and could be relevant when those tabs are mass-loaded by the turbine blades. Further research should also be performed to understand how to more accurately predict the damping of the assembly.

The measurements obtained in this case study are located on the Dynamic Substructuring Wiki.[10] Data from several other tests are also available, as well as tutorials, a bibliography of relevant literature, etc. The wiki is an excellent place for a new engineer to learn more about dynamic substructuring and for experienced researchers to find sample measurements and FEA models that can be used to test new substructuring concepts.

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