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A Note on Sonic Fatigue Life Estimation

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Structures such as those of engine test cells or aerospace components may be exposed to extended periods of intense sound. It is of interest to obtain an estimate of the fatigue lives of such structures early in the design process. This article addresses an approach to conservative fatigue life estimation of simple structural elements, which approach may provide guidance for dealing with more complex components.

Upper-Bound Stress in Beams. A uniform beam that is exposed to a complex sound field may be expected to experience its greatest flexural stress if one of its modes is excited at resonance – that is, at the mode's natural frequency – by a sound pressure whose spatial distribution matches at least approximately the mode's deflection distribution. For this situation an upper bound to the greatest anticipated amplitude of the oscillatory bending stress has been shown to obey:[1]

$$\sigma_{OSC} = \frac{0.21 C c_L b p}{r A \eta f} \tag{1}$$

Here C denotes the distance from neutral axis to the farthest fiber of the beam cross-sectional area. A signifies that area and r its radius of gyration. Further, c_L represents the longitudinal wave-speed in the beam's material, η the loss factor of the beam and b the effective width of the beam surface on which sound pressure acts. Also, p denotes the amplitude of sound pressure at frequency f which here also is the natural frequency of the beam mode under consideration.

Matching of a sound field's pressure distribution along the entire beam at least approximately to the beam's modal deflection distribution can occur at or above the "trace-matching" or "critical" frequency f_c. At this frequency the wavelength of sound matches the wavelength of the beam deflection; it obeys:

$$f_c = c_a^2 / 2\pi r c_L \tag{2}$$

where c_a denotes the speed of sound in the gaseous medium enveloping the beam.

Another situation in which the pressure distribution associated with an incident sound field acts in the same direction as a beam mode deflection over the entire length of the beam occurs at low frequencies, at or below the frequency at which the half-wavelength of the sound is equal to the length L of the beam. This "length-matching" frequency is given by

$$f_L = c_a / 2L \tag{3}$$

At frequencies where resonance and trace matching or length matching does not occur the oscillating stress may be expected to be much smaller than the value one obtains from Eq. (1).

In the foregoing discussion it was tacitly assumed that the sound pressure acts on only one surface of the beam, with the beam deflecting in the direction perpendicular to this surface, as would occur if the beam were part of a barrier or enclosure. In such a situation the effective with b of the beam surface (measured in the direction perpendicular to the beam's length) on which the sound field acts may be taken to include the contributing width of structural elements (membranes or plates) connected to the beam that are exposed to sound and transmit pressure-related forces to the beam. For example, in the case where a series of beams constitute ribs attached to a thin plate, b would include the width of the membrane or plate portions associated with a beam. If the plate portions associated with a beam in an arrangement as described above are substantial, they should be considered as part of the beam, whose section properties then would consist of a combination of these of the beam and of the plate portions.

If all of the parameters that characterize the beam are not frequency-dependent to any significant degree, one may conclude from Eq. (1) and the foregoing discussion that the greatest stress may be expected at the resonance and matching condition at which there occurs the greatest value of p/f.

Upper-Bound Stress in Plates. The greatest amplitude of the oscillatory bending stress induced in a plate occurs under conditions analogous to those discussed above for beams and is given by:[1]

$$\sigma_{OSC} = \frac{0.37\ c_L p}{h \eta f} \tag{4}$$

Here, h denotes the plate's thickness, all other symbols are defined as for beams. For plates, Eq. 2 applies with r equal to $h/\sqrt{12}$.

As for beams, the greatest stress may be expected at the greatest values of the ratio p/f, assuming all other parameters do not vary appreciably with frequency.

Fatigue Evaluation.[2] Fatigue analysis and fatigue life estimation can involve many complexities, some of which are not well understood, nor easily summarized. Since also the fatigue behaviors of different materials vary widely, the discussion presented here – intended merely to provide basic guidance – addresses only some of the general considerations in broad outline and focuses on structures made of steel.

For fatigue evaluation one needs to consider the oscillatory stress that occurs at stress concentrations, such as those due to surface flaws or discontinuities (e.g., notches, welds, and holes.) The amplitude of the oscillatory stress at a stress concentration is obtained by multiplying the stress σ_{osc} pertaining to a uniform and flaw-free structure by a stress concentration factor k. Values of stress concentration factors may be found in handbooks and design guides. (The stress concentration factor corresponding to a hole, for example, is equal to 2.0 or less, with smaller values corresponding to greater ratios of hole diameter to material thickness.)

Fatigue Life.[3] A structure subject to an oscillatory stress of amplitude $k\sigma_{osc}$ may be expected to last indefinitely without fatigue failure if the effective oscillatory stress amplitude $k\sigma_{osc}$ does not exceed the so-called endurance limit $S_{endurance}$. "Indefinitely" conventionally is taken to mean 10^9 cycles.

If the effective oscillatory stress $k\sigma_{osc}$ exceeds the endurance limit, the number N of stress cycles that a structural element can endure without failure may be estimated from

$$N = 10^9 (S_{endurance} / k\sigma_{OSC})^\beta \tag{5}$$

For most steels and other ductile materials $\beta = 9$ may be considered as representative, but for other materials greatly different exponents may apply.

For the case where a structural component of steel is subject to a steady stress σ_{st} on which an oscillatory stress is superposed the relevant endurance limit should be taken as

$$S_{endurance} = S_{fatigue}(1 - \sigma_{st} / S_{ult}) \tag{6}$$

where $S_{fatigue}$ represents the fatigue limit – that is, the endurance limit in absence of a steady stress – and S_{ult} denotes the steel's ultimate strength. The fatigue limit in general depends not only on the specific material, but also on its manufacture and surface processing (e.g., plating, nitriding, induction hardening, rolling, shot peening, grinding, welding, flame cutting) and its environment (water, salt, chemical-bearing atmospheres). Limited quantitative information is readily available in handbooks regarding some of these effects. For example, the cited references indicate that cold-rolling reduces the fatigue limit by a factor of about 0.7; exposures to fresh or salt water result in reductions by factors of about 0.6 and 0.4, respectively; temperatures between –100 °F and +400 °F typically have little effect.

The fatigue life L of a structural component – the number of seconds that the structure is expected to last without suffering a fatigue failure if it is exposed to stress at frequency f – obeys:

$$L = N / f \tag{7}$$

For the case where the effective oscillatory stress $k\sigma_{osc}$ exceeds the endurance limit one finds from equations (5) and (1) or (4) that L is proportional to f^8/p^9 . Thus, to obtain a lower bound on the fatigue life, one needs to find the smallest value of this ratio for the frequencies at which there can occur the upper-bound stresses considered in the first section of this discussion. If the sound pressure does not vary greatly with frequency, one may expect to obtain this smallest value at the lowest relevant frequency.

Concluding Remarks. The predicted upper-bound oscillatory stresses result from resonances and thus depend directly on the magnitude of the structural damping. This magnitude cannot be determined from first principles; one generally needs to estimate it on the basis of experience. Consequently, there is considerable uncertainty in the estimates of these stresses and in the results based on these, and minor inaccuracies in the estimation relations and calculations are of relatively little consequence. In view of this uncertainty, the suggested approach can only provide general guidance for sonic fatigue life estimation, rather than precise predictions.

1. E. E. Ungar, "Estimation of Upper Bounds to Stresses Induced by Sound," Transactions of 22nd National Conference on Noise Control Engineering, October 2007, pp 677-681.
2. S. M. Tipton, "Static and Fatigue Design," Section 7 of *Mechanical Design Handbook*, H. A. Rothbart, Ed. , McGraw-Hill, 1996
3. C. Lalanne "Fatigue Damage," of *Mechanical Vibration & Shock*, Volume IV, Hermes Penton Ltd., 2002

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