Fatigue Damage Related Descriptor for Random Vibration Test Environments

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The dynamic loads delivered by random vibration exciters to test items are commonly described by a power spectrum of the shaker table acceleration. For test items with resonances, the potential damage to a test item delivered by two vibration tests with different power spectra is difficult to predict. This article proposes a new method for describing the shaker table motion that relates directly to the possible damage experienced by a resonant test item, independent of its natural frequency. The new descriptor, called the damage potential spectrum, is easily computed from an acceleration power spectrum. It is applicable to all random signal driven electrodynamic and electrohydraulic shakers, but is not rigorously applicable to pneumatic hammer-driven (repetitive shock) shakers that are commonly used for environmental stress screening vibration tests, except at frequencies above about four times the repetition rate of the hammer impacts.

The dynamic loads delivered by random vibration testing machines (shakers) to test items are commonly specified and measured in terms of a power spectral density (PSD) function (also called an autospectrum) for the shaker table motion. To assure that two different tests will produce the same potential damage to a test item, it is necessary to make both the power spectrum and duration of the two tests similar. This is easy to accomplish when the vibration tests are performed using random signal-driven electrodynamic or electrohydraulic shakers, because the power spectrum for the table motion of such shakers is easy to control. However, if tests have already been performed with two different power spectra, the relative damage potential to a test item caused by the two tests is difficult to assess. Furthermore, there is a growing interest in the use of pneumatic hammer-driven shakers, also called repetitive shock (RS) shakers or multiple degree-of-freedom shakers, for so-called environmental stress screening (ESS) tests.[1] The vibration generated by RS shaker tables is technically a repetitive transient, but because of its complexity, it appears to the test item much like a quasi-random vibration with a non-Gaussian probability density function. However, although the overall vibration delivered to a test item by RS shakers can be altered by changing the pneumatic power to the actuators, it is difficult to tailor the spectrum of the table vibration, which may vary substantially from one type of RS shaker to another. Hence, there is a problem in relating the severity of the vibration provided by two different RS shakers, as well as between an RS shaker and a random signal-driven electrodynamic or electrohydraulic shaker that delivers near-Gaussian vibration with a carefully controlled spectrum.

One method for describing the severity of the vibration produced by tests with two different power spectra and/or two different types of shakers is to assess the table motion in terms related to the potential damage that will be experienced by a test item subjected to that motion. A number of damage potential descriptors of random vibration environments have been proposed,[2] but they typically relate the potential damage from one environment to another at a specific frequency and do not relate the potential damage from one frequency to another. A potential damage descriptor that applies to both different environments and different frequencies is desired.

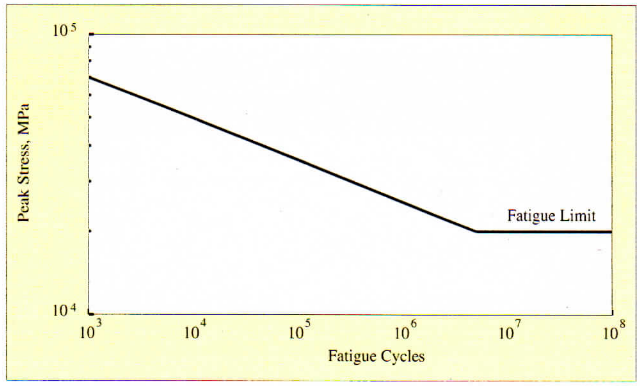

Figure 1. Idealized S-N curve for typical aluminum alloy.

Simplified Fatigue Damage Model for Test Items

The damage experienced by a test item during a vibration exposure can involve many different mechanisms,[3,4] but classical fatigue damage is generally the most common. Structural fatigue is a very complicated subject involving the principles of fracture mechanics.[5] However, in highly simplified terms, if a structure is subjected to repeated applications of a load producing an adequate stress level, cumulative damage occurs that ultimately causes a crack to initiate and propagate until the structure fails. The adequate stress level required to allow the accumulation of damage is referred to as the fatigue limit or endurance limit of the material. Fatigue data for various materials are commonly presented as peak stress versus the total number of loading cycles to failure (the number of cycles needed to cause both the initiation of a crack and its propagation to a critical length). Such data plots are referred to as *S-N* curves, and are widely published for many different materials, e.g., References 5-8. Various functional forms have been proposed for the *S-N* curves of metals,[9] but as a first order of approximation, an idealized *S-N* curve involving two straight lines on a plot of log S versus log N is commonly assumed for steel and aluminum alloys, as illustrated in Figure 1. A further simplification is achieved if the fatigue limit of the material is ignored. For this case, the idealized *S-N* curve in Figure 1 can be defined by a single straight line on a plot of log S versus log N,[10] leading to the equation

$$N = cS^{-b} \tag{1}$$

where:

S = peak value of cyclical stress (Pa)
N = number of stress cycles to failure
b = fatigue parameter
c = constant of proportionality.

Of course, ignoring the fatigue limit in Equation (1) leads to an overestimate of the damage caused by low level vibration inputs to a test item, but this will have little impact on the predicted damage produced by the high level vibration tests most likely to cause a test item failure.

To determine the fatigue damage caused by repeated loads producing different peak stress levels, Miner[11] suggests that the damage at each peak stress level can be summed in a linear manner, as follows:

$$D = \sum_i \frac{n_i}{N_i} \tag{2}$$

where:

n_i = number of cycles applied with peak stress S_i

N_i = number of cycles with peak stress S_i needed to cause failure

D = total damage (failure occurs when $D \approx 1$).

Substituting from Equation (1), the total damage can then be approximated by

$$D \approx \sum_i \frac{n_i}{c S_i^{-b}} \approx \sum_i \frac{1}{c} n_i S_i^b \propto \sum_i n_i S_i^b \tag{3}$$

For random vibration environments with a continuous stress time history, Equation (3) can be written as[12]

$$D \approx v_m^+ T \int_0^\infty [p(S)/(cS^{-b})] dS \propto v_m^+ T \int_0^\infty p(S) S^b dS \tag{4}$$

where:

v_m^+ = number of positive maxima per unit time in the stress time history

T = total time of exposure to the stress environment

$p(S)$ = probability density function of the stress maxima.

The value of b in Equations (3) and (4) can vary from 4 to 25 depending on the specific material and its geometry, i.e., notch factor. A value of $b = 8$ ($M = b/2 = 4$) is recommended in MIL-STD-810[13] for the structural materials commonly used in transportation vehicles when subjected to complex or random loading. This value of b provides a reasonable approximation for the fatigue characteristics of a number of common construction metals, including unnotched specimens of A36 steel, and 2024 and 6061 aluminum alloys. However, a value of $b = 4$ is also recommended in Reference 13 and elsewhere in Reference 14 for the accelerated testing of complex structural assemblies and equipment items.

Estimates of Stress in Test Items

Shaker vibration inputs to test items are normally measured using accelerometers, where the table motion is described in terms of a power spectral density (PSD) function (also called an autospectrum) with the units of g^2/Hz or $(m/s^2)^2$/Hz. However, it is velocity rather than acceleration that has a direct relationship to stress.[15,16] Velocity is rarely measured directly, but given an acceleration PSD denoted by $G_{aa}(f)$, a velocity PSD, denoted by $G_{vv}(f)$, is easily computed by[17]

$$G_{vv}(f) = \frac{1}{(2\pi f)^2} G_{aa}(f) \tag{5}$$

where $G_{aa}(f)$ must have the units $(m/s^2)^2$/Hz to obtain $G_{vv}(f)$ with the units $(m/s)^2$/Hz.

Equation (5) defines only the velocity input to the test item from the shaker table. To estimate the stress related response of a test item to the input vibration, let the following assumptions apply:

1. The shaker table motion represents a physical realization of a "strongly mixed" random process. This assumption is reasonable for random signal-driven electrodynamic and electrohydraulic shakers, but is not rigorously correct for pneumatic hammer-driven RS shakers since the table motion for such shakers is produced by repetitive hammer impacts with a fixed repetition rate. The validity of this assumption for RS shakers is evaluated later.
2. The dynamic response of the test item subjected to the shaker table motion is linear and dominated by a single resonant mode (natural frequency); i.e., the test item behaves as a base excited linear oscillator (single degree-of-freedom system). This assumption may or may not be reasonable, depending on the complexity of the test item.
3. The dynamic responses of the test item along the three orthogonal axes, as well as the three rotational axes, are not coupled; i.e., the response along any one axis is determined solely by the excitation along that same axis. This assumption may or may not be reasonable, depending on the complexity and nonlinear behavior of the test item.
4. The velocity PSD of the shaker table motion is approximately uniform over the half-power point bandwidth of the test item resonance, which can be approximated by[18]

$$B_r \approx 2\zeta f_n \tag{6}$$

This assumption is reasonable for typical test items with lightly damped resonances ($\zeta \le 0.1$).

With the above assumptions, the stress response of the test item to the shaker vibration along each axis will be narrowband and almost Gaussian, no matter what probability density function the shaker table vibration may exhibit along that axis.[19] Hence, the probability density function for the peaks in the stress response of the test item will be approximately Rayleigh in form;[20] i.e.,

$$p(S) \approx \frac{S}{\sigma_S^2} e^{-S^2/2\sigma_S^2} \tag{7}$$

where S is peak stress value and σ_S is the standard deviation of the stress time history. It follows that the fatigue damage in the test item can be estimated by solving Equation (4) with the peak probability density function given in Equation (7). The solution is derived to be[21]

$$D \approx \frac{\upsilon_0^+ T}{c} [\sqrt{2}\sigma_S]^b \Gamma[1 + b/2] \tag{8}$$

where:

υ_0^+ = number of upward zero crossings per second (average frequency) of the stress time history

T = total duration of exposure to the dynamic environment

σ_S = standard deviation of the stress time history

$\Gamma[1 + b/2]$ = Gamma function of [ ]

b,c = material constants.

From References 15 and 22, the standard deviation for the stress in the test item can be approximated by:

$$\sigma_S \approx k\sigma_v = k\sqrt{\frac{\pi f_n G_{vv}(f_n)[1 + 4\zeta^2]}{4\zeta}} \tag{9}$$

where:

σ_S = standard deviation of stress time history

σ_v = standard deviation of velocity time history

f_n = natural frequency of test item

$G_{vv}(f_n)$ = PSD of input velocity at the test item resonance frequency

ζ = damping ratio of test item resonance

k = constant of proportionality.

Substituting Equation (5) into Equation (9) and assuming the damping ratio is relatively small, say $\zeta < 0.1$,

$$\sigma_S \approx k\sqrt{\frac{G_{aa}(f_n)}{16\pi f_n \zeta}} \tag{10}$$

where $G_{aa}(f_n)$ = PSD of input acceleration at the test item resonance frequency in $(m/s^2)^2$/Hz, and all other terms are as defined in Equation (9). Finally, substituting Equation (10) into Equation (8) and noting that $\upsilon_0^+ \approx f_n$ for the response of a lightly damped oscillator, it follows that:

$$D \approx \frac{f_n T}{c}\left[\frac{k^2 G_{aa}(f_n)}{8\pi f_n \zeta}\right]^{b/2} \Gamma[1 + b/2] \propto f_n T \left[\frac{G_{aa}(f_n)}{f_n \zeta}\right]^{b/2} \tag{11}$$

The last expression in Equation (11) is defined as the "damage potential spectrum" given by:

$$DP(f_n) = f_n T \left[\frac{G_{aa}(f_n)}{f_n \zeta}\right]^{b/2} \tag{12}$$

where $DP(f_n)$ is proportional to the fatigue damage in a test item with any natural frequency f_n when the input vibration is measured in terms of an acceleration PSD. Specific values for the material constant b and the damping ratio ζ must be assumed to establish a potential damage spectrum for a given item of hardware. For example, with the commonly assumed values of $b = 4$ and $\zeta = 0.05$,

$$DP(f_n) = \frac{400T}{f_n}[G_{aa}(f_n)]^2 \qquad (13)$$

The following characteristics of the $DP(f_n)$ function defined in Equation (12) should be noted:

1. Since the $DP(f_n)$ evolves from a proportional relationship, the acceleration PSD can now be measured in the more common units of g^2/Hz.
2. The $DP(f_n)$ is not bounded by unity, as is the actual damage D estimated in the first half of Equation (11).
3. The $DP(f_n)$ defines the damage to the test item as a function of the natural frequency of the test item, which may or may not be known. For those cases where the natural frequency of the test item is unknown, the maximum value of the $DP(f_n)$ can be interpreted as the worst possible damage that can be caused by that test.

Applications to Repetitive Shock Machines

The damage potential spectrum, $DP(f_n)$, defined in Equation (12) provides a simple way to assess the potential fatigue damage to test items produced by a random vibration test, assuming the four critical assumptions detailed in the preceding section are complied with. The first and most fundamental of these assumptions concerns the randomness of the shaker table input vibration to the test item. Specifically, if the input vibration represents a "strongly-mixed" random process, the instantaneous probability density function (PDF) for the test item response will be Gaussian and, hence, the PDF of peaks for the test item response will closely approximate the Rayleigh distribution given by Equation (7), as long as the test item response is linear and narrowband; i.e., the test item represents a simple oscillator. This will be true even when the instantaneous PDF for the input vibration is not Gaussian, since the narrowband character of the test item response will suppress deviations from the Gaussian form,[19] as long as the input vibration is random. However, although the vibration produced by pneumatic hammer-driven RS shakers is very complex in character, it does not constitute a "strongly-mixed" random process in the strict sense. In this case, the instantaneous PDF for the response of the test item may not be Gaussian.

To check the validity of the random assumption for RS shakers, two small beams were mounted on the table of an RS shaker, one with a resonance frequency of f_n = 270 Hz, and the other with a resonance frequency of f_n = 1308 Hz. Vibration was applied to the beams, and the response at the end of each beam was measured with a small accelerometer. The test setup is shown in Figure 2. The normalized, instantaneous probability density functions (PDFs) for the RS shaker table excitation and the response of the two beams are shown in comparison to the Normal (Gaussian) distribution in Figure 3. Note in Figure 3 that the table excitation is very nonGaussian, displaying the classical characteristics of what statisticians call "large kurtosis," i.e., an unusually large fourth moment. However, this deviation from normality diminishes in the PDFs for the beam responses as the natural frequency of the beam increases, i.e., the PDF for the response of the f_n = 1308 Hz beam is much closer to normal than for the response of the 270 Hz beam. However, even for the f_n = 1308 Hz beam, a close inspection of the tails of the PDF reveals higher probability densities than predicted by the normal distribution. This is important because it is the extreme values of stress that cause most of the fatigue damage to test items.

The explanation for the results in Figure 3 is believed to be as follows. Pneumatic hammer-driven RS shakers actually produce a repetitive transient excitation that has dominant spectral components at the repetition rate of the hammer impacts and all harmonics thereof, e.g., the RS shaker used to generate the data in Figure 3 had a repetition rate of about 35 Hz. These excitation harmonics are somewhat stochastic in magnitude, but nevertheless are concentrated around discrete frequencies. For structures with low natural frequencies, the bandwidth of

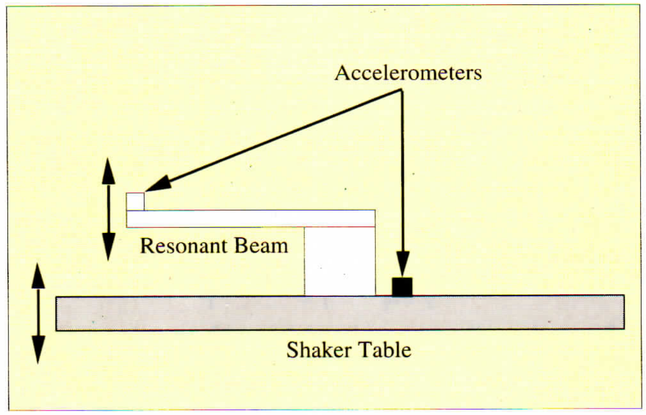

Figure 2. Test setup for a repetitive shock shaker vibration experiment.

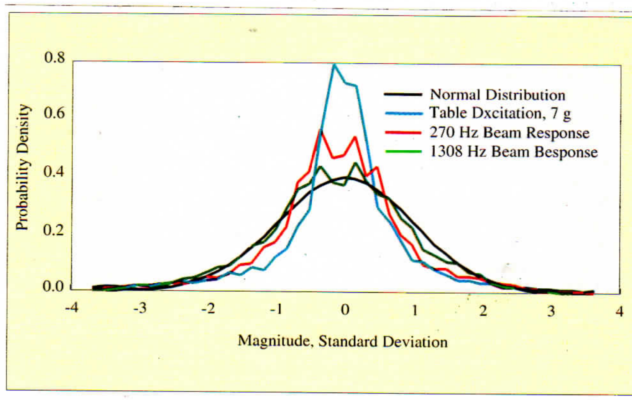

Figure 3. Normalized probability density functions for a repetitive shock shaker vibration experiment.

their resonant response, which can be approximated by Equation (6), is too narrow to accept more than one of the excitation harmonics. For example, at f_n = 270 Hz and assuming 5% damping, $B_r \approx$ 27 Hz, which means that resonant response of the beam is dominated by a single harmonic of the excitation (this also explains why RS shakers deliver very little excitation to test items at low frequencies). On the other hand, at f_n = 1308 Hz and again assuming 5% damping, $B_r \approx$ 131 Hz, which means that resonant response of the beam is dominated by at least three harmonics of the excitation. As the number of harmonics driving the response increases, the excitation producing the response appears more stochastic, as required in[19] for narrowband filtering operations to suppress deviations from the Gaussian form. Since a normal distribution for the response of test items is required to make Equation (12) valid, it follows that the damage potential spectrum can be applied to pneumatic hammer-driven RS shakers only at frequencies above, say, four times the repetition rate of the hammer impacts. To be more specific, letting R be the repetition rate of the hammer impacts, the lowest frequency where Equation (12) might apply is:

$$f_n \geq 2R/\zeta \qquad (14)$$

where ζ is the estimated damping ratio of the test item at its natural frequency.

Conclusions

A descriptor of the vibration input to a test item produced by random vibration testing machines has been formulated that relates to the damage potential of the vibration input as seen by the test item. The descriptor is called the damage potential spectrum, and is given by:

$$DP(f_n) = f_n T\left[\frac{G_{aa}(f_n)}{f_n \zeta}\right]^{b/2} \qquad (12)$$

where:

$G_{aa}(f_n)$ = PSD of shaker table acceleration in g^2/Hz

f_n = dominant natural frequency of test item in Hz

ζ = damping ratio of test item at its dominant natural frequency

b = fatigue curve slope parameter

T = duration of test in sec.

In words, the damage potential spectrum, $DP(f_n)$, can be estimated as follows:

1. Compute the acceleration PSD of the shaker table motion in g^2/Hz at the attachment point of the test item using conventional FFT procedures.
2. Estimate a fatigue curve slope parameter for the test item (use $b = 8$ for load carrying structures and $b = 4$ for complex equipment items as default values).
2. Estimate a damping ratio for the test item (use $\zeta = 0.05$ as a default value).
3. For any given test duration T and acceleration PSD, compute damage potential as a function of the natural frequency of the test item using Equation (12).

The damage potential spectrum is applicable to tests performed using random signal-driven electrodynamic and electrohydraulic shakers, but is not rigorously applicable to tests performed on pneumatic hammer-driven repetitive shock shakers, except perhaps at frequencies above about four times the repetition rate of the hammer impacts (commonly above 1500 Hz). An important aspect of the damage potential spectrum is that it allows a comparison of the severity of a vibration environment from one frequency to another, as well as from one environment to another.

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