Multi-Shaker Control
A Review of the Evolving State-of-the-Art

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Serious vibration testing has been taking place in the U.S. and around the world for more than seventy years. However, it is only in the past 20 that the use of multiple shakers for testing many kinds of structures, large and small, has been seriously developed. This has required simultaneous development of couplings and fixtures; test specifications such as MIL-STD-810G, Method 527; recommended practices such as DTE-022.1, multi-shaker test and control; and modern control systems that can implement the many new concepts and functions that have been developed. This article looks at many of these developments in light of the currently available MIMO DTE-022 RP and the current MIL-STD, which covers MIMO testing, and suggests ways in which the current recommended practices can be strengthened and expanded to include recent developments

There are many reasons for wanting to employ more than one shaker when performing a vibration test on an important test structure. Some of these are listed in the IEST Recommended Practice DTE-022.1, first published in 2014. These may include the need for more force than is available from a single shaker, the need to control different test profiles at several points on the test structure, or the desire to simultaneously excite the structure in multiple directions.

One of the first attempts to create three-dimensional motion simultaneously took place at White Sands Proving Ground, now White Sands Missile Range, in 1958. Although telemetered vibration data from missile firings was rudimentary at best in those days, it was clear that there was certainly measurable acceleration in all three major axes simultaneously. A system consisting of three small shakers and an aluminum cube, about 10 inches on a side, was assembled. Oil film plates were used to permit simultaneous motion in all axes until the film broke away. This typically occurred at levels over 1 g or frequencies above a few hundred Hz. Although the system proved impractical for testing Army ordnance, some interesting insight into component vector motion was developed.

In the late 1960s, both Chadwick-Helmuth Inc. and Spectral Dynamics Corporation developed "controlled" multi-shaker analog swept-sine control systems. These operated quite well until a major resonance was encountered. Since there was very little, if any, cross-coupling compensation available in these computerless systems, the only strategy available to the controllers was to shut down the drive to the offending control point. (The fact that each drive affects every control point may have been known, but no data to support this was measured, nor could it have been used, even if it was available.)

In one famous test on a quarter-scale model of a Saturn rocket attachment ring, eight hydraulic exciters, each rated at 50,000 lbs force, were employed to create a 1 g swept-sine test to about 150 Hz. When the first resonance was encountered, the control levels started to exceed 1 g. So the shakers were shut down one by one until there was only one shaker still driving, and most of the responses were still over 1 g! The test was considered only a partial success. The background of this test was written by Dick Arone and Paul Brock in 1967.[1]

Modern Control Developments

One of the first published attempts to control two shakers using computer-generated, real-random signals and closed-loop code, was done by Dave Smallwood in 1978 while he was working at Sandia Laboratories in Albuquerque. Since available computers were then very limited compared to today's devices, he took a cautious approach to random control. He first tried to control the magnitudes of the two shakers individually. Then if there was "sufficient CPU time" available, he worked on phase control. If there was "still more CPU time" available, he took a crack at coherence control. This became known as the Smallwood Algorithm.[2] Over the next decade or more, several organizations, including some commercial instrumentation corporations, used this algorithm as the basis for developing multi-shaker controllers.

By the early 1990s Dr. Marcos Underwood had developed and registered several patents on fundamental elements of multi-shaker control. Some of these are shown in the references.

Beginning around 1999, a series of regular improvements and developments in the area of multi-shaker control were being made. Many of these were documented and published as a series of ongoing papers at shock and vibration symposia, aerospace testing symposia, International Modal Analysis Conferences (IMAC) and Institute of Environmental Sciences and Technology (IEST) annual meetings. A review of these papers will form the basis of this treatise on the evolving state-of-the-art of multi shaker control.

Also, on May 1, 2000, the first meeting of the IEST Working Group on Multi-Shaker Test & Control, DTE-022, was held in Newport, RI, in conjunction with ESTECH 2000. The first version of the recommended practice, DTE-022.1, which evolved from this committee, was released in October, 2014.[3]

It has been said that "multi-shaker testing is not for the faint of heart." Part of the reason for this statement is that sometimes, even a carefully designed multiple input/multiple output (MIMO) test may not run. Factors such as shaker orientation and placement, fixturing, attachments to the device under test (DUT), size, shape and center of gravity of the DUT, test levels, seismic mass, etc., can all contribute to the success or failure of the MIMO endeavor. Naturally, a flexible, powerful, control system is also a very important necessity for successfully completing the desired test.

A. Square Control

Almost from the beginning of attempts to perform testing with multiple exciters, the configuration most often attempted was one where the number of control transducers is equal to the number of test exciters. This configuration is termed "square control."

An example of four-shaker square control is shown below.[4] All multi-shaker tests are of necessity described in matrix form. A typical four-shaker, four-control-point matrix may appear as:

$$\begin{Bmatrix} c_1(f) \\ c_2(f) \\ c_3(f) \\ c_4(f) \end{Bmatrix} = \begin{bmatrix} h_{11}(f) & h_{12}(f) & h_{13}(f) & h_{14}(f) \\ h_{21}(f) & h_{22}(f) & h_{23}(f) & h_{24}(f) \\ h_{31}(f) & h_{32}(f) & h_{33}(f) & h_{34}(f) \\ h_{41}(f) & h_{42}(f) & h_{43}(f) & h_{44}(f) \end{bmatrix} \begin{Bmatrix} d_1(f) \\ d_2(f) \\ d_3(f) \\ d_4(f) \end{Bmatrix} \quad (A.1)$$

where $\{C(f)\}$ and $\{D(f)\}$ are the vector of spectra of the respective control and drive signals, and $[H(f)]$ is the matrix of the frequency response functions between the various drive and control channels known as the frequency response matrix.

For a particular four-shaker random test at a 1 g_{rms} test level, the PSDs (Power Spectral Densities) of the four control channels may appear as in Figure A.1.

If the correct test conditions exist, square control is often the first choice in a control strategy.

B. Rectangular Control

Most current multi-shaker, MIMO tests are performed on the basis of a "square" arrangement, in which the number of control transducers equals the number of controlled exciters. However, cases exist where it is desirable to have more control transducers than exciters. This has been termed "rectangular" control.[5]

While there are many tests where multi-axis or even six-DOF simulation are necessary, perhaps the largest current desire for MIMO testing is where two electrodynamic shakers will be used in a single, usually vertical, axis. This is a realistic simulation, for

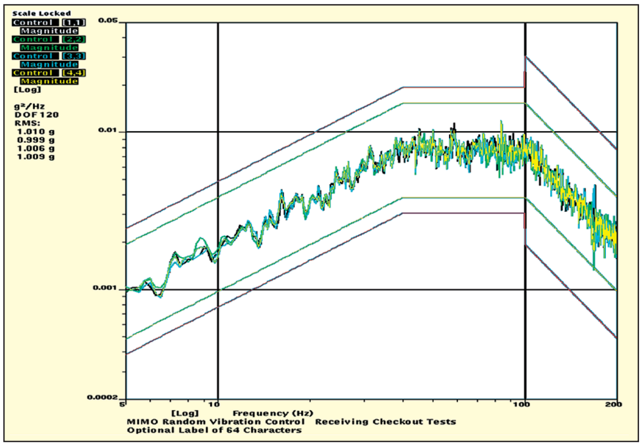

Figure A.1. 4. Control PSDs for typical 1 grms, four-exciter test on square plate.

example, for many missile transportation or airborne applications.

In this type of test the traditional control technique was to use two control accelerometers that were placed either on the two shaker heads, the two attachment points or two locations on the missile near the attachment points. With this type of testing, many response points could also be measured giving a good picture of the dynamics of the entire test system. It is also possible, in this scenario, to define several or many limit channels in conjunction with the two control channels to avoid severe overtesting of the test article.

However, as with any limit channel test scenario, the use of limit control will often reduce the excitation level dramatically and create uneven motion on the test article. One reason for attempting rectangular control, rather than limit control, has been to create a more uniform motion on the shaker head/head expander and on the test article.[5] Recent testing experience on two shaker loaded missile container tests has verified this concept.

The next three figures show the differences between Square Control with Response measurements, Square Control with Limit channels and Rectangular Control, all using the exact same transducers and transducer locations.

Figure B.1 shows excellent square control with two accelerometers, but response measurements that exceed 50 dB in dynamic range. Figure B.2 shows what happens when the two response measurements are used for LIMIT control. Notice that the desired test level is never exceeded, but the range of response channels is still about 25 dB, and the original control channels vary by about 15 dB. Also the test levels at the control locations have been reduced by about 20%.

Figure B.3 shows the result of using four-channel rectangular control. Note that the energy is more evenly distributed among the same four-transducer locations. The two original control locations deviate by about a maximum of 10 dB.

The fundamental control problem, in rectangular control, is to find a drive vector with a spectral density matrix $[G_{dd}(f)]$ that minimizes the following matrix error norm:

$$\|E(f)\|^2 = \left\|\left[G_{rr}(f)\right] - \left[H(f)\right]\left[G_{dd}(f)\right]\left[H(f)\right]^*\right\|^2 \qquad \text{(B.1)}$$

where $[E(f)]$ is the error matrix between the test-specified reference spectral density matrix, $[G_{rr}(f)]$, and the spectral density matrix of the control transducers, $[G_{cc}(f)]$. Note that (Eq. B.1) uses the linear approximation: $[G_{cc}(f)]=[H(f)][G_{dd}(f)][H(f)]^*$ instead. $[G_{rr}(f)]$ is the reference spectral density matrix that has a row and column dimension equal to the number of control transducers (the control signals); $[H(f)]$ is the frequency response matrix between the drive vector and the response of the control transducers, which has a row dimension equal to the number of control transducers (M) and a column dimension equal to the number of shakers (N); and $[G_{dd}(f)]$ is the spectral density matrix of the signals used to drive the shakers (the drive signals), which has a row and column dimension equal to the number of shakers (N). Also it is assumed that $N<M$; i.e., we have more control channels than drive signals.

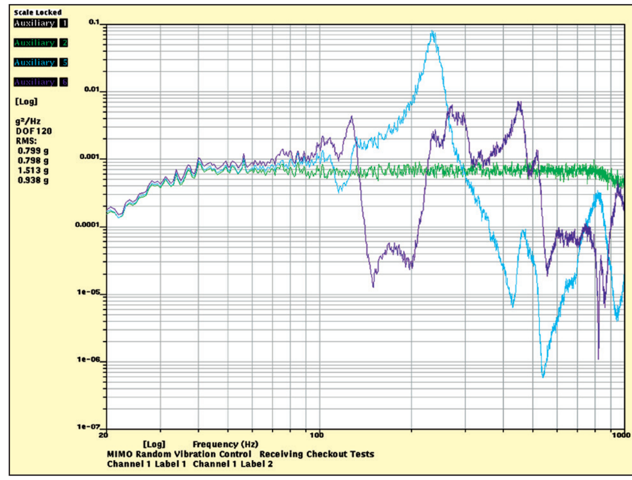

Figure B.1. Control (Ch. 1,2) and response (Ch. 5,6) accelerometers, square control.

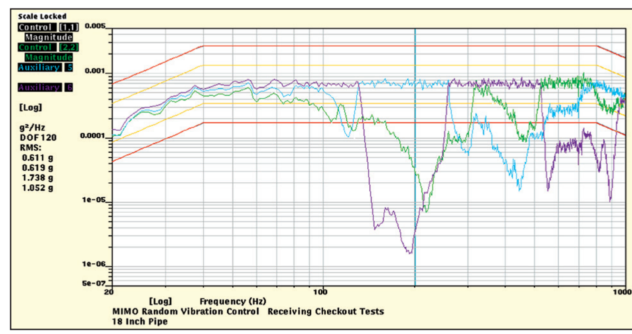

Figure B.2. Control and limit channel responses during square control with two limit channels.

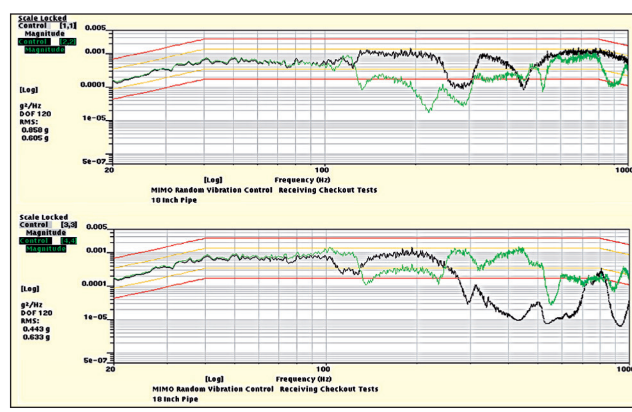

Figure B.3. Control responses during four-channel rectangular control.

C. Coordinate Transformations

Due to the dynamic characteristics of the tables used to couple multiple exciters to achieve multi-degree of freedom (MDOF) test performance and of the test article, the number of desired degrees of freedom for the MDOF operation might not match the number of exciters or control transducers being used. Thus it is often necessary to transform the response from multiple control transducers and the actuation capability of the multiple actuators that are being used from actuator space to MDOF space and vice-versa to effectively perform these MDOF tests.[6]

Often test parameters will be specified in the three translational axes, X, Y and Z. Profiles for roll, pitch and yaw are typically specified or designated to be "kept to a minimum."[6,11,16,19] For many seismic applications, six, seven or eight hydraulic actuators will be attached to the test table, and it will be necessary to transform these actuators and their control points into a series of control and drive parameters suitable for controlling X, Y, Z, roll, pitch and yaw – the classic six-degree-of-freedom parameters.[6] This coordinate transformation, which traditionally was done in hardware, by summing or subtracting control signals, can now be done much more efficiently in software, optimally right in the digital vibration control system.

For an eight-actuator seismic test using eight control accelerometers, a typical Input Coordinate Transformation may look like:

$$\begin{Bmatrix} X \\ Y \\ R_z \\ Z \\ R_y \\ R_x \end{Bmatrix} = \begin{pmatrix} 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & -0.5 & 0 & 0 & 0 & 0 \\ -0.41 & -0.41 & -0.41 & -0.41 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0 & -0.41 & -0.41 & 0.41 & 0.41 \\ 0 & 0 & 0 & 0 & 0.41 & -0.41 & -0.41 & -0.41 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \\ Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{Bmatrix} \quad \text{(C.1)}$$

In Eq. C.1, the responses of four vertical, two longitudinal and two lateral accelerometers are being "mapped" into X, Y, Z, roll, pitch and yaw control vectors.

While this is the classical application of using the transformation to create standard 6 DOF controls, there are other applications that can benefit from such transformations.

$$\begin{Bmatrix} z \\ r \\ p \\ t \end{Bmatrix} = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & -0.25 & 0.25 & -0.25 \\ 0.25 & -0.25 & -0.25 & 0.25 \end{pmatrix} \begin{Bmatrix} A1 \\ A2 \\ A3 \\ A4 \end{Bmatrix} \quad \text{(C.2)}$$

In Eq. C.2, the 4 ×4 matrix is an example of mounting four vertical accelerometers on a square plate and creating four control vectors – vertical, roll, pitch and torsion. This can be incredibly powerful if reducing torsional motion is an important part of the test requirement.

Figure C.1 shows the control result of using the 4 × 4 control matrix to help reduce the plate torsional motion. The vertical control from four-corner mounted accelerometers is virtually perfect at 1.000 g_{rms}, and the torsional motion has been suppressed by more than two decades compared to the vertical motion. Also, as shown in Rererence 6 and Figure C.2, reducing torsional motion also had the effect of greatly improving the controlled phase and coherence between the control transducers.

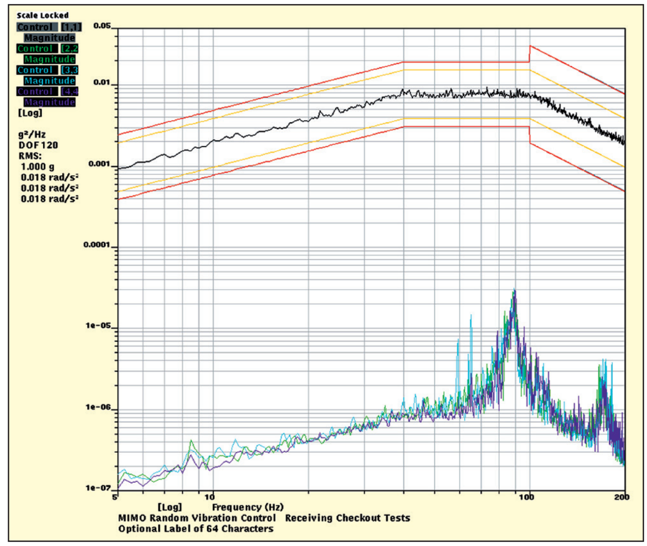

Figure C.1. Vertical, roll, pitch and torsion control vector components; 4DOF control.

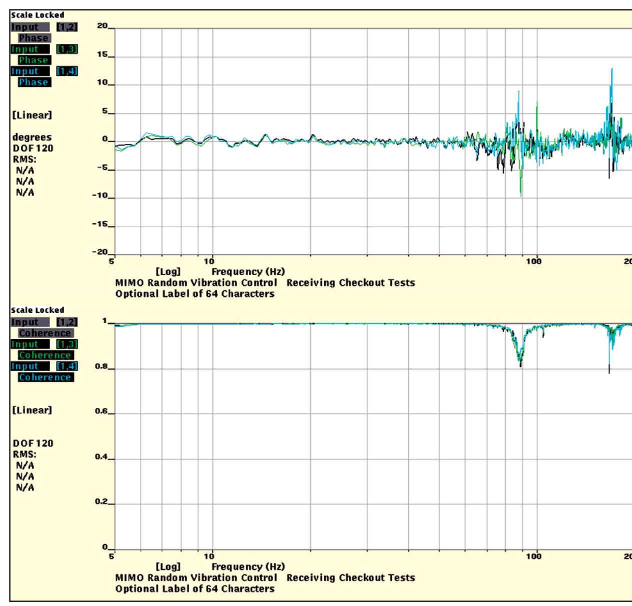

Figure C.2. Phase and coherence between four vertical accelerometers for 4DOF control.

D. Spectral Density Matrix

The spectral density matrix (SDM)[7] is the generalization of the PSD or ASD (Auto Spectral Density), which is a frequency-domain function to a matrix of frequency domain functions. Just as the PSD or ASD, which is a positive function of frequency that characterizes the intensity or power that an individual stationary random process may have as a function of frequency, the SDM with its diagonal elements also characterizes the intensity or power that a vector of stationary random processes, like the control-response and drive signals associated with a MIMO test, have as a function of frequency, where each diagonal element is a PSD or ASD of the particular control-response and/or drive signal.[7,24,25]

In the MIMO test setting, as contrasted to single-shaker testing, the relative phase and coherence (joint characteristics) between the respective control-responses and/or drive signals are very important, which are characterized by the off-diagonal elements of the SDM as a function of frequency. These joint characteristics are the relative coherence and phase between each of the control-response and/or drive signal vector elements. In this manner, the SDM characterizes both the temporal and spatial characteristics that a vector of stationary random processes like the control-response and/or drive signals has. The SDM's ability to describe the overall spatial motion is used in such areas as MIMO random control, operational deflection shape (ODS) and experimental modal analysis (EMA). We'll provide an example in what follows of how this information can be used to visualize the motion that an SDM describes.[7,24.25]

The cross-spectral density matrix (CSDM), which is also important for MIMO theory, is the generalization of the cross-spectral density function (CSD) to matrix terms. Just as the CSD, which is a complex function of frequency, characterizes the joint "power" or intensity that two stationary processes have as a function of frequency, the CSDM also characterizes the joint power that two sets of stationary processes have as a function of frequency. These characterize the statistical similarity that these two sets of random processes have, both in the temporal as well as in a spatial sense. It contains information that allows us to understand if there is a linear system between these two sets of processes and also to determine what the frequency response matrix, $[H(f)]$, between them may be.[7,24,25]

The two matrices, SDM and CSDM, have numerous other applications like allowing us to determine other quantities of interest such as the partial and total coherence[8] that may exist between two sets of random processes; e.g. the drive vector used to excite a structure and the control-response vector that thus results. In a MIMO control setting, one is typically concerned with the SDMs of the control-responses, $[G_{cc}(f)]$, and of the drive-signals, $[G_{dd}(f)]$. The CSDM of interest in that case is the CSDM between the control-response vector and the drive-signal vector, $[G_{cd}(f)]$. For MIMO linear systems, the various SDM and CSDM matrices are related by the following generally useful matrix equations:[7,24,25]

$$\left[G_{cd}(f)\right] = \left[H(f)\right]\left[G_{dd}(f)\right] \quad \text{(D.1)}$$

and

$$\left[G_{cc}(f)\right] = \left[H(f)\right]\left[G_{cd}(f)\right]^{*} \quad \text{(D.2)}$$

where $[H(f)]$ is typically the unknown frequency response matrix that characterizes the linear output characteristics of the MIMO system under test, and $[G_{cd}(f)]^*$ represents the complex conjugate

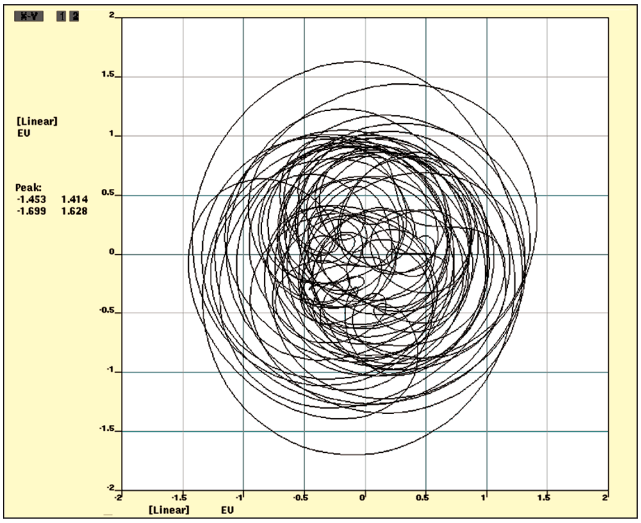

Figure D.1. Response X and Y plotted against each other with 90° between each other.

transpose of the matrix $[G_{cd}(f)]$. These equations can be used for system identification of the unknown $[H(f)]$ associated with the system under test and are the matrix generalizations of Type 1 and Type 2 FRF estimations associated with obtaining ordinary frequency response functions to be used with experimental modal analysis. Eq. D.1 is sensitive to noise in the $[G_{dd}(f)]$ estimation, and Eq. D.2 is sensitive to noise in the $[G_{cc}(d)]$ estimate, which is why Eq. D.1 is recommended as the system ID for MIMO control.[7,24,25] As an example from Ref. 7 of the relative coherence and phase between response signals in a MIMO random vibration test, consider the case of a bi-axial accelerometer mounted at the center of a square table to be tested with pre-specified X and Y axes MIMO random excitation. Also assume that that we have two shakers mounted horizontally so that they can excite the table in the X and Y axis in such a way that the shakers can operate independently from each other, perhaps through the use of hydrostatic bearings. Assume also that the coherence between the signals obtained from the X and Y accelerometers have a coherence of nearly 1 and a relative phase of 90^o.

In that case, the motion will be nearly circular, in the X and Y plane, as shown in Figure D.1, which is a Lissajous between simultaneous X and Y time histories. If the relative phase is 0, then the motion will be along the diagonal between the positive X- and Y-axes. If on the other hand, the relative phase is 180^o between them, then the motion will be along the diagonal between the negative-X axis and the positive-Y axis.

If we consider the previous cases, but nearly zero coherence between the signals obtained from the X and Y accelerometers, then the motion will tend to fall within a circle in the X-Y plane, but with the predominant motion occurring with a probability given by a two-dimensional Gaussian distribution. It's unlike the case of coherence of nearly 1. In that case, the motion occurs along the circumference of a circle, while in the case of 0 coherence, the motion occurs within the interior of a circle, of radius 6σ, where σ represents the RMS of the signals coming from each accelerometer that has been integrated twice to obtain the associated displacement at each accelerometer location. Figure D.1 shows the time history acceleration-response of a high-coherence case with 90^o between responses for the bi-axial accelerometer test-case that we've discussed.

E. Ordinary and Partial Coherence

As was discussed in Ref. 7&8, the ordinary coherence, at a given frequency f, and between the i^{th} and j^{th} time histories, is given by:

$$\gamma_{ij}^2(f) = \frac{\left|g_{ij}(f)\right|^2}{g_{ii}(f)\,g_{jj}(f)} \quad \text{(E.1)}$$

In this manner, the off-diagonal elements of the SDM, $[G_{XX}(f)]$, contain the relative phase and coherence between the elements of the vector of time histories: $\{X(t)\}$. So the off-diagonal elements of $[G_{XX}(f)]$, in combination with its diagonal elements, can be simply used to determine the ordinary coherence between the elements of $\{X(t)\}$, as described in Ref. 7&8.

However, when dealing with the case where a selected response time history results from multiple driving-time histories, ordinary coherence analysis is not appropriate, because ordinary coherence cannot distinguish between the effects of multiple inputs on a single response signal and treats these other effects as noise, which can produce unreasonably low values for coherence. So ordinary coherence may not always be suitable to determine the degree of linear dependence between two time histories if there are other effects on the selected time histories. In these cases, it may be better to consider the partial coherence between two time histories as a measure of linear dependence between two time histories. (Ref. 8 and its references have a good discussion on partial and ordinary coherence and its applications.)

As an example of how these many complex parameters interact in a MIMO control test, consider Eq. E.2:

$$\gamma_{c_1,d_1\cdot d_2}^2(f) = \frac{\left|\sqrt{\gamma_{c_1d_1}^2(f)}\,e^{j\theta_{c_1d_1}(f)} - \sqrt{\gamma_{c_1d_2}^2(f)\,\gamma_{d_1d_2}^2(f)}\,e^{j\left(\theta_{c_1d_2}(f)-\theta_{d_1d_2}(f)\right)}\right|^2}{\left(1-\gamma_{d_1d_2}^2(f)\right)\left(1-\gamma_{c_1d_2}^2(f)\right)} \quad \text{(E.2)}$$

As can be seen, Eq. E.2 simply relates the ordinary coherences between control-response #1 and drives #1 and #2; the ordinary coherence between drives #1 and #2; and the relative phase between the drives and between the drives and the control-response to the partial coherence between control-response #1 and drive #1.

Understanding the importance of each of these parameters during a MIMO random test may not be intuitively obvious, but each can contribute to a successful multi-shaker test.

F. Response Limit Control

Control Constraints. In many multi-shaker and multi-axis MIMO vibration tests, many of the shakers may be restricted in what drive motions they can impart upon the common structure to which they are all connected. In many cases, some of the shakers are arranged in push-pull and push-push orientations with respect to the common structure to which they are connected. In some cases, the shakers are connected in an over-determined manner, where there are more shakers than there are rigid-body degrees of freedom.

For example, in a typical four-shaker, single-axis testing system, the shakers are arranged as shown in the Figure F.1. The shaker arrangement is over-determined, since there are more shakers than available rigid-body degrees of freedom. There are three possible rigid body degrees of motion in this case: pure Z, roll and pitch motions. If the test objective is to conduct a single-axis test, then there are additional constraints, where the responses of the control channels are required to be in phase and with an ordinary coherence of nearly 1 between the control response signals. If these structural constraints are not maintained, then there is potential for damage to the system under test.

Furthermore, if only one of the drive signals were to be reduced, to reduce the response of a given limit channel, then a "rocking" condition coupled with bending forces would occur on the square aluminum plate structure that connects all four shakers together, perhaps exceeding some overturning moment constraint. Depending on the magnitude of the various drive signals that are involved, damage could occur to: the armatures of the shakers; the plate; the attachment mechanisms; or all of the subsystems.

Even more complex interconnections of shakers and the system under test are possible.[9] So generally response-limiting control can't be achieved by simply reducing the drive amplitude to one of the shakers to limit the response at a particular limit response location.

Because of situations like what have been discussed, as well as other test conditions that require important structural constraints be maintained, there is a need that a more general and better-understood, limit-control method be provided. To satisfy these inherent constraints on the motions that are possible, the chosen control method must maintain the control-response to prespecified phase and coherence between the control points on the system under test.

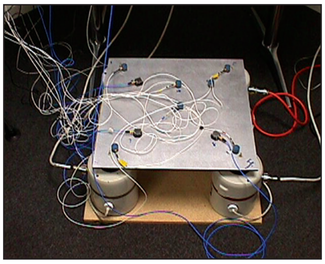

Figure F.1. Square aluminum plate driven by four small vertical shakers.

Control Method. In view of the previous concerns, an effective means for the control of limit channels was found; one that does not violate important structural constraints. It was determined that to preserve the prespecified coherence and phase relationships between control-channels, it was sufficient to reduce the magnitude of either the reference vector or the reference spectral density, which is being used for control, at those frequencies where a limit level is being exceeded. Additionally, this limit control method can be used for both MIMO random and MIMO sine applications, but in the interest of simplicity, only MIMO random results will be presented.

In other words, the presented control method reduces the magnitude of the reference the controller is using in proportion to the particular limit exceedance, thereby reducing the response at the limit-channel location but without violating the phase and coherence specifications that are present in the reference spectrum or spectral density matrix test specification. So the approach provides the following benefits:

- The phase and coherence relationships of the control channels are preserved.
- The method reduces the drive energy that is being sent to the test article.
- The response at any given limit channel location is reduced to just below the respective limit channel's response limit
- The entire system matrix is updated in each control loop in which limiting occurs.

The basic idea of this control method is to have two control loops that:

- Monitor *in real time* the limit channel responses and reduce the reference spectrum within the frequency range or ranges where a particular limit-channel response exceeds its respective limits
- Monitor the control responses and compare them to their respective references and adjust the drive amplitudes to maintain proper amplitude, phase, and coherence control (if applicable).

It has been found and field proven that the described control method is effective in maintaining limit control of one or more limit channels while maintaining proper amplitude, phase, and coherence control and without violating any important structural constraints.

Figure F. 2 includes a powerful and unique display of *limit channel* vs. frequency. This display, which can have more than 90 channels included, shows at a glance which limit channel is having the most influence on test control at every frequency.

Conclusions.

- Excellent phase and coherence control can be maintained when using limit control in conjunction with multi-shaker square control.
- This can be crucial in applications where there are important structural constraints or when there are more shakers than there are allowable degrees of freedom.
- Rectangular control, with or without limit control, distributes energy and phase.
- This can give it a response-limiting characteristic that is related to force response limiting with an appropriate choice of control channels.
- Shaker drive phase is more affected by rectangular than by limit control, because limit control explicitly maintains the prespecified phase and coherence between control channels, while rectangular control maintains the prespecified phase and coherence only in the least-mean-squared sense.
- In either case, limit control during MIMO testing can be very effective.

G. Controlling Unusual Waveforms

The implementation of multi-exciter testing has evolved around the world into two basic approaches for conducting tests:[10]

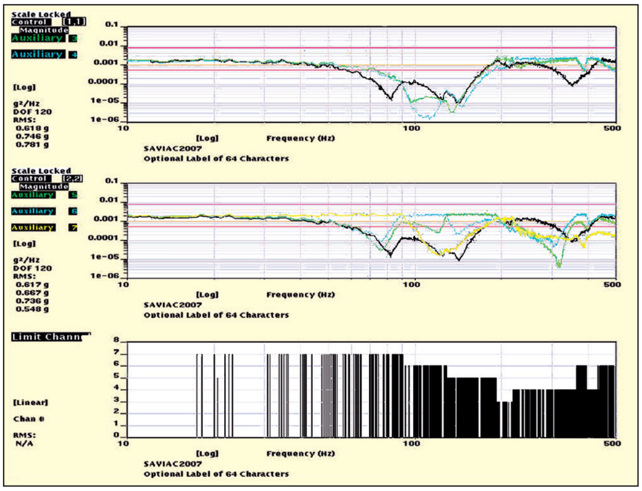

Figure F.2. All channels for square control and five limit channels.

- The use of multiple exciters and multi-channel time waveform replication using measured field data as the inputs. Care must be taken to assure that a realizable test setup has been implemented.
- Where real field data are not available, multi-shaker tests can be created by designing test profiles for random, swept sine or shock signals.

The design of such tests must be done on a matrix basis and care must be taken that physically realizable phase and coherence values are specified, in addition to achievable magnitudes. The design of many such tests follows the concepts of some single-shaker, SIMO, testing with the introduction of logical, achievable phase values.[10]

With the rapid evolution of multi-shaker testing along the above two lines, there has also evolved the need to design multi-shaker test profiles that mirror such single shaker tests as sine on random or swept random on random. Those who have conducted single-shaker tests using swept-sine-on-random or mixed-mode profiles know the difficulties that can be encountered in trying to accurately control moving and sometimes overlapping sine tones and harmonics. Now expand this concept to multiple shakers and even multiple axes and throw in the need for realizable and achievable phase control with cross-coupling compensation. Or consider a tractor-trailer hauling containerized missiles over a highway or partially paved road and experiencing potholes of various sizes. This could create the need for a combination of a shock pulse on random or a shock pulse on top of a moving sine tone at multiple locations.

Although the first choice is to design a multi-exciter test to measure field data as much as possible, this is not always practical. For example:

- Cross properties, such as phase, may not have been measured, and all you have is a set of PSDs.
- The field excitation is something like continuous wind loads on a wing-mounted missile, which does not lend itself to convenient, known, excitation particulars.
- The field data are really nonstationary such as with certain gunfire or tracked-vehicle excitations.

So there is an evolving need[10] to be able to create complex waveforms, which may involve combinations of random, swept sine, fixed sine, harmonics, shock pulses and sweeping bands of random. These signals would have to be created in such a way to include achievable magnitudes, phases and in the case of random, coherence values for a combination of shakers, fixtures and test articles. Control transducer placement may prove critical and could be patterned after actual flight, seaborne or road transportation measurements.

Once a set of such waveforms is defined a "reference" signal set could be created that may last for minutes or even hours. This data set would then form the reference waveforms for use in a time waveform replication, multi-shaker test. Using a MIMO test scenario, the creation of just such a set of waveforms can be called "MIMO Waveform Generation." Ref. 10 describes just such a testing

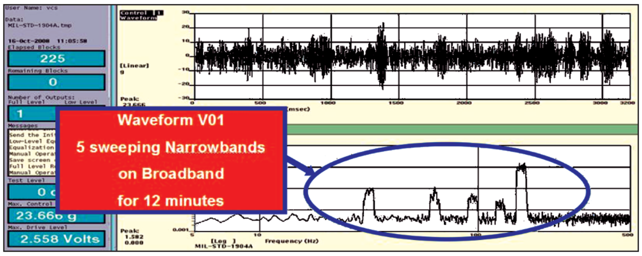

Figure G.1. Five sweeping narrowbands on static broadband MIL-STD-1904 compliant waveforms.

tool and gives some examples of developing required waveforms and using them to control multiple exciters attached to complex test hardware.

Figure G.1 shows how a MIMO waveform generator is used to create sweeping narrowband on random waveforms that are compliant with MIL-STD-1904A.

H. MIMO Testing Methods

This article discusses some MIMO testing that was performed to compare the results that are possible with: under-defined square control, optimal square control, rectangular control, and I/O transformation control. Two types of test profiles were used for this study: Haystack references for X, Y, and Z; and MIL-STD 810G profiles for X, Y, and Z.

In all of these tests, the objective is to drive the X, Y, and Z axes so that the respective axis response agrees with their prespecified reference PSD and so that the respective axis responses are incoherent with each other.

The tests discussed in Ref. 11 were conducted using MIMO random because of the increased visibility that this testing methodology provides in assessing performance. The focus was in determining what the performance envelope of the Team Model III Cube was in this application and the levels at which it could run a 3D version of the newest MIL-STD-810G composite-wheeled profiles that are discussed in its section: METHOD 514.6 ANNEX C, for the traditional one-axis-at-a-time testing. We knew that we would be pushing the physical limits of the Cube employed for this test sequence and were interested to see if we could optimize various parameters of the tests to further enhance the Cube's testing capabilities.

A further focus was to see what type of 6-DOF testing performance could be obtained with the Cube and the use of MIMO control. For this, we decided to compare the test results obtained with the use of square control, both under defined and optimal; rectangular control; and I/O transformation control. Ref. 11 also provides much-needed discussion on the pros and cons of the various MIMO testing methodologies and the trade-offs that each entails.

I. High-Kurtosis Replication

Ref. 12 discusses the use of waveform replication to accomplish high-kurtosis random vibration tests. Since the central limit theorem restricts the occurrence of high-kurtosis responses at other than control points, a nonstationary method, such as waveform replication, should be used for these tests. (Ref. 12 discusses the effects of the central limit theorem and the many trade-offs associated with this type of testing.)

These test results from Ref. 12 lead to the following conclusions:

- Multi-axis vibration testing must consider Φ and γ^2 values as well as PSD, since they could severely influence g_{rms} values at noncontrol locations.
- If nonstationary road data with high kurtosis are run on controlled SIMO or MIMO tests using waveform replication, the high kurtosis will propagate to noncontrol test article locations, because the drive signals are not stationary.
- If high kurtosis values are added to stationary random signals and controlled with waveform replication, control locations can reproduce the high kurtosis. But noncontrol locations will tend to exhibit Gaussian amplitude distributions with greatly reduced kurtosis due to the effect of the central limit theorem.

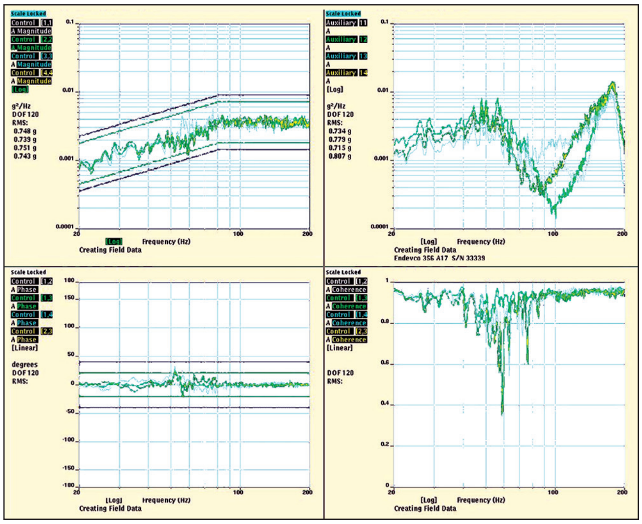

Figure J.1. Control magnitude, phase, coherence and four response readouts from Test 1.

- For these and many other reasons, great care must be taken when specifying the use of waveforms with high kurtosis for SIMO and MIMO testing

J. Using Field Data; Random

Figure J.1[13] shows the result of the square control test #1 of a plate driven by four vertical shakers at each of its four corners. Each of the control points have the same test g_{rms} level, within less than 2% and the same general PSD shape. The auxiliary response channels show some general plate resonant activity near 180 Hz and still show g_{rms} levels within 8% of the four-corner test levels. These results are for high-coherence and zero-phase test conditions.

To demonstrate how random data can be converted to an SDM reference, the data from the center auxiliary accelerometers were acquired and stored on a local storage for further processing to simulate field data. The data were then analyzed, and a resulting SDM of the recorded data was obtained. This "field data" SDM was then imported into a Jaguar MIMO control system to be used as the reference SDM for test #2.

Figure J.2 shows the test results from using the SDM obtained from "field data" as the reference for MIMO random test #2. The SDM obtained from our field data matches its reference SDM very well.

Figure J.3 compares the individual control PSDs from test #2, which uses the measured field data SDM from the center of the plate as a set of new reference values, and auxiliary channel PSDs measured on the center of the plate during test #1. This is demonstrated by its four overlaid plots that show: (1) control [1,1] with auxiliary 11; (2) control [2,2] with auxiliary 12; (3) control [3,3] with auxiliary 13; and (4) control [4,4] with auxiliary 14; where the control channels are from test #2 and the respective auxiliary channels are from test #1.

In all cases, the test RMS levels are within 1% of the original measured response levels. This shows that the field data were "replicated" faithfully in a MIMO random sense. Additionally, looking back at the original control locations, 1 and 3, the reciprocal measurements for these locations during test #2, show an RMS value that is within 5% of the original PSD data, indicating fairly linear system behavior over the test frequency range and supporting the fact that the field data were reproduced faithfully.

Conclusions.

- Multi-exciter, MIMO, tests can be developed either from measured field data, which may have to be tailored, or by defining physically realizable control profiles. For the case of a random test, the SDM test definition should include supportable values of magnitude, phase and coherence. Match field boundary conditions as best possible, modifying test requirements as needed.
- If: 1) the measured field data includes the cross spectral densities between control points (coherence and phase), 2) a controller capable of matching the entire SDM is used, 3) the boundary

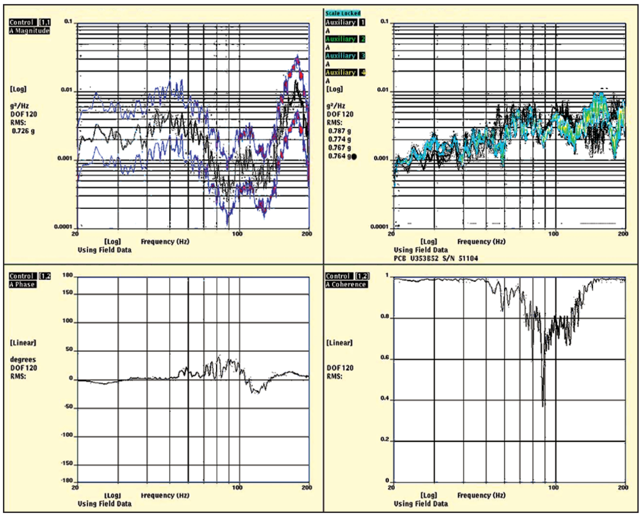

Figure J.2. Control and measurement results for "swapped" control points – Test 2.

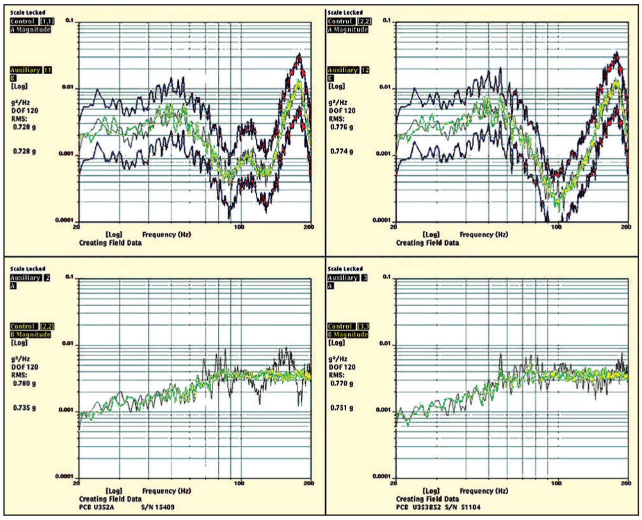

Figure J.3. Overlay of Tests 1 and 2 shows excellent dynamic reciprocity.

conditions in the field are suitably matched in the lab, and 4) the system under test is approximately linear, then other response (noncontrol) points on the structure that were present when the data were gathered, will respond with good agreement to what they were when the field data were acquired for the chosen control points.

- This allows locations that are not situated conveniently for instrumentation or control purposes to be so controlled indirectly by instead controlling other related locations and using dynamic reciprocity to cause these inconveniently located response points to also respond as they do in the field. The test results[13] show that this result is ensured if the previous conditions are satisfied.
- If the measured field data *do not* include the cross-spectral densities between the control points, then even if the remaining above conditions are satisfied, the other response (noncontrol) points on the structure, that were present when the data were gathered, will not respond in agreement to what they were when the field data were acquired for the chosen control points and may be significantly larger.

K. Using Field Data; Time Waveform Replication (TWR)

Although multi-shaker testing has been attempted for more than 40 years, it is only in the past 8 to 10 years that the correct combination of instrumentation, computers and software has become available to make this technique a reality. In the absence of a set of comprehensive multi-exciter test specifications, many organizations have tried to take the advice of MIL-STD-810 and use measured or borrowed field vibration data, to create a laboratory simulation. Ref. 14 has indicated that when using actual measured field vibration for multi-exciter simulation, care must be taken to assure good phase and coherence duplication or unexpected results may occur. So it is up to the test engineer to assure that he has taken into account, or at least considered:

- Quality of the multi-channel test data to be used for replication.
- Boundary condition differences, field to lab.
- Number and location of lab shakers.
- Test lab fixture arrangement and possible effect on system dynamics.
- Control approach to be implemented: square, rectangular, overdetermined etc.
- Position and number of control transducers.
- Approach to error analysis for proof of performance.

Ref. 14 covers many of these considerations with examples where possible. It discusses the use of field data with time waveform replication testing, which may be unfamiliar.

Conclusions.

- TWR control performance, while employing the field response data discussed above, was very good, with waveform errors less than 1% in virtually all cases.
- Comparing reference and control waveforms during TWR testing, is an extremely sensitive procedure, much more so than, for example, comparing PSD levels. So extra precautions should be taken to assure that factors such as loose connections or slapping are not present during the test.
- The use of adaptive control during TWR testing is very effective in addressing time varying and nonlinear characteristics in the test setup.
- Satisfying field boundary conditions in the lab, as much as possible, may have the largest single effect on assuring a quality laboratory simulation.
- More work needs to be done on selecting a range of acceptable error metrics when performing TWR tests. However, the point-to-point error comparison used in this article is the most severe.
- There should some consideration for using spectral methods.
- Measuring and comparing noncontrol points in the field and in the lab during TWR tests will help assure a quality TWR simulation.

L. Maintaining Positive Definite Characteristics

MIL-STD-810G, Method 527, recommends use of measured field data for the basis of MIMO random tests. After the measured data have been recorded, processed and formatted into an acceptable spectral density matrix (SDM) for MIMO random testing, the resulting SDM may not be positive definite (PD) or positive semi-definite (PSD). If it is not PD or at least PSD, a physically unrealizable test problem is created.

Remedies for this problem are needed. Ref.15 examines what the mathematical requirements are for such SDM matrices and composites to be PD or at least PSD. It describes what types of adjustments can be made to such matrices to satisfy those conditions. Ref. 15 also presents spectral averaging procedures that minimize the possibility of such SDMs being indefinite. Related data format presentations will be discussed that can also minimize the possibility that an imported SDM be Indefinite.

M. Overcoming Overturning Moments in Control of 6-DOF Satellite Testing

MIMO swept sine testing for satellites has so far been primarily limited to vertical testing using four shakers. In those cases, the requirement is to keep the four Z responses in phase and at the same level. Examples have been used in Japan, and at ESTEC in the Netherlands.[16] However, only the installation in Japan used modern MIMO swept-sine control, while the installation at ESTEC used the older idea of analog phase control to keep the armature currents in phase, which may not be able to operate correctly past the first flexible body resonance. This article discusses the first known use of full 6-DOF MIMO swept-sine control for this purpose. Swept sine control needs to use high-performance digital tracking filters, which are described in Ref. 16 and its references.

Starting in December 2010, Thales Alenia Space Italia (TAS-I) issued a contract to a local company in Rome to construct a new concept, 6-DOF, MIMO hydraulic shaker system, which is called the MultiShaker system. This was initially reported by Di Pietro, Olivierio, and Tiani from TAS-I; and Belotti and Rinaldi from Belotti Sistemi at the 27th Aerospace Testing Seminar in October, 2012. There were several purposes for embarking on this project:

- A system was needed to provide sufficient excitation force commensurate with requirements that were coming on line in the near future.
- Typical "final qualification" tests are performed up to 100 Hz, so it was not necessary to employ larger electrodynamic shakers with 2-kHz capabilities.
- Testing in all three axes, even if done sequentially, should be able to be performed with a minimum of satellite repositioning or moving (preferably no repositioning at all).
- As discussed in Ref. 16, there is a need to test satellites with a high center of gravity with respect to the vibration platform. These test articles create large overturning moments during horizontal testing that in turn cause excessive off-axis acceleration responses. So, an effective means to compensate, either passively or actively, for these excessive off-axis responses is needed.
- MIMO control, in conjunction with a suitably designed multi-shaker system can provide the capability to actively reduce excessive off-axis responses. This creates a firm requirement for true 3-DOF or even 6-DOF testing capability.
- Using multiple exciters in orthogonal directions is shown to be an excellent way to reduce unwanted excessive off-axis responses when horizontally testing satellites with a center of gravity meters above the test-table platform by actively resisting the high-overturning moments that can result.
- A carefully designed hydraulic excitation system can support significantly more weight, with a much smaller footprint than an "equivalent" ED (Electro-Dynamic) system.

As part of acceptance testing, full-scale, Y-axis testing was performed at a 1.04-g peak control level with X and Z axes set to levels 26 dB lower. Testing was performed at four octaves per minute sweep rate from 5 to 100 Hz. In Figure M.1, the middle traces, Y, show excellent 1.04-g control for all four Y-axis control transducers, channels 5 to 8, which are located at points P1, P2, P3, and P4 and shown in Figure 2b of Ref. 16. The upper four traces, X, and the lower four traces, Z, behave very much like Y and Z did during X-axis testing shown in Figure 8 of Ref. 16. Some visible rocking was evident in the 40 to 50 Hz range, again due to the fact that the Z actuators are inefficient around 42 Hz with respect to pushing on the seismic base.

Figure M.2 shows the phase of the drives used to drive the eight actuators during Y-axis testing. The upper traces show the drive phases for the X axis, the middle traces show the drive phases for the Y axis, and the lower traces show the drive phases for Z axis. Note that the Z-axis drives are now out of phase in pairs Z1, Z2 and Z4, Z3 with respect to each other but are now out of phase with respect to the roll responses shown in Figure 7 of Ref. 16. These pairs of out-of-phase drives are what is largely responsible for significantly reducing the off-axis responses, since they induce antirolling to cancel the rolling induced by the high-overturning moments that the Y-axis excitation causes due to the high center of mass of the test article.

Ref. 16 should be consulted for the other test results that were obtained as well as a more complete description of the multi-shaker design and of the dummy mass tested.

N. Applications of MIMO Coherent Output Power

Ref. 17 discusses extensions to SIMO-coherent output power analyses that provide estimates for MIMO-coherent output power (MCOP) and applications. There is also a matrix that plays the role of ordinary coherence that is developed and discussed in Ref. 17. The use of MCOP analysis depends on the relationship:

$$\left[G_{\hat{c}\hat{c}}(f)\right] = \left[G_{cd}(f)\right]\left[G_{dd}(f)\right]^{-1}\left[G_{cd}(f)\right]^{*} \tag{N.1}$$

where $\left[G_{\hat{c}\hat{c}}(f)\right]$ is the coherent SDM due to the drive SDM $[G_{dd}(f)]$. The MCOP analysis method is to compare corresponding diagonal terms of $\left[G_{\hat{c}\hat{c}}(f)\right]$ and $[G_{cc}(f)]$ for frequencies between f_{min} and f_{max} and to f_{nyq} for aliasing. Other frequency ranges can be used to spot areas that may be caused by non-linear effects of the actuators, fixtures, bearings, and other such linkages. In general, the MCOP analysis method can detect nonlinear responses, uncorrelated noise, and aliasing that may be present in $[G_{cc}(f)]$ but not in $\left[G_{\hat{c}\hat{c}}(f)\right]$.

Measured drive SDMs and control-drive CSDMs can be used for MIMO-coherent output power analysis. These matrices are used to estimate the coherent control SDM: $\left[G_{\hat{c}\hat{c}}(f)\right]$. This SDM can be used to estimate response components of $[G_{cc}(f)]$ that are incoherent with the drive's SDM $[G_{dd}(f)]$. These drive-incoherent responses may be due to: measurement noise, nonlinear response components, and/or aliasing. The method is simple to use, since it can be performed post-test with the drive and control response vectors from a MIMO random test, by a straightforward comparison of corresponding diagonals from $[G_{cc}(f)]$ and $\left[G_{\hat{c}\hat{c}}(f)\right]$.

O. Use of Quaternions to Compensate for Geometric Distortion

Seismic testing sometimes requires significant roll, pitch, and yaw rotations from their MDOF vibration test systems when reproducing certain reference waveform vectors on seismic tables that are associated with particular seismic environments. When these reference waveforms (typically accelerations) are used for such testing, significant rotational motion of the test system's shake table can occur. As a result, the X, Y, and Z control transducers that are mounted on the surface of the table will no longer be aligned with the world (fixed) coordinate system as the table rotates about X, Y, and Z, but rather with respect to the shaker table's body (moving) coordinate system.

This lack of alignment with respect to the world coordinate system in turn causes distortion in the output of the control accelerometers that are required to control such MDOF tests. These errors in turn can cause further errors in the control system's ability to reproduce the motion specified by the test's reference waveforms with respect to the world coordinate system as required by the seismic test specifications. Similar problems will also occur in other applications of MDOF testing[16] with reference acceleration waveforms that similarly encounter significant roll, pitch, and/or yaw rotations with their tests. Ref. 19 presents methods based on the use of quaternions to compensate for the resulting control transducer measurement distortions.

The natural question that arises, why use quaternion methods instead of the more traditional matrix methods? Unfortunately, methods that use matrices to represent rotations, though simple, have several intrinsic problems[19] that are due to certain singularities and numerical problems that are inherent in their formulation. For this reason, the method used to correct the accelerometer measurements[19] will not use these matrices directly. Instead, the method will implement the required rotations of the frames of reference using results from quaternion algebra.[19]

To fix these problems we need an approach for the representation of rotations so that:

- It yields a method that is easy to normalize and that does not suffer excessively from numerical sensitivities or singularities in its representation of rotations.
- Provides for a method that allows us to perform linear interpolation of rotations in the correct space: the space of orthogonal linear transformations, which consists of only rotations.

Ref. 19 presents methods for dealing with these problems from research that has resulted in the consensus that the use of quaternions to represent rotations provides such robust methods. This is the primary reason we've also chosen to use quaternions to implement the needed rotational transformations to correct the accelerometer readings for the effects of the rotation of the shake table (aligned with the world axes).

P. Using MIMO for Direct Field Acoustic Testing

High-intensity acoustic testing is a critical component of acceptance testing for satellites and other space hardware. Ref. 18 discusses how MIMO control is used to spectrally shape the acoustic

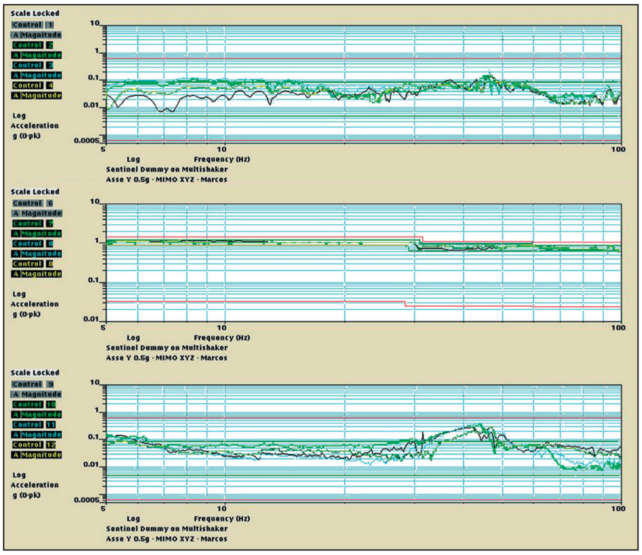

Figure M.1. Testing in Y-axis only at 1.04 g, with X (top), Y (middle), Z (bottom) results shown.

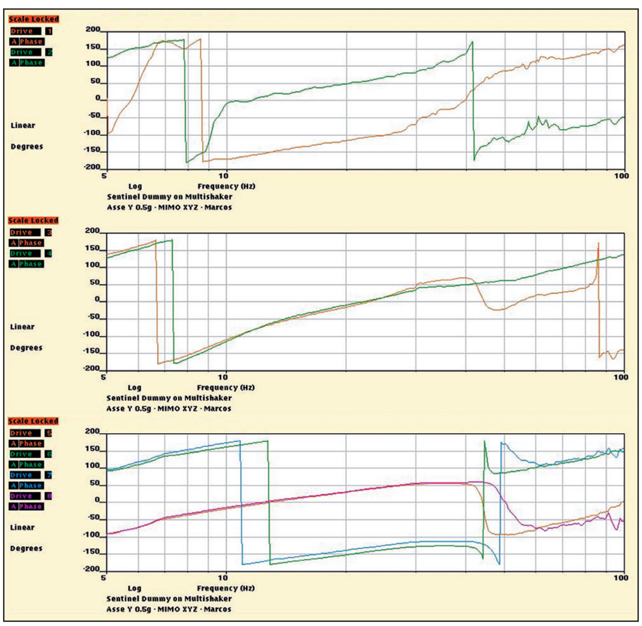

Figure M.2. Testing in Y-axis only, at 1.04 g, with drive phases for: X (top), Y (middle), Z (bottom) shown.

response of multiple microphones to direct field acoustic excitation from multiple loud-speaker stacks. The reference also discusses their relative coherence and phase, subject to the acoustic modal constraints as a result of chamber dimensions and microphone and speaker stack placement.

Q. Conclusions

As can be seen by the presented material, we have come a long way technologically and have also developed a reservoir of shared experience. We hope that the presented excerpts from past literature on the subject of multi-actuator control and applications will prove helpful to users of these new technologies and empower them to make contributions to this field.

We intend that this article be used in conjunction with both the latest revision of MIL-STD-810G and the current release of Recommended Practice DTE-022 Multi-Shaker Test and Control. The MIL-STD describes single and multi-shaker tests that must be performed under certain conditions. The recommended practice gives a broad look at many things that should concern the test engineer facing perhaps his or her first MIMO test. But we touch on many things learned from actual experience gleaned over the past 20 years or so. The combination of all three resources can only help test engineers, either experienced or new to MIMO, in performing successful, meaningful environmental simulations.

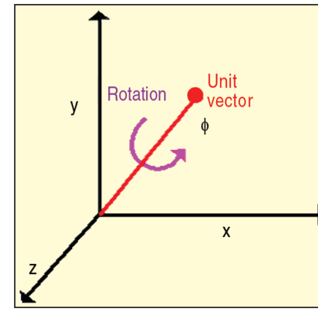

Figure O.1. Rotation about a unit vector.

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